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ON THE CORRECTNESS OF ONE ASPECT OF THE EQUIVALENCE OF MECHANISMS

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One of methods used in the kinematic analysis of space mechanisms, developed recently and based on the modern mathematics (matrix-tensor method) is the method put together by *D.* Mangeron and *C.* Dragan. Without going into details of the first part of that paper, which contains the theoretical background, we shall discuss here the second part of the Mangeron — Dragan paper, which deals with applications of the method concerned to some concrete objects — mechanisms. The basic idea and the starting point in this part of the paper is the substitution of a realistic mechanisms with a mechanism equivalent to it but comprising kinematic pairs of the fifth class only. This means that a spherical pair is transformed into three rotationary pairs, the axes of which are mutually perpendicular, while a spherical pair with a restraint is tranformed into two rotationary pairs the axes of which are mutually perpendicular, and finally a sliding pair is tranformed into a single rotationary pair and a translantory pair. Since there exists a possibility that the axes change their orientations (although this fear was not explained, nor what it actually means!), the authors introduced a fictitious pair of a variable length instead of one rotationary pair. What the transformation of a four-member space mechanism of a general type into an equivalent mechanism looks like, is shown in Fig. 1

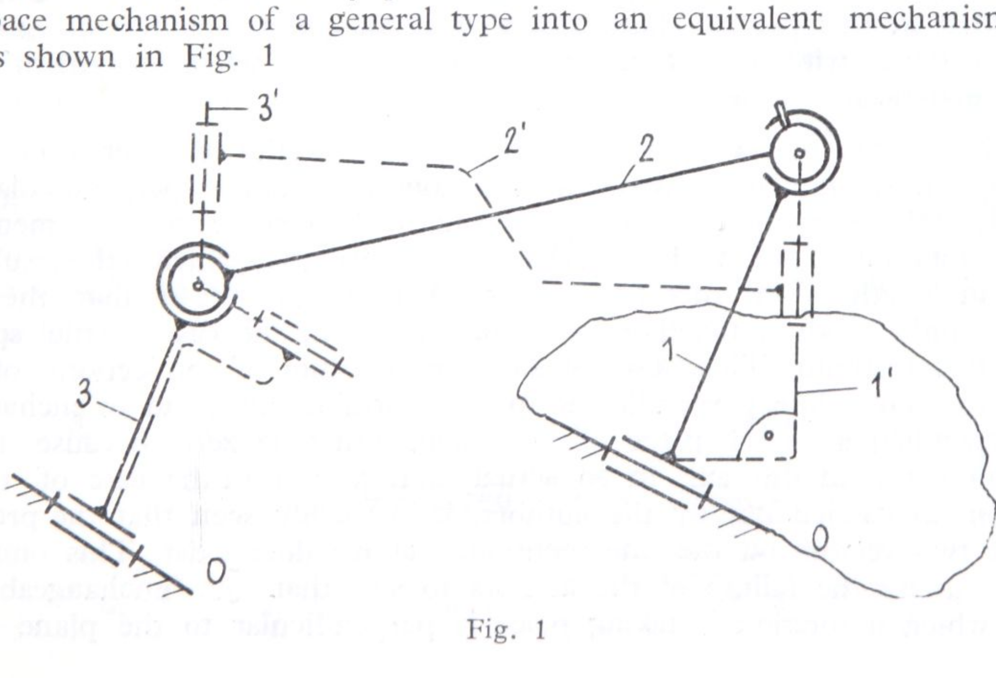

Fig. 1

In our critical treatment that follows, by analysing the essential correctness of this procedure, we shall first discuss the introduction of fictitious members. Due to a certain fear of the change of orientations of the axes in the spherical pair as well as in the case of a spherical pair with a restraint, one rotational pair each, in the first substitution, is replaced by one fictitious member. Without entering into a deeper analysis of this procedure, it is possible to note certain inconsistencies. If in the case of the spherical pair with a restraint there exists a fear that the coordinate axes may change their orientation, then such a fear still exists in the case of the spherical pairs, or, conversely, if in the case of the spherical pair, when in it there are two rotationary pairs, there is no fear of the axes changing orientation, then there should be no fear in the case of a spherical pair with a restraint either. On the basis of this consideration, it is apprehensible that the fear of a change of the orientation of the coordinate axes is not quite clear, and, therefore, the reason for the infroduction of fictitious members also remains unclear.

From the foroing remark, there follows still another essential quetion which is related to the correctness of the substitution of kinematic pairs by pairs equivalent to them. Following the substitution of kinematic pairs by pairs equivalent to them, all properties of joints, geometrical and kinematical, must be preserved completely. By considering a spherical pair with a restraint we can reach the conclusion that this basic requirement with respect to the equivalency is not satisfied. After the first substitution, the spherical pair with a restraint consists of theree members of constant length, and of two rotational pairs, while, however, by a later introduction of a member of a variable length, it consists of one member of the constant length, one member of variable length and one rotational pair. But, later on, due to the vicinity of a spherical pair and a spherical pair with a restraint it occurs that one member of the variable length from the spherical pair makes part of the spherical pair with a restraint, this finally leading to the fact that at the end of the complete substitution, in the spherical pair with a restrain there appear two members of variable lengths as well as one rotationary pair. According to the author' claims, one rotational pair is substituted by a fictitious member; this means that in this pair we have three rotational pairs, i. e. we do not have a spherical pair with a restraint but a spherical pair, which statement can be proved analytically as well.

The motion of the member 2 with respect to the member 1 in a realistic mechanism is composed of two rotations around two perpendicular axes. Since the relative motion of the member 2' with respect to the member 1' consists of a rotation around the axis DB, and of the rotation which is the result of the change in length of the member 2 (Fig. 2), it would mean that these two motions could produce together two rotations, as in the case of the spherical pair with a restraint. This, also, means that the sum of projections of these two vectors on a line perpendicular to the restrainer axis, and an unchangeable axis perpendicular to the plane of the cut-in, must be zero, because there is no rotation around this axis in an actual motion. But in the case of the substitution, as carried out by the authors, it is readily seen that the projection of these two vectors on the line mentioned above does exist. This ommission came about by the failure of the authors to see that the unchangeable axis around which a rotation is taking place is perpendicular to the plane of the cut-in of the restraint. The authors, however, used the axis of the vertical pair R_1 (Fig. 3) as an unchangeable axis. This resulted in the possibility of the existence of a projection on a non-permissible line.

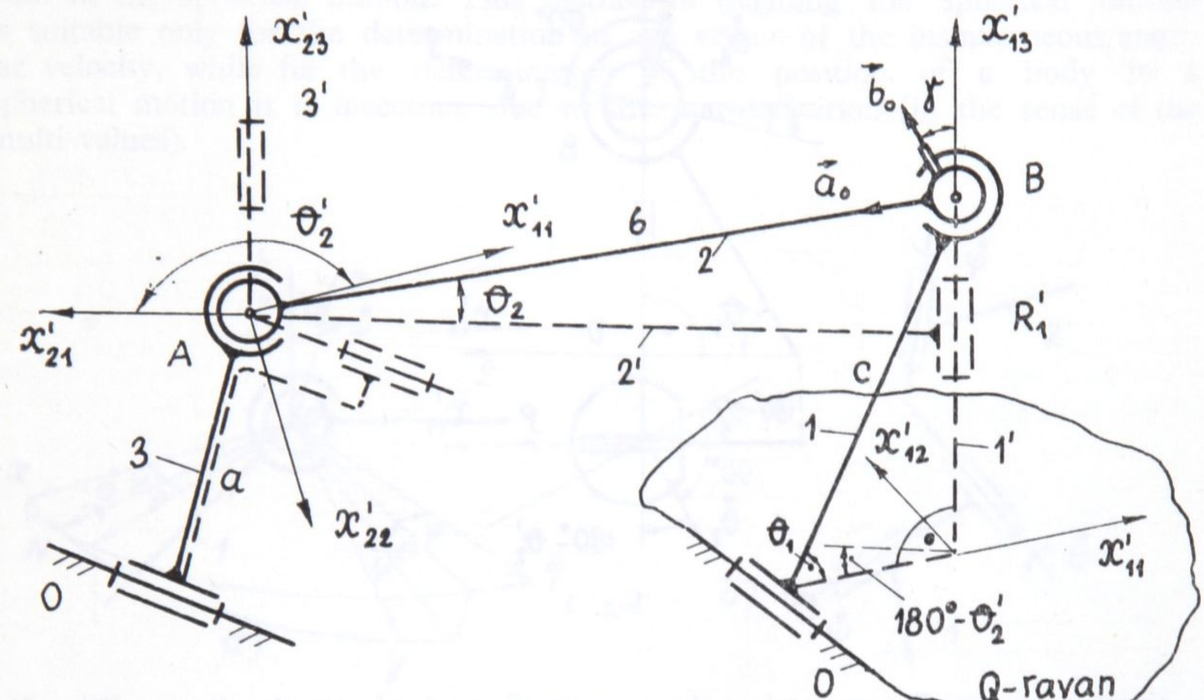

Fig. 2

Now, we propose to proceed with an analytical proof of our remarks stated so far. In order to achieve this, we must find out a relatioship between the angle γ and angles θ_2 and θ'_2, which are the measure of the relative motion of the member 2' with respect to the member 1'. We shall obtain the angle γ as the function of angles θ_2 and θ'_2 by using the construction of the pair „B", since the angle betwen the restraint and the member 2 is equal to 90 degrees, that is, the scala product of unit vectors $\vec{a}_0$ and $\vec{b}_0$ must be equal to zero. If the vectors $\vec{a}_0$ and $\vec{b}_0$ are expressed in terms of their projections no the coordinate axes of a conditionally stationary coordinate system $D\,x'_{11}\,x'_{12}\,x'_{13}$ (Fig. 3), we have:

$$\vec{b}_0 = -\sin\gamma\,\vec{i} + \cos\gamma\,\vec{j}; \quad \vec{a}_0 = -\cos\theta_2\cos(180-\theta'_2)\,\vec{i} +$$

$$+\cos\theta_2\sin(180-\theta'_2)\,\vec{j} - \sin\theta_2\,\vec{k}$$

$$\vec{a}_0\cdot\vec{b}_0 = |\vec{a}_0|\;|\vec{b}_0|\cos(\vec{a}_0,\vec{b}_0) = 0 \quad \rightarrow \quad tg\gamma = -\frac{tg\,\theta_2}{\cos\theta'_2} \tag{1}$$

In order to project the vectors of the angular velocities of the relative motion of the member 2' with respect to the member 1' we shell draw the spherical pair with a restraint in the coordinate plane $D\,x'_{11}\,x'_{13}$ which will make the

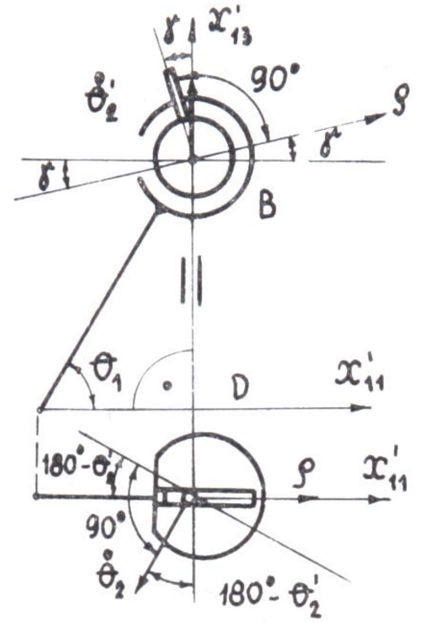

Fig. 3

projection easier. By projecting the vectors $\dot{\theta}_2$ and $\dot{\theta}'_2$ on the line ρ (Fig, 3), we obtain:

$$\dot{\theta}'_2 \sin\gamma - \dot{\theta}_2 \sin(180 - \theta'_2)\cos\gamma = 0 \tag{2}$$

$$\dot{\theta}'_2 \sin\beta - \dot{\theta}_2 \sin\theta'_2 \cos\gamma = 0$$

$$\dot{\theta}'\, tg\gamma = \dot{\theta}_2 \sin\theta'_2$$

$$\int_{\theta'_{20}}^{\theta'_2} \frac{d\,\theta'_2}{\cos\theta'_2\,\sin\theta'_2} = -\int_{\theta_{20}}^{\theta_2} \mathrm{ctg}\,\theta_2\, d\,\theta_2$$

$$tg\,\theta'_2 \sin\theta_2 = \text{const.} \tag{3}$$

To enable the joint in the pair „B" to retain all its properties even after the substitution, the Equation 3 must be satisfied, but in the Mangeron--Dragan paper, there is no such equation ,which is essential for an accurate analysis. If this euation is not satisfied, there will appear also a component of the angular velocity along the line ρ, and this will in effect produce a spherical pair but not a spherical pair with a restraint.

Finally, there also appears an essential question; is it possible to replace a spherical pair (a spherical motion) by three rotational pairs, the axes of which are mutually perpendicular, and on the grounds of these three rotations is it posible to formulate a tensor of location? It is obvious that such a substitution (with respect to the determination of position) is incorrect in principle. This method of determination of spherical motion by means of three rotations about mutually perpendicular axes (moving, at that) ξ. η, ζ, is a classical example of quasi-coordinates. By means of these angles (Fig. 4b), used as coordinates, it is not possible to define uniquely the position of a bodi in the spherical motion. This method of defining the spherical motion is suitable only for the determination of the vector of the instanteneous angular velocity, while for the determination of the position of a body in a spherical motion it is inaccirate due to the non-definition (in the sense of the multi-values).

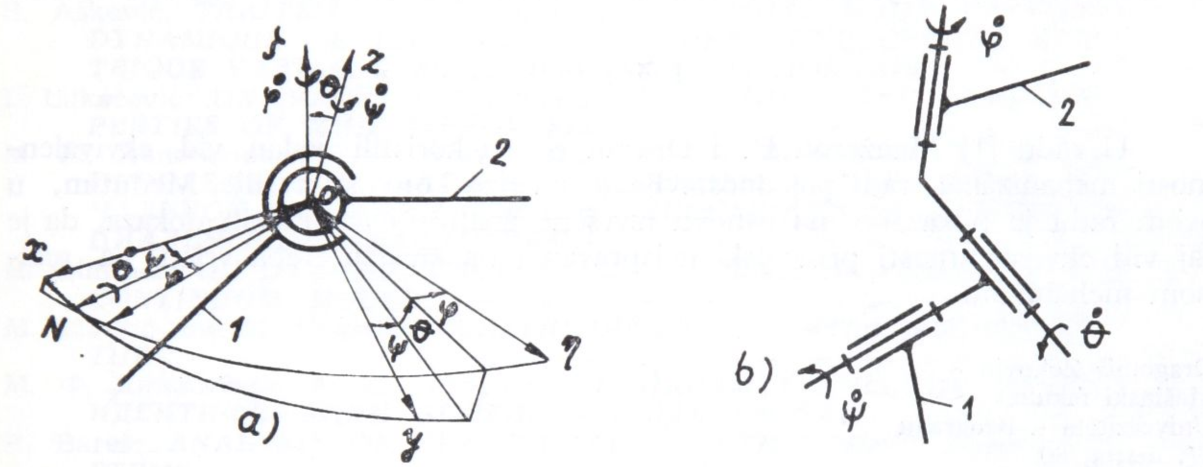

Fig. 4

On the grounds of the analyses given above it can be stated that this aspect of equivalency of mechanisms is incorrect in principle!

REFERENCES

[1] Манжерон, Д., Драган, К., *Кинематическое исследование новым матрично-тензорним методом четырзвеных пространственных механизмов*, Букурешт, Révue de Mécanique Appliquée, Acad. R. P., 7, 3, 539–552, (1962).

[2] Zeković, D., *Tenzorsko-matrični postupak u kinematičkoj analizi prostornih mehanizama*, magistarski rad, Mašinski fakultet, Beograd, (1978).

[3] Лебедев, П., *Кинематика пространственых механизов*, Машиностроение, Москва, Ленинград, (1966).

[4] Неймарк, Ю, и Фуфаев, Н., *Динамика неголономных систем*, Наука, Москва (1967).

О ТОЧНОСТИ ОДНОГО ВИДА ЭКВИВАЛЕНТНОСТИ МЕХАНИЗОВ

Резюме

В работе [1] пользюмся один вид эквивалентности механизов, для упрощения проводимого анализа. Однако, в этой работе показано, на основании выполненоого анализа и аналитического доказательства что етот вид эквивалентности механизмов в принципе ошибочный и поэтому неопходимо выполнять анализ на реальном механизме.

O ISPRAVNOSTI JEDNOG VIDA EKVIVALENTNOSTI MEHANIZMA

Izvod

U radu [1] Manžeron D. i Dragan K. su koristili jedan vid ekvivalentnosti mehanizama, radi pojednostavljenja analize koju su vršili. Međutim, u ovom radu je pokazano, na osnovu izvršene analize i analitičkih dokaza, da je taj vid ekvivalentnosti principski neispravan i da analizu treba vršiti na realnom mehanizmu.

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