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STATIONARY AND UNSTATIONARY FORCED NONLINEAR OSCILLATION MODES OF THREE DISC ON LIGHT ELASTIC SPINDLE

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This article deals with three disc forced oscillations on light elastic spindle by means of differential equation sistem of the first approximation for amplitudes and phases of forced and two-frequency oscillation mode in the case of nonlinear dependence of the spring elasticity force moment between the first and the second discs. The differential equations of the first approximation for amplitudes and phases are derived from the basic ideas of asymtomic method Крылов-Богоюбов-Митропольский for one-frequency oscilation mode.

The spindle model with discs is shown on fig. 1. The forced moments $E_1 \cos \theta_1$ and $E_2 \cos \theta_2$ affect the second and the third disc. E_1, E_2 are the amplitudes of forced moments and $d\,\theta_1/d\,t = \nu_1(\tau)$, $d\,\theta_2/d\,t = \nu_2(\tau)$ are frequencies of forced moments.

Conditions φ_1, φ_2, φ_3 are generated coordinations of given system. The spring elasticity force moment between the first and the second disc iz a function of relative angle of the first and the second disc rotation and is presented by following function:

$$F(\varphi_2 - \varphi_1) = c_1'(\varphi_2 - \varphi_1) + \varepsilon f(\varphi_2 - \varphi_1) \quad (1)$$

The spring elasticity force moment between the second and the third disc is linear function,

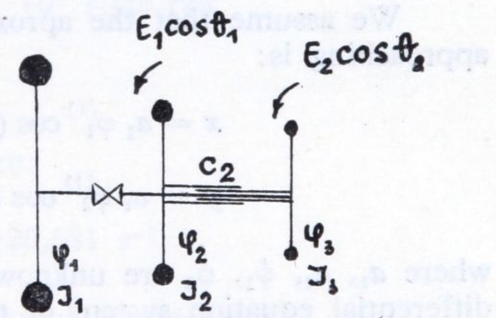

Fig. 1.

$$M_t = c_2(\varphi_3 - \varphi_2). \quad (2)$$

There is, between the second and the third disc, an internal resistance in material. Due to this, the internal resistance force moment is caused and it is in proportion to the second disc relative speed rotation in regard to the second disc, α is the coefficient of proportion.

The differential equations of oscillator system are:

$$J_1 \ddot{\varphi}_1 - F(\varphi_2 - \varphi_1) = 0$$

$$J_2 \ddot{\varphi}_2 + F(\varphi_2 - \varphi_1) - c_2(\varphi_3 - \varphi_2) = E_1 \cos \theta_1 + \alpha(\dot{\varphi}_3 - \dot{\varphi}_2) \tag{3}$$

$$J_3 \ddot{\varphi}_3 + c_2(\varphi_3 - \varphi_2) = -\alpha(\dot{\varphi}_3 - \dot{\varphi}_2) + E_2 \cos \theta_2$$

If the new variables are introduced,

$$x = \varphi_2 - \varphi_1, \quad y = \varphi_3 - \varphi_2 \tag{4}$$

the differential equations (3) are now;

$$J_1 J_2 \ddot{x} + c_1'(J_1 + J_2) x - c_2 J_1 y = -(J_1 + J_2) \varepsilon f(x) + E_1 J_1 \cos \theta_1 + \alpha J_1 \dot{y}$$

$$J_2 J_3 \ddot{y} + c_2(J_2 + J_3) y - c_1' J_3 x = -\alpha(J_2 + J_3) \dot{y} - E_1 J_3 \cos \theta_1 +$$

$$+ E_2 J_2 \cos \theta_2 + J_3 \varepsilon f(x) \tag{5}$$

Forming approximation for differential equation sistem we assume that the coefficient of internal resistance α, non-linearity coefficient of the spring ε elasticity force moment and amplitudes of forced moments E_1 and E_2 are small values in comparison to other system parameters. The non-linearity of the elasticity force moment of the spring between the first and the second discs is given by following function:

$$F(x) = \begin{cases} c_1' x & -x_1 \leqslant x \leqslant x_1 \\ c_1' x + c_1'' & x_1 \leqslant x \leqslant +\infty \\ c_1' x + c_1'' & -\infty \leqslant x \leqslant -x_1 \end{cases} \tag{6}$$

We assume that the aproximation for differential equation in the first approaching is:

$$x = a_1 \varphi_1^{(1)} \cos(\theta_1 + \psi_1) + a_2 \varphi_1^{(2)} \cos(\theta_2 + \psi_2)$$

$$y = a_1 \varphi_2^{(1)} \cos(\theta_1 + \psi_1) + a_2 \varphi_2^{(2)} \cos(\theta_2 + \psi_2)$$

where a_1, a_2, ψ_1, ψ_2 are unknown time functions which we define from the differential equation system of the first approximation:

$$\frac{d a_1}{d t} = -\frac{\alpha}{2 m_1} [(J_2 + J_3) \varphi_2^{(1)2} - J_1 \varphi_1^{(1)} \varphi_2^{(1)}] a_1 - \frac{E_1 (J_1 \varphi_1^{(1)} - J_3 \varphi_2^{(1)})}{m_1 (\omega_1 + \nu_1(\tau))} \sin \psi_1$$

$$\frac{d a_2}{d t} = -\frac{\alpha}{2 m_2} [(J_2 + J_3) \varphi_2^{(2)2} - J_1 \varphi_1^{(2)} \varphi_2^{(2)}] a_2 - \frac{E_2 J_2 \varphi_2^{(2)}}{m_2 (\omega_2 + \nu_2(\tau))} \sin \psi_2 \tag{8}$$

$$\frac{d \psi_1}{d t} = \omega_1 - \nu_1(\tau) - \frac{3 \upsilon_1}{8 m_1 \omega_1} [(\varphi_1^{(1)} a_1)^2 + 2 (a_2 \varphi_1^{(2)})^2] -$$

$$- \frac{E_1 (J_1 \varphi_1^{(1)} - J_3 \varphi_2^{(1)})}{m_1 a_1 (\omega_1 + \nu_1 (\tau))} \cos \psi_1$$

$$\frac{d \psi_2}{d t} = \omega_2 - \nu_2 (\tau) - \frac{3 \upsilon_2}{8 m_2 \omega_2} [2 (\varphi_1^{(1)} a_1)^2 + (a_2 \varphi_1^{(2)})^2] -$$

$$- \frac{J_2 E_2 \varphi_2^{(2)}}{m_2 a_2 (\omega_2 + \nu_2 (\tau))} \cos \psi_2$$

In the differential equations (8) the following marks are introduced:

$$\upsilon_1 = c_1'' [J_3 \varphi_2^{(1)} \varphi_1^{(1)} - (J_1 + J_2) \varphi_1^{(1)2}]$$

$$\upsilon_2 = c_1'' [J_3 \varphi_2^{(2)} \varphi_1^{(2)} - (J_1 + J_2) \varphi_1^{(2)2}]$$

$$m_1 = J_1 J_2 \varphi_1^{(1)} \varphi_1^{(1)} + J_2 J_3 \varphi_2^{(1)} \varphi_2^{(1)}$$

$$m_2 = J_1 J_2 \varphi_1^{(2)} \varphi_1^{(2)} + J_2 J_3 \varphi_2^{(2)} \varphi_2^{(2)}$$

For the following physical-geometrical system characteristics:

$$J_1 = 487{,}87 \text{ Ncms}^2 \qquad c_1' = 10{,}31\ 10^5 \text{ Ncm/rad}$$

$$J_2 = 99{,}89 \text{ Ncms}^2 \qquad c_1'' = 7{,}65\ 10^6 \text{ Ncm/rad}$$

$$J_3 = 31{,}61 \text{ Ncms}^2 \qquad c_2 = 1{,}65\ 10^5 \text{ Ncm/rad}$$

$$\alpha = 10 \text{ Ncm/rad}$$

the cyclic frecquencies „undisturbed system" are:

$$\omega_1 = 67{,}792\ s^{-1} \qquad \omega_2 = 120{,}681\ s^{-1}$$

If the dimensionless amplitudes are introduced in the form:

$$\overset{*}{a}_1 = a_1 \varphi_1^{(1)} / x_1 \qquad \overset{*}{a}_2 = a_2 \varphi_1^{(2)} / x_1 \qquad x_1 = 0{,}005 \text{ ran},$$

the differential equation system of the first approximation is:

$$\frac{d \overset{*}{a}_1}{d t} = - 0{,}295529 \cdot \overset{*}{a}_1 - \frac{271{,}956}{(67{,}792 + \nu_1 (\tau))} \sin \psi_1$$

$$\frac{d \overset{*}{a}_2}{d t} = - 0{,}802748 \cdot \overset{*}{a}_2 + \frac{339{,}665}{(120{,}681 + \nu_2 (\tau))} \sin \psi_2 \qquad (10)$$

$$\frac{d\psi_1}{dt} = 67{,}792 - \nu_1(\tau) + 0{,}037485 \cdot (a_1^{*2} + 2\, a_2^{*2}) - \frac{271{,}956}{\overset{*}{a}_1 (67{,}792 + \nu_1(\tau))} \cos \psi_1$$

$$\frac{d\psi_2}{dt} = 120{,}681 - \nu_2(\tau) + 0{,}06885 \cdot (2\, a_1^{*2} + a_2^{*2}) + \frac{339{,}665}{\overset{*}{a}_2 (120{,}681 + \nu_2(\tau))} \cos \psi_2$$

The system passing through the resonance and stationary oscillation system mode are exemined by mean; of equations (10). We determine the stationary oscillation system mode when the side of equations (10) is equalled to zero. Then we can easily eliminate ψ_1 from the first and the third equation, and ψ_2 from the second and the fourth. Therefore we get the following equation system:

$$0{,}349351\ a_1^{*2} \nu_1^2 + a_1^{*2} [67{,}792^2 - \nu_1^2 + 0{,}07497 \cdot \nu_1 \cdot (a_1^{*2} + 2\, a_2^{*2})]^2 = 271{,}956^2$$

$$2{,}577614\, a_2^{*2} \nu_1^2 + a_2^{*2} [120{,}681^2 - \nu_2^2 + 0{,}1377 \cdot \nu^2 \cdot (2\, a_1^{*2} + a_2^{*2})]^2 = - 339{,}665^2 \tag{11}$$

If we determine $\overset{*}{a}_2$ from the first equation (11) and change it in the second we'll get the imlicit equation in which $\overset{*}{a}_1$ depends on ν_1 and ν_1. If we fix the frequency values $\nu_2 = \text{const}$ we'll get the stationary mode curves:

$$\overset{*}{a}_1 = f_1(\nu_1) \qquad \overset{*}{a}_2 = f_2(\nu_1)$$

$$\psi_1 = f_3(\nu_1) \qquad \psi_2 = f_4(\nu_1)$$

Similarly if we fix $\nu_2 = \text{const}$ we'll get the stationary mode curves:

$$\overset{*}{a}_1 = h_1(\nu_2) \qquad \overset{*}{a}_2 = h_2(\nu_2)$$

$$\psi_1 = h_3(\nu_2) \qquad \psi_2 = h_4(\nu_2)$$

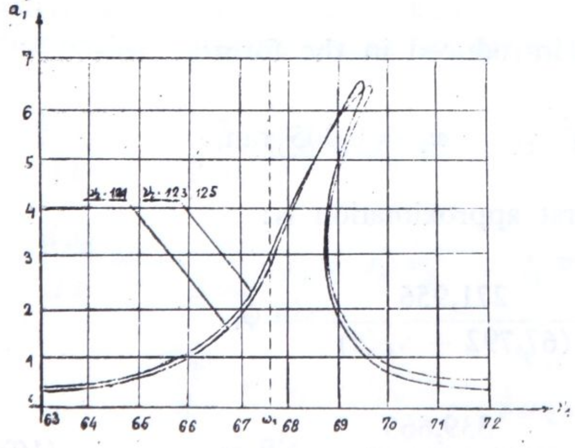

Fig. 2a

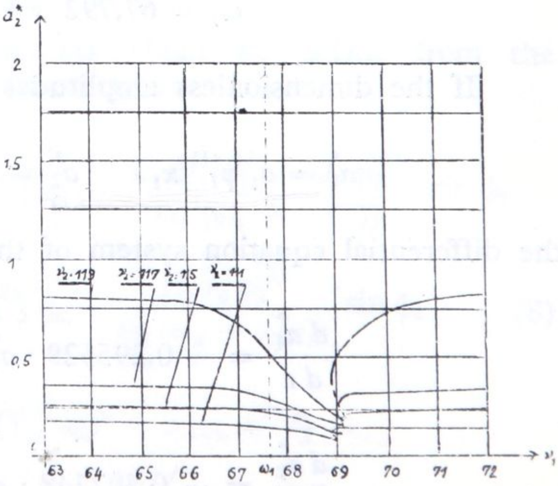

Fig. 2b

On the figures 2, a, b, 3, a, b and 4, a, b we can see the families of the first and the second harmonics of the amplitude-frequency curves in the stationary resonance mode for the continual change of discrete freguence values ν_1 and ν_2 of forced moments in the resonance bands. On the figures 2,a and 2,b we can see that amplitude-frequency curves $\overset{*}{a}_1(\nu_1, \nu_2)$ of the first harmonic for stationary mode during continual change of discrete frequency values ν_1 in the area $\nu_1 \in (\sim 64\ s^{-1}, \sim 72\ s^{-1})$ slightly alter with discrete frequency chage $\nu_2 \in (\sim 114\ s^{-1}, \sim 130\ s^{-1})$.

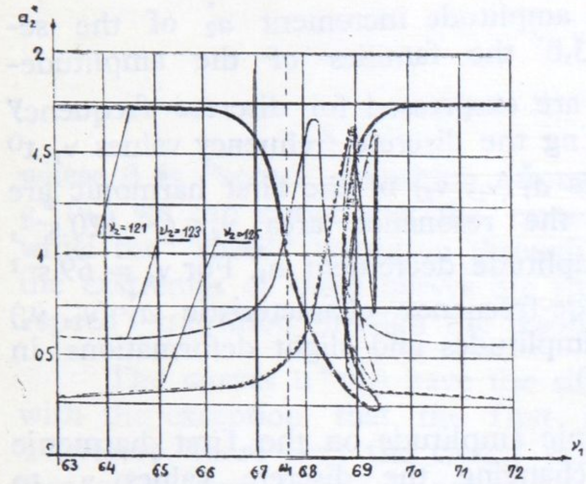

Fig. 3a

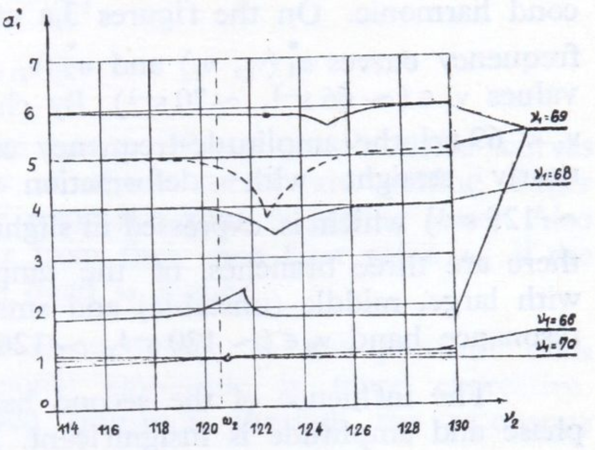

Fig. 3b

On the figures 3,a and 3,b we can see amplitude-frequency curves of the second harmonic $\overset{*}{a}_2(\nu_1, \nu_2)$ for stationary mode during continual change of discrete frequency values ν_1 in the band $\nu_1 \in (\sim 64\ s^{-1}, \sim 72\ s^{-1})$, with discrete frecquency change $\nu_2 \in (\sim 114\ s^{-1}, \sim 130\ s^{-1})$ slightly alter both in size and form of the amplitude.

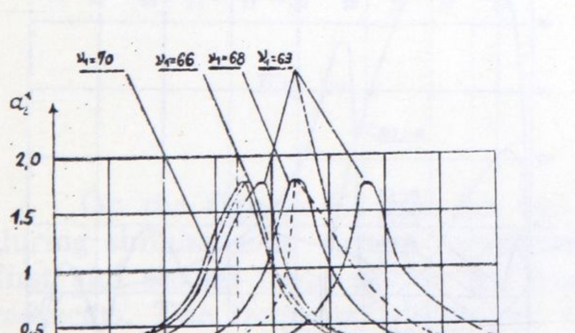

Fig. 4a

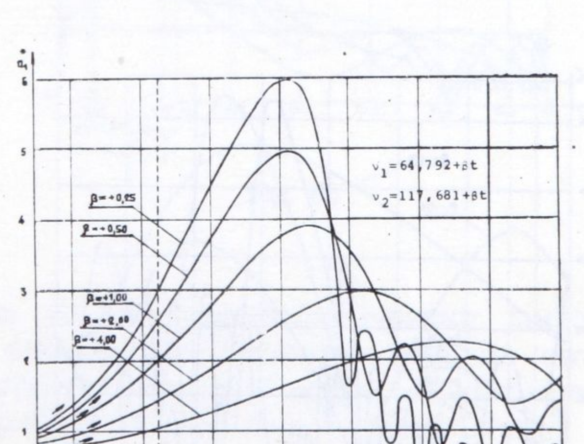

Fig. 4b

We can conclude that continual change of discrete frequency values of the first harmonic essentially influence during simultaneous selection of the values ν_1 and ν_2 from the „critical" band of the coresponding resonance frequencies is specially expressed. For $\nu_2 \in (\sim 122\,s^{-1}, \sim 126\,s^{-1})$ the curves $(\overset{*}{a}_2, \nu_1)$ change character in the area $\nu_1 \in (\sim 66\,s^{-1}, \sim 69\,5\,s^{-1})$. The influence of the first harmonic on the second one in the area $\nu_2 < 122\,s^{-1}$ is expressed in amplituda decrement $\overset{*}{a}_2$ and at the same time in amplituda increment $\overset{*}{a}_1$. In the area $\nu_2 \in (\sim 122\,s^{-1}, \sim 126\,s^{-1})$ the first harmonic influence on the curve $(\overset{*}{a}_1, \nu_1)$ is expressed in amplituda decrement and increment of the „irregular" curve part. For the higher values $\nu_2 > 126\,s^{-1}$ the amplitude increment of the first harmonic causes the amplitude increment $\overset{*}{a}_2$ of the second harmonic. On the figures 3,a and 3,b the families of the amplitude-frequency curves $\overset{*}{a}_1(\nu_1, \nu_2)$ and $\overset{*}{a}_2(\nu_1, \nu_2)$ are exspressed for discrete frequency values $\nu_1 \in (\sim 66\,s^{-1}, \sim 70\,s^{-1})$. By charg'ng the discrete frehuency values ν_1 to $\nu_1 < 69\,s^{-1}$ the amplitude-frequency curves $\overset{*}{a}_1(\nu_1, \nu_2)$ of the first harmonic are nearly straight with deformation in the resonance area $\nu_2 \in (\sim 120\,s^{-1}, \sim 125\,s^{-1})$ which is expressed in slight amplitude decrement $\overset{*}{a}_1$. For $\nu_1 = 69\,s^{-1}$ there are three branches of the amplitude-frecuency characteristic $\overset{*}{a}_1(\nu_1, \nu_2)$ with large, middle (unstable) and small amplitudes and slight deformations in resonance band $\nu_2 \in (\sim 120\,s^{-1}, \sim 126\,s^{-1})$.

The influence of the second harmonic amplitude on the first harmonic phase and amplitude is insignificant. By changing the discrete values ν_1 to $\nu_1 < 69\,s^{-1}$ the amplitude-frequency curves $\overset{*}{a}_2(\nu_1, \nu_2)$ of the second harmonic are with one characteristic branch and amplitudes maximum which are moved to higher frequencies ω_2. When $\nu_1 = 69\,s^{-1}$ three branches of amplitude-frequency curve appear and the middle one is unstable. When $\nu_1 > 69\,s^{-1}$ amplitude-frequency curves $\overset{*}{a}_2(\nu_1; \nu_2)$ are with one branch and amplitudas maximum move to lower frequencies closer to ω_2.

We can conclude that the influence of the first harmonic on the second harmonic phase and amplitude is significant in the narrow frequency band ν_1

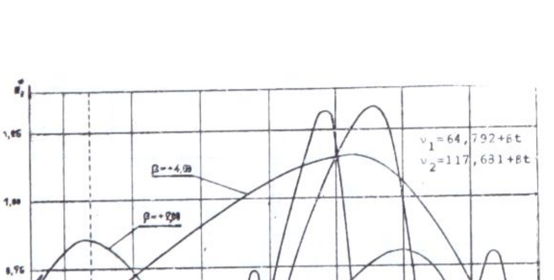

Fig. 5a

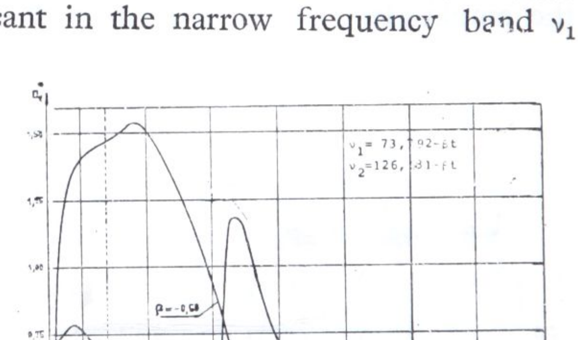

Fig. 5b

whose values are grater than ω_1. The influence of the second harmonic on the first one is insignificant. We can get the unstationary amplitude-frequency characteristics by means of numerical integration of the differential equation sysem (10) by Runge-Kuta's method.

On the figures 5.a and 5.b we can see the amplitude-frequency characteristics during simultaneous system transition through resonance with its own first and second frequency in the frequency increment direction of disturbed moments for several different "speed" transition. The frequency increment is performed according to linear laws:

$$\nu_1 = \nu_{10} + \beta t$$

$$\nu_1 = \nu_{20} + \beta t$$

where β is "speed" transition through resonance. If we watch these curves $a^*(\nu_1)$ we can notice that the exstremes of these curves are getting larger while the "speed" transition through the resonance zone is getting lesser. Also the exstremes of these curves are further from their own first value ω_1 if the "speed" transition through the resonance band is higher.

The curves $a^*(\nu_2)$ have the similar characteristic as the previous ones with the exception that the first harmonic amplitude is more expressive. Therefore, there isn't the clearly expressed transition through the resonance area of its own second frequency ω_2.

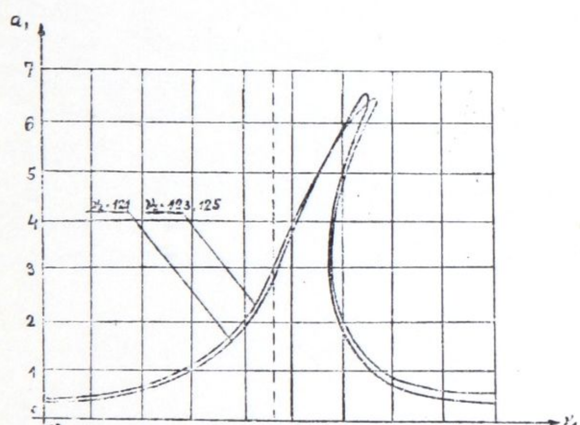

Fig. 6a

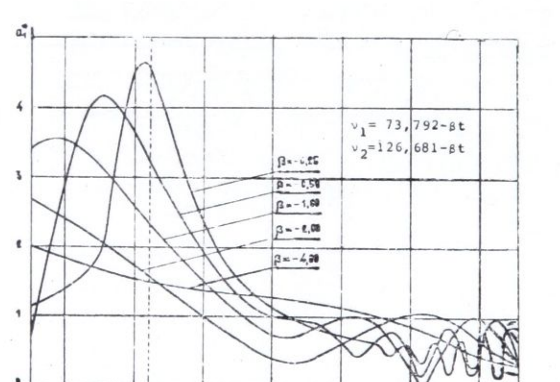

Fig. 6b

On the figures 6.a and 6.b we can see the amplitude-frequency curves during simultaneous system transition through the resonance with its own first and second frequency in the frequency increment direction of disturbed moments. The frequency decrement is performed according to linear laws:

$$\nu_1 = \nu_{10} - \beta t$$

$$\nu_1 = \nu_{20} - \beta t$$

The curve extremes $a_1^*(\nu_1)$ are on the left side of their own first value ω_1 and they are further from it if the "speed" transition through the resonance state is higher. For the lower "speed" transition thourgh the resonance area, the amplitudes increase and they are closer to their own first value ω_1 if the "speed" transition through the resonance area is lower. With the curves $a_2^*(\nu_2)$ we can notice the exception. For the lower "speed" transition through the resonance area of its own second frequency, the amplitude do not increase. This is the result of the more significant influence of the first harmonic amplitudas on the second harmonic amplitudas.

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СТАЦИОНАРНЫЕ И НЕСТАЦИОНАРНЫЕ РЕЖИМЫ НЕЛИНЕЙНЫХ КОЛЕБАНИЙ ТРИ ДИСКА НА ЛЕГКОМ УПРУГОМ ВАЛЕ

Р е з ю м е

В статье рассмотрены вынужденные нелинейные колебания три диска на легком упругом вале со связкой нелинейной характеристики. Уравнения первого приближения построены асимтотическим методом для амплитуды и фазы двухчастотного режима колебания. Обработана серия числовых примеров для случая стационарных и нестационарных условий и нарисованы амплитудно-частотные крывы как стационарных так и нестационарных двухчастотных режимов колебания вала для случая изменения частот вынужденых моментов.

STACIONARNI I NESTACINARNI REŽIMI NELINEARNIH OSCILACIJA TRI DISKA NA LAKOM ELASTIČNOM VRATILU

Izvod

U radu se izračunavaju prinudne nelinearne oscilacije tri diska na lakom elastičnom vratilu sa spojnicom, nelinearne karakteristike. Asimptotskom metodom su postavljene jednačine prve aproksimacije za amplitude i faze dvofrekventnog režima oscilacija. Obrađena je serija numeričkih primera za izabrano vratilo sa diskovima za slučaj stacinarnih i nestacionarnih uslova i nacrtane amplitudno-frekventne krive kako stacionarnih tako i nestacionarnih dvofrekventnih režima oscilacija vratila za slučaj promene frekvencija prinudnih momenata.

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