

**THE GENERALIZATION OF STEVENSON'S THEORY OF PLATES**

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The problem of exact solving of three equilibrium equations, six compatibility equations of Beltrami-Michell and six equations of Hook's law is based in the Stevenson's theory [3] on the assumption that the transverse load  $q(x, y)$  is a harmonic function. This assumption which namely leads to a more realistic theory than Kirchhoff's, introduces some restrictions on applicability of Stevenson's theory. In this, article we treat the bending of elastic, homogeneous and isotropic plates by the assumption that the transverse load is a biharmonic function. Based on this, and the assumption that the weight is the only body force, exact solution of the above mentioned equations is obtained. The generalized theory is therefore exactly applicable to the extended set of transverse loads. In this article we do no represent the complete results, but only the differences between the generalized theory and Stevenson's theory. Complete results of the Stevenson's theory may be found in [1].

**1. The Generalization of Stevenson's Theory**

Let a homogeneous plate, with density  $\rho$ , be loaded with transverse load  $q(x, y)$ . The thickness of plate  $2h$  is constant. The component of the stress tensor  $\sigma_z$  must satisfy the following boundary conditions:

$$\sigma_z(Z = h) = -q, \quad \sigma_z(Z = -h) = 0, \quad \frac{\partial \sigma_z}{\partial Z}(Z = \pm h) = \rho g \quad (1.1)$$

The last equation results from the third equilibrium equation, while the equations of Beltrami-Michell imply:

$$\Delta \Delta \sigma_z = 0, \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial Z^2} = \Delta_1 + \frac{\partial^2}{\partial Z^2}. \quad (1.2)$$

*Theorem 1.1*

By the assumption that  $\sigma_z$  is the polynomial of degree five with respect to the coordinate  $Z$ , from equations (1.1) and (1.2) it follows:

$$\sigma_z = -\frac{q}{2} - \left( \frac{3q}{4h} + \frac{h\Delta_1 q}{40} + \frac{\rho g}{2} \right) Z + \left( \frac{q}{4h^3} + \frac{\Delta_1 q}{20h} + \frac{\rho g}{2h^2} \right) Z^3 - \frac{\Delta_1 q}{40h^3} Z^5 \quad (1.3)$$

and

$$\Delta_1 \Delta_1 q(x, y) = 0. \quad (1.4)$$

The transverse load is a biharmonic function and therefore there exist two analytic functions  $P_1(z)$  and  $P_2(z)$  ( $z = x + iy$ ) in the following sense:

$$q(x, y) = 2 \operatorname{Re} [P_1'(z) + \bar{z} P_2''(z)] = 2 \operatorname{Re} [P_1(z)] + \bar{z} P_2''(z) + \overline{z P_2''(z)}. \quad (1.5)$$

The component of the stress tensor  $\sigma_z$  is known and from the equilibrium equations and from the equations of Beltrami-Michell one can determine the stress combinations  $A, B, C$ , which are defined as follows:

$$A = \sigma_x + \sigma_y, \quad B = \sigma_y - \sigma_x + 2i\tau_{xy}, \quad C = \tau_{xz} + i\tau_{yz}. \quad (1.6)$$

### Theorem 1.2

From (1.3) and from the equations of equilibrium and Beltrami-Michell it follows, that the stress combinations are expressed in the form:

$$\begin{aligned} A &= \underline{a}_0 + \underline{a}_1 Z + \underline{a}_2 Z^2 + \underline{a}_3 Z^3 \underline{a}_5 Z^5, \\ B &= \underline{b}_0 + \underline{b}_1 Z + \underline{b}_2 Z^2 + \underline{b}_3 Z^3 + \underline{b}_4 Z^4 + \underline{b}_5 Z^5 + \underline{b}_7 Z^7, \\ C &= (\underline{c}_0 + \underline{c}_1 Z + \underline{c}_2 Z^2 + \underline{c}_4 Z^4) (Z^2 - h^2), \end{aligned} \quad (1.7)$$

where:

$$\begin{aligned} \underline{a}_0 &= a_0 - v \operatorname{Re} [\bar{z} P_2''(z)], \\ \underline{a}_1 &= a_1 + \operatorname{Re} \left[ \frac{3(1+v)}{8h^3} \bar{z}^2 P_2'(z) + \frac{3(2-3v)}{10h} \bar{z} P_2''(z) + \right. \\ &\quad \left. + \frac{h}{5} P_2'''(z) \right] + \frac{\rho g}{2} \left[ 1 + \frac{3(1+v)}{2h^2} z \bar{z} \right], \\ \underline{a}_2 &= 2(1+v) \operatorname{Re} [P_2'''(z)], \end{aligned} \quad (1.8)$$

$$\underline{a}_3 = a_3 + \operatorname{Re} \left[ -\frac{2+v}{2h^3} \bar{z} P_2''(z) + \frac{3v+1}{5h} P_2'''(z) \right] - \frac{(2+v)\rho g}{2h^2},$$

$$\underline{a}_5 = \frac{3+v}{10h^3} \operatorname{Re} [P_2'''(z)],$$

$$\underline{b}_0 = b_0 - \frac{v}{2} \left[ P_2'(z) + \frac{\bar{z}_2}{2} P_2'''(z) \right] + \frac{2h^2}{3} \bar{z} P_2^{IV}(z),$$

$$\underline{b}_1 = b_1 - \frac{3(1-v)}{8h^3} \left[ z \overline{P_2(z)} + \frac{1}{6} \bar{z}^3 P_2''(z) \right] -$$

$$\frac{3(2+3v)}{20h} \left[ \overline{P_2'(z)} + \frac{1}{2} \bar{z}^2 P_2'''(z) \right] - \frac{h}{10} \bar{z} P_2^{IV}(z) -$$

$$- \frac{3(1-v)}{8h^2} \rho g \bar{z}^2,$$

$$\bar{b}_2 = b_2 - (1 - \nu) \bar{z} P_2^{\text{IV}}(z) - \frac{4 h^2}{3} P_2^{\text{V}}(z), \quad (1.9)$$

$$\underline{b}_3 = b_3 + \frac{2 - \nu}{4 h^3} \left[ \overline{P'_2(z)} + \frac{1}{2} \bar{z}^2 P_2'''(z) \right] + \frac{3 \nu - 1}{10 h} \bar{z} P_2^{\text{IV}}(z) +$$

$$+ \frac{h}{15} P_2^{\text{V}}(z),$$

$$\underline{b}_4 = \frac{2 - \nu}{3} P_2^{\text{V}}(z),$$

$$\underline{b}_5 = b_5 - \frac{3 - \nu}{20 h^3} \bar{z} P_2^{\text{IV}}(z) + \frac{4 - 3 \nu}{50 h} P_2^{\text{V}}(z),$$

$$\underline{b}_7 = \frac{4 - \nu}{210 h^3} P_2^{\text{V}}(z),$$

$$\begin{aligned} \underline{c}_0 &= c_0 - \frac{3}{8 h^3} \left[ \bar{z} P'_2(z) + \frac{1}{2} z^2 \overline{P''_2(z)} \right] - \\ &- \frac{1}{20 h} [P''_2(z) + z \overline{P'''_2(z)}] - \frac{h}{5} \overline{P_2^{\text{IV}}(h)} - \frac{3 \rho g}{4 h^2} z, \end{aligned}$$

$$\underline{c}_1 = - \frac{2}{3} \overline{P_2^{\text{IV}}(z)}, \quad (1.10)$$

$$\underline{c}_2 = c_2 + \frac{1}{4 h^3} [P''_2(z) + z \overline{P'''_2(z)}] - \frac{1}{10 h} \overline{P_2^{\text{IV}}(z)},$$

$$\underline{c}_4 = - \frac{1}{20 h^3} \overline{P_2^{\text{IV}}(z)}.$$

The quantities  $a_0, a_1, \dots, c_2$  are in agreement with the same quantities in [1] if we put:  $P(z) = 2P_1(z)$ . From equations (1.8), (1.9), (1.10) it appears that only the function  $P_2(z)$  contributes to the differences between the results of the Stevenson's and the generalized theory.

From equations (1.6), (1.7), (1.8), (1.9), (1.10) one can determine the equations for the components of the stress tensor. The components  $\sigma_x, \sigma_y, \tau_{xy}$  are the polynomials of degree seven, the components  $\tau_{xz}$  and  $\tau_{yz}$  are the polynomials of degree five with respect to the coordinate  $Z$ . After integration of the components of the stress tensor, we can determine the equations for unit forces and moments and for the main force and the main moment. The equations are similar as in the Stevenson's theory [1]; the additional terms are contributed by the function  $P_2(z)$ .

When the stress combinations  $A, B, C$  are known, one can determine the displacements  $u, v, w$  from Hooke's law.

*Theorem 1.3*

From equations (1.8), (1.9), (1.10) and from Hooke's law, it follows that the displacements are expressed in the form:

$$\begin{aligned} u(x, y) + iv(x, y) &= \underline{D}_0 + \underline{D}_1 Z + \underline{D}_2 Z^2 + \underline{D}_3 Z^3 + \underline{D}_4 Z^4 + \underline{D}_5 Z^5 + \\ &\quad + \underline{D}_7 Z^7, \\ w(x, y) &= \underline{w}_0 + \underline{w}_1 Z + \underline{w}_2 Z^2 + \underline{w}_3 Z^3 + \underline{w}_4 Z^4 + \underline{w}_6 Z^6, \end{aligned} \quad (1.11)$$

where

$$\begin{aligned} 8\mu\underline{D}_0 &= 8\mu D_0 + \left[ \bar{z} P'_2(z) + \frac{1}{2} z^2 \overline{P''_2(z)} \right] + \frac{4h^2}{3} [P''_2(z) - z \overline{P'''_2(z)}] \\ 8\mu\underline{D}_1 &= 8\mu D_1 + \frac{3(1-\nu)}{8h^3} \left[ \bar{z}^2 P_2(z) + \frac{1}{3} z^3 \overline{P'_2(z)} \right] + \\ &\quad + \frac{3(2+3\nu)}{10h} \left[ \bar{z} P'_2(z) + \frac{1}{2} z^2 \overline{P''_2(z)} \right] + \frac{h}{5} [P''_2(z) + \\ &\quad + z \overline{P'''_2(z)}] + \frac{3(1-\nu)}{4h^2} \rho g \bar{z} z^2 + \rho g z, \\ 8\mu\underline{D}_2 &= 8\mu D_2 + (2-\nu) [P''_2(z) + z \overline{P'''_2(z)}] + \frac{8h^2}{3} \overline{P''_2(z)}, \\ 8\mu\underline{D}_3 &= 8\mu D_3 - \frac{2-\nu}{2h^3} \left[ \bar{z} P'_2(z) + \frac{1}{2} z^2 \overline{P''_2(z)} \right] + \\ &\quad + \frac{1-3\nu}{5h} [P''_2(z) + \overline{P'''_2(z)}] - \frac{2h}{15} \overline{P''_2(z)} - \frac{2-\nu}{h^2} \rho g z, \quad (1.12) \\ 8\mu\underline{D}_4 &= -\frac{2(2-\nu)}{3} \overline{P''_2(z)}, \\ 8\mu\underline{D}_5 &= 8\mu D_5 + \frac{3-\nu}{10h^3} [P''_2(z) + z \overline{P'''_2(z)}] - \frac{4-3\nu}{25h} \overline{P''_2(z)}, \\ 8\mu\underline{D}_7 &= -\frac{4-\nu}{105h^3} \overline{P''_2(z)}, \\ 2\mu\underline{w}_0 &= 2\mu w_0 + Re \left[ -\frac{3(1-\nu)}{96} \bar{z}^3 P_2(z) + \frac{3(8-3\nu)}{80h} \bar{z}^2 P'_2(z) + \right. \\ &\quad \left. + \frac{h}{20} \bar{z} P''_2(z) + \frac{2h^3}{5} \overline{P'''_2(z)} \right] + \frac{5\rho g}{8} z \bar{z} - \\ &\quad - \frac{3(1-\nu)\rho g}{64h^2} z^2 \bar{z}^2, \end{aligned}$$

$$\begin{aligned}
 2\mu \underline{w}_1 &= 2\mu w_1 + Re[-(1-\nu) \bar{z} P_2''(z)], \\
 2\mu \underline{w}_2 &= 2\mu w_2 + Re\left[-\frac{3\nu}{16h^3} \bar{z}^2 P_2'(z) - \frac{3(5-3\nu)}{20h} \bar{z} P_2''(z) - \right. \\
 &\quad \left. - \frac{h}{10} P_2'''(z)\right] - \frac{3\nu\rho g}{8h} z \bar{z} - \frac{\rho g}{4}, \\
 2\mu \underline{w}_3 &= -\frac{2\nu}{3} Re[P_2'''(z)], \\
 2\mu \underline{w}_4 &= 2\mu w_4 + Re\left[\frac{1+\nu}{8h^3} \bar{z} P_2''(z) + \frac{2-3\nu}{20h} P_2'''(z)\right] + \\
 &\quad + \frac{(1+\nu)\rho g}{8h^2}, \\
 2\mu \underline{w}_6 &= -\frac{2+\nu}{60h^3} Re[P_2''(z)].
 \end{aligned} \tag{1.13}$$

The quantities  $D_0, D_1, \dots, w_4$  are in agreement with the same quantities in [1] if we put:  $P(z) = 2P_1(z)$ .

It is also evident here that the additional terms are contributed only by the function  $P_2(z)$ .

## 2. Boundary Value Problems

We have already mentioned that the comparison between the results of the first chapter and the corresponding results of the Stevenson's theory show that the differences between both theories are only in the contributions of function  $P_2(z)$ . On the basis of this statement we expect that the boundary value equations for the first and for the second generalized boundary value problem [1] will be similar as in the Stevenson's theory. The differences will appear only because of the function  $P_2(z)$  which is equal to zero in Stevenson's theory.

Boundary value equations which we get by the generalized theory for the first boundary value problem are:

$$\varphi(t) + t \overline{\varphi'(t)} + \overline{\psi(t)} - \frac{2}{3} \sigma h^2 \overline{\varphi''(t)} = f(t) + \underline{Q}_1(t) + \alpha_k, \tag{2.1}$$

$$\kappa' \Omega(t) - t \overline{\Omega'(t)} - \overline{\omega'(t)} + \kappa'' h^2 \overline{\Omega''(t)} = \tilde{f}(t) + \widetilde{Q}_1(t) + i r_k t + \beta_k. \tag{2.2}$$

The functions  $f(t)$  and  $\tilde{f}(t)$  are defined in the same manner as in [1] but for the functions  $\underline{Q}_1(t)$  and  $\widetilde{Q}_1(t)$  we find:

$$\begin{aligned}
 \underline{Q}_1(t) &= Q_1(t) + \nu \left[ t P_2'(t) + \frac{1}{2} t^2 \overline{P_2''(t)} \right] - \frac{2(1+\nu)h^2}{3} [t \overline{P_2'''(t)} + \\
 &\quad + P_2''(t)] - \frac{2(14+3\nu)h^4}{45} \overline{P_2^{IV}(t)}.
 \end{aligned} \tag{2.3}$$

$$\begin{aligned}
\underline{\underline{Q}}_1(t) = & \underline{\underline{Q}}_1(t) + \frac{3}{8 h^3} \left[ \bar{t}^2 P_2(t) + \frac{1}{3} t^3 \overline{P_2''(t)} \right] + \frac{6 \nu}{5(1-\nu) h} \left[ \bar{t} P_2'(t) + \right. \\
& \left. + \frac{1}{2} t^2 \overline{P_2''(t)} \right] - \frac{(471 + 141 \nu)}{350(1-\nu)} P_2''(t) + \\
& + \frac{(157 - 141 \nu)}{350(1-\nu)} t \overline{P_2'''(t)} - \frac{2(127 - 43 \nu) h^3}{1575} \overline{P_2^{\text{IV}}(t)} + \\
& + \frac{(1 + 3 \nu) \rho g}{5(1-\nu)} t - \frac{3 \rho g}{4 h^2} t^2 \bar{t} - \frac{3}{(1-\nu) h^3} \int_0^S \left[ \frac{1}{2} \bar{t}^2 P_2'(t) + \right. \\
& \left. + ght \bar{t} - \int_0^{\tau} \bar{t} P_2'(\tau) d\tau - \tau \overline{P_2'(\tau)} d\tau + \rho gh (\tau d \bar{t} - \bar{t} d \tau) \right] dt. \quad (2.4)
\end{aligned}$$

The functions  $\underline{Q}_1(t)$  and  $\underline{\underline{Q}}_1(t)$  are the same as in [1] if we put:  $P(z) = 2P_1(z)$ .

Boundary value equations, which we get by the generalized theory for the second generalized boundary value problems are:

$$\kappa \varphi(t) - t \overline{\varphi'(t)} - \overline{\psi(t)} = g(t) + \underline{Q}_2(t), \quad (2.5)$$

$$\Omega(t) + t \overline{\Omega'(t)} + \overline{\omega'(t)} - \frac{4}{1-\nu} h^2 \overline{\Omega''(t)} = \tilde{g}(t) + \underline{\underline{Q}}_2(t). \quad (2.6)$$

Here also the functions  $g(t)$  and  $\tilde{g}(t)$  are defined in the same manner as in [1] but for the functions  $\underline{Q}_2(t)$  and  $\underline{\underline{Q}}_2(t)$  we find:

$$\underline{Q}_2(t) = Q_2(t) - \nu \left[ \bar{t} P_2'(t) + \frac{1}{2} t^2 \overline{P_2''(t)} \right] - \frac{4 h^2}{3} [P_2''(t) - t \overline{P_2'''(t)}], \quad (2.7)$$

$$\begin{aligned}
\underline{\underline{Q}}_2(t) = & \underline{\underline{Q}}_2(t) - \frac{3}{8 h^3} \left[ \bar{t}^2 P_2(t) + \frac{1}{3} t^3 \overline{P_2'(t)} \right] + \\
& + \frac{3(8-3\nu)}{10(1-\nu)h} \left[ \bar{t} P_2'(t) + \frac{1}{2} t^2 \overline{P_2''(t)} \right] + \frac{h}{5(1-\nu)} [P_2''(t) + \\
& + t \overline{P_2'''(t)}] + \frac{8 h^3}{5(1-\nu)} \overline{P_2^{\text{IV}}(t)} + \frac{5 \rho g}{1-\nu} t - \frac{3 \rho g}{4 h^2} t^2 \bar{t}. \quad (2.8)
\end{aligned}$$

The functions  $\underline{Q}_2(t)$  and  $\underline{\underline{Q}}_2(t)$  are the same as in [1] if we put:  $P(z) = 2P_1(z)$ .

### 3. Solution of a Concrete Case

Let us analyse a circular plate, with radius  $R$ , by the generalized theory. A plate is stiffly clamped along the boundary and loaded with a parabolic load:

$$q(x, y) = q_0 \left(1 - \frac{r^2}{R^2}\right), \quad q_0 = \text{constant}, \quad r^2 = x^2 + y^2. \quad (3.1)$$

We can not solve this problem by the Stevenson's theory because the transverse load is not a harmonic function.

From equation (1.5) it follows:

$$P_1'(z) = \frac{1}{2} q_0, \quad P_2''(z) = -\frac{q_0}{2 R^2} z; \quad P_1(z) = \frac{q_0}{4} z^2, \quad P_2(z) = -\frac{q_0}{12 R^2} z^3. \quad (3.2)$$

The plate is stiffly clamped and therefore we must take into account the following boundary conditions:

$$u_0(r = R) = 0, \quad v_0(r = R), \quad w_0(r = R) = 0, \quad \left. \frac{dw_0}{dr} \right|_{R=0} = 0. \quad (3.3)$$

The functions  $g(t)$  and  $\tilde{g}(t)$  are therefore equal to zero but for the functions  $\underline{\Omega}_2(t)$  and  $\tilde{\underline{\Omega}}_2(t)$  we get:

$$\begin{aligned} \underline{\Omega}_2(t) &= -\frac{2-\nu}{2} q_0 t, \quad \tilde{\underline{\Omega}}_2(t) = \frac{3 q_0}{20(1-\nu) h} \left[ 2 + 3\nu - \frac{4 h^2}{R} - \right. \\ &\quad \left. - \frac{25(1-\nu) R^2}{12 h^2} \right] t + \rho g \left( \frac{5}{1-\nu} - \frac{3 R^2}{4 h^2} \right) t. \end{aligned} \quad (3.4)$$

Now we can solve the boundary value equations (2.5) and (2.6) by Muskhelishvili's method [2] and we find:

$$\begin{aligned} \varphi(z) &= -\frac{(2-\nu)(1+\nu)}{4(1-\nu)} q_0 z, \quad \psi(z) = 0, \quad \omega(z) = \text{constant}, \\ \Omega(z) &= \frac{3 q_0}{40(1-\nu) h} \left[ 2 + 3\nu - \frac{4 h^2}{3 R^2} - \frac{25(1-\nu) R^2}{12 h^2} \right] z + \\ &\quad + \frac{\rho g}{2} \left( \frac{5}{1-\nu} - \frac{3 R^2}{3 h^2} \right) \bar{z}. \end{aligned} \quad (3.5)$$

The components of the stress tensor and the displacements can be simply calculated from known stress functions  $\varphi(z)$ ,  $\psi(z)$ ,  $\omega(z)$  and by the application of theorems 1.2 and 1.3.

For example, we put down only the bending  $w$  and the component of the stress tensor  $\sigma_r$ :

$$w = w_0 + w_1 Z + w_2 Z^2 + w_3 Z^3 + w_4 Z^4 + w_6 Z^6, \quad (3.6)$$

where:

$$\begin{aligned}
 w_0 &= -\frac{q_0}{576 DR^2} (7 R^6 + 9 R^2 r^4 - 15 R^4 r^2 - r^6) - \\
 &\quad - \frac{\rho g h}{32 D} (R^2 - r^2)^2 - \frac{(8 - \nu) q_0 h^2}{(1 - \nu) 160 DR^2} (R^2 - r^2)^2, \\
 w_1 &= \frac{q_0 h^3}{3 D} \left[ \frac{4 \nu - 2 - \nu^2}{2(1 - \nu)^2} + \frac{r^2}{R^2} \right], \\
 w_2 &= -\frac{\nu q_0}{40 D (1 - \nu)} \left[ 5 r^2 - \frac{5}{4 R^2} r^4 - \frac{25}{12} R^2 + \frac{2(5 - 3 \nu) h^2}{\nu R^2} \right] - \\
 &\quad - \frac{q_0 h^2}{120 D (1 - \nu)^2} \left[ 3(10 - 8 \nu + 3 \nu^2) - 4 \frac{h^2}{R^2} \right] + \frac{\rho g h}{24 D (1 - \nu)^2} [ - \\
 &\quad - 4(1 + 4 \nu) h^2 + 3 \nu(1 - \nu) (R^2 - 2 r^2)], \tag{3.7}
 \end{aligned}$$

$$\begin{aligned}
 w_3 &= \frac{2 \nu q_0 h^3}{9 D (1 - \nu) R^2}, \\
 w_4 &= \frac{q_0}{120 D (1 - \nu)} \left[ 5(1 + \nu) \left( 1 - \frac{r^2}{R^2} \right) - \frac{2(2 - 3 \nu) h^2}{R^2} \right] + \\
 &\quad + \frac{(1 + \nu) \rho g h}{12 D (1 - \nu)}, \\
 w_6 &= \frac{(2 + \nu) q_0}{180(1 - \nu) R^2} \quad \left( D = \frac{2 E h^3}{3(1 - \nu)^2} \right), \\
 \sigma_r &= \alpha_0 + \alpha_1 Z + \alpha_2 Z^2 + \alpha_3 Z^3 + \alpha_5 Z^5, \tag{3.8}
 \end{aligned}$$

where:

$$\begin{aligned}
 \alpha_0 &= -\frac{\nu q_0}{8} \left( \frac{3 - \nu}{1 - \nu} - \frac{r^2}{R^2} \right), \\
 \alpha_1 &= \frac{q_0}{64 h^3} \left[ 6(3 + \nu) r^2 - (5 + \nu) \frac{r^4}{R^2} - 5(1 + \nu) R^2 \right] + \frac{3 \rho g}{16 h^2} [(3 + \\
 &\quad + \nu) r^2 - (1 + \nu) R^2] - \frac{9(2 - \nu) q_0}{80 h R^2} r^2 + \frac{3(12 - 5 \nu + 3 \nu^2) q_0}{80(1 - \nu) h} - \\
 &\quad - \frac{h q_0}{10(1 - \nu) R^2} + \frac{(3 + 2 \nu) \rho g}{2(1 - \nu)}, \\
 \alpha_2 &= -(1 + \nu) \frac{q_0}{2 R^2}, \\
 \alpha_3 &= \frac{q_0}{16 h^3} \left[ (6 + \nu) \frac{r^2}{R^2} - 2(2 + \nu) \right] - \frac{(3 \nu + 1) q_0}{20 h R^2} - \frac{(2 + \nu) \rho g}{4 h^2}, \\
 \alpha_5 &= -\frac{(3 + \nu) q_0}{40 h^3 R^2}.
 \end{aligned}$$

If we compare these results with the results that we find for the same case by the Kirchhoff's theory, we can state that by the Kirchhoff's theory there are only two terms in equations for the bending  $w$  which are the same as the first two terms in the equation for  $w_0$ . For the component of the stress tensor  $\sigma_r$  we also state that by the Kirchhoff's theory we get only two terms which are the same as the first two terms in equation for  $\alpha_1$ . The differences between the results obtained by the generalized Stevenson's theory and by the Kirchhoff's theory are connected with the influence of plate thickness. The differences also increase with the plate thickness.

#### R E F E R E N C E S

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#### VERALLGEMEINERUNG DER STEVENSONSCHEN THEORIE DER PLATTEN

##### Z u s a m m e n f a s s u n g

Stevenson'sche Plattentheorie ist auf der Hypothese, dass die Querlast eine harmonische Funktion darstellt, begründet. Diese Hypothese, die zwar zu einer viel sachlicheren Theorie, als die Kirchoffsche Theorie, führt stellt eine bestimmte Beschränkung hinsichtlich Verwendbarkeit der Stevensonschen Theorie dar. In der Arbeit wird eine exakte Lösung der Gleichgewichtsgleichungen, der Beltrami-Michell Gleichung und des Hookesgesetzes behandelt, dass alles unter einer Hypothese, das die Querlast eine biharmonische Funktion darstellt. Verallgemeinerte Theorie ist deshalb bei mehreren Arten von Belastung exakter als die Stevensonsche Theorie verwendbar.

## POSPLOŠITEV STEVENSONOVE TEORIJE PLOŠČ

### Povzetek

Stevensonova teorija plošč temelji na predpostavki, da je prečna obtežba harmonična funkcija. Ta pretpostavka, ki sicer vodi do mnogo realnejše teorije kot je Kirchhoffova, predstavlja določeno omejitev glede uporabnosti Stevensonove teorije. V članku obravnavamo eksaktne rešitev ravnotežnih enačb, Beltrami-Michellovih enačb in Hookeovega zakona ob predpostavki, da je prečna obremenitev biharmonična funkcija. Poslošena teorija je zato eksaktne uporabne pri bogatejši množici obremenitev kot Stevensonova.

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