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MICROPOLAR THEORY OF ELASTIC-VISCOPLASTIC POROUS MEDIA

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1. Introduction

In the present study a micropolar theory of elastic-viscoplastic porous media is developed. The consideration of possible rotation of grains during the motion of granular material is the main contribution of the present study. The basic equations of balance of media with microstructure are presented and based on thermodynamical considerations a set of constitutive equations are derived. The theory naturally gives rise to the generation of couple stress tensor and anisotropic stress tensor. The viscoplastic flow of porous media is studied in detail and the possible application to soil mechanics is also discussed. Since soil is a granular plastic material and micropolar continuum theories have been proved to be a proper model for structured materials, it is conceived that the present theory could be an interesting model of soil.

2. Governing Equations

According to a continuum theory of Goodman and Cowin [1, 2], a granular media can be characterized by the bulk density of the distributed solid volume ν. Clearly the solid volume distribution function is one minus the porosity (void volume) function. If ρ_0 is the granules mass density then

$$\rho = \rho_0 \nu, \qquad 0 \leqslant \nu \leqslant 1. \tag{2.1}$$

The distributed solid body must satisfy the laws of motion of a continuum mechanics. Accordingly, the following field equations must be satisfied for a micropolar granular continuum:

conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{V}) = 0; \tag{2.2}$$

for incompressible granular material, i. e., if ρ_0 is constant equation (2.2), becomes

$$\frac{\partial \nu}{\partial \tau} + \nabla \cdot (\nu \underset{\sim}{V}) = 0; \tag{2.3}$$

balance of Linear Momentum

$$\tau_{ij,i} + \rho\, b_j = \rho\, \dot{V}_j; \tag{2.4}$$

balance of Angular Momentum

$$m_{ji,j} + e_{ijk}\, \tau_{jk} + \rho\, C_i = \rho\, J \dot{\nu}_i; \tag{2.5}$$

balance of equilibrated force

$$h_{i,i} + \rho\, l + \hat{g} = \rho\, K \ddot{\nu}; \tag{2.6}$$

conservation of equilibrated inert'a

$$\frac{dK}{dt} - 2\, \dot{\nu} K = 0; \tag{2.7}$$

conservation of energy

$$\begin{aligned} \rho\, \dot{e} &= \tau_{ij} D_{ij} + m_{ij}\, \nu_{j,i} + q_{k,k} + \rho\, h \\ &+ h_k\, \dot{\nu}_{,k} - \hat{g}\, \dot{\nu}; \end{aligned} \tag{2.8}$$

Entropy Inequality (Clausius-Duhem)

$$\rho\, \dot{\eta} - (q_k/\theta)_{,k} - \rho\, h/\theta \geqslant 0; \tag{2.9}$$

in (2.8), the microdeformation rate tensor D_{ij} is defined by

$$D_{ij} = V_{i,j} - e_{jik}\, \nu_k. \tag{2.10}$$

Throughout this paper the regular cartesian tensor notation is employed with superposed dot indicating the material time derivative and indices following a comma denoting partial differentiations. In equations (2.2) – (2.10) $V_k = \dot{u}_k$ is the velocity vector, u_k is the displacement vector τ_{ij} is the stress tensor, b_i is the body force per unit mass, m_{ij} is the couple stress tensor, C_i is the body couple per unit mass, J is the micro inertia, $\nu_i = \dot{\Phi}_i$ is the microgyration vector, Φ_i is the microrotation vector, h_i is the equilibrated stress vector, l is equilibrated force per unit mass, $\hat{g}$ is the internal equilibrated force, K is the equilibrated inertia, e is the internal energy density per unit mass, q_k is the heat flux vector pointing outward, h is the internal heat source per unit mass, η is the entropy per unit mass, and θ is the absolute temperature.

Introducing the Helmholtz free energy ψ for the distributed soil

$$\psi = e - \eta\theta \tag{2.11}$$

and eliminating $\rho\, h$ between (2.8) and (2.9), we find an alternative form of the Clausius-Duhem inequality

$$\begin{aligned} -\rho\,(\dot{\psi} + \eta\dot{\theta}) &+ \tau_{kl} D_{lk} + m_{kl}\, \nu_{l,k} + h_k\, \dot{\nu}_{,k} \\ &- \hat{g}\, \dot{\nu} + q_k\, \theta_{,k}/\theta \geqslant 0. \end{aligned} \tag{2.12}$$

3. Elastic-Viscoplastic Porous Media

In the following, our attention will be restricted to infinitesimal strain theory and we introduce the plastic parts of strain and microdeformations through the kinematic decompositions

$$e_{ij}^{P} = e_{ij} - e_{ij}^{e},$$
$$\Phi_{ij}^{P} = \Phi_{i,j} - \Phi_{i,j}^{e},$$
$$\varepsilon_{ij}^{P} = \varepsilon_{ij} - \varepsilon_{ij}^{e}, \tag{3.1}$$

where the total strain tensor e_{ij} and the total microstrain tensor ε_{ij} are defined by

$$e_{ij} = {}^{1/2}(u_{i,j} + u_{j,i}), \tag{3.2}$$
$$\varepsilon_{ji} = u_{i,j} - e_{ijk}\,\Phi_{k}, \tag{3.3}$$

and superscript e and P correspond to elastic and plastic deformations, respectively.

We define our micropolar elastic-plastic porous media as a medium which posseses a Helmholtz free energy function of the form

$$\psi = \bar{\psi}\,(\varepsilon_{ij}, \varepsilon_{ij}^{P}, \nu, \nu_{,i}, \Phi_{i,j}, \Phi_{i,j}^{P}, \theta, k). \tag{3.4}$$

The parameter k is the work hardening parameter. Because plastic materials respond differently in hydrostatic solution than deviatoric deformations, we shall separate the stresses and strains, so that $\bar{\psi}\,(\varepsilon_{ij}) = \bar{\psi}\,(\bar{\varepsilon}_{ij}, \varepsilon_{kk})$ where $\bar{\varepsilon}_{ij}$ is the deviatoric part of the total strain tensor i.e,

$$\bar{\varepsilon}_{ij} = \varepsilon_{ij} - \varepsilon_{kk}\,\delta_{ij}/3,$$
$$\bar{\varepsilon}_{ii} = 0. \tag{3.5}$$

The stresses are similarly divided such that

$$\bar{\tau}_{ij} = \tau_{ij} - \tau_{kk}\,\delta_{ij/3},$$
$$\bar{\tau}_{ii} = 0. \tag{3.6}$$

Furthemore. it is assumed the free energy, ψ, explicitly contain $\varepsilon_{ij}^{P}, \Phi_{i,j}^{P}$, with

$$\frac{\partial\psi}{\partial\varepsilon_{ij}^{P}} \neq 0, \qquad \frac{\partial\psi}{\partial\Phi_{i,j}^{P}} \neq 0. \tag{3.7}$$

The conditions (3.7) are required for the micropolar theory of porous plasticity. Without those assumptions, the micropolar theory of thermoelasticity of porous media will be recovered.

Employing (3.4) in Clausius-Duhem inequality (2.12) after some rearrangements we find

$$
\begin{aligned}
&-\rho\left(\frac{\partial\psi}{\partial\theta}+\eta\right)\dot{\theta}+\left[\bar{\tau}_{(kl)}+\rho\frac{\partial\psi}{\partial\nu_{,k}}\nu_{,l}+\rho\frac{\partial\psi}{\partial\Phi_{m,k}}\Phi_{m,l}+\rho\frac{\partial\psi}{\partial\Phi^{P}_{m,k}}\Phi^{P}_{m,l}\right.\\
&\left.-\rho\frac{\partial\psi}{\partial\bar{\varepsilon}_{kl}}\right]V_{(k,l)}+\left[t_{kk/3}-\rho\frac{\partial\psi}{\partial\varepsilon_{kk}}+\rho\frac{\partial\psi}{\partial\nu_{,l}}\nu_{,l}+\rho\frac{\partial\psi}{\partial\Phi_{m,l}}\Phi_{m,k}\right.\\
&\left.+\rho\frac{\partial\psi}{\partial\Phi^{P}_{m,l}}\Phi^{P}_{m,l}\right]\dot{\varepsilon}_{jj}+\left\{\tau_{[kl]}-\rho\frac{\partial\psi}{\partial\bar{\varepsilon}_{kl}}\right\}D_{[kl]}+\left[\rho\frac{\partial\psi}{\partial\nu_{,k}}\nu_{,l}\right.\\
&\left.+\rho\frac{\partial\psi}{\partial\Phi_{m,k}}\Phi_{m,l}+\rho\frac{\partial\psi}{\partial\Phi^{P}_{m,k}}\Phi^{P}_{m,l}\right]V_{[k,l]}+\left(m_{kl}-\rho\frac{\partial\psi}{\partial\Phi_{k,l}}\right)\nu_{l,k}\\
&+\frac{1}{\theta}q_k\theta_{,k}-\left(g+\rho\frac{\partial\psi}{\partial\nu}\right)\dot{\nu}_{,k}+\left(h_k-\rho\frac{\partial\psi}{\partial\nu_{,k}}\right)\dot{\nu}_{,k}-\rho\frac{\partial\psi}{\partial\varepsilon^{P}_{kl}}\dot{\varepsilon}^{P}_{kl}\\
&-\rho\frac{\partial\psi}{\partial\Phi^{P}_{m,k}}\dot{\Phi}^{P}_{m,k}-\rho\frac{\partial\psi}{\partial\kappa}\dot{\kappa}\geqslant 0,
\end{aligned}
\tag{3.8}
$$

where $\bar{\tau}_{(kl)}$ and $\bar{\tau}_{[kl]}$ are the symmetric and antisymmetric parts of the derivatoris stress tensor and $V_{(t,k)}$ and $V_{[l,k]}$ are the symmetric and antisymmetric part of velocity gradiant tensor $V_{l,k}$. In the derivation of inequality (3.8), the following identities have been employed.

$$\frac{d}{dt}(\nu_{,k})=\dot{\nu}_{,k}-\nu_{,l}V_{l,k}, \tag{3.9}$$

$$\frac{d}{dt}(\Phi_{m,k})=\dot{\Phi}_{m,k}-\Phi_{m,l}V_{l,k}, \tag{3.10}$$

$$\frac{d}{dt}(\Phi^{P}_{m,k})=\dot{\Phi}^{P}_{m,k}-\Phi^{P}_{m,l}V_{l,k}. \tag{3.11}$$

The entropy inequality (3.8) must hold for all independent variations of $\dot{\theta}$, $V_{(k,l)}$, $\dot{\varepsilon}_{jj}$, $D_{[kl]}$. $\dot{\nu}$, $\dot{\nu}_{,k}$, $V_{[k,l]}$ and $\nu_{l,k}$. These variables appear linearly in the inequality and thus their coefficients must vanish. It then follows that

$$\eta=-\frac{\partial\psi}{\partial\theta}, \tag{3.12}$$

$$\bar{\tau}_{kl}=\rho\left(\frac{\partial\psi}{\partial\bar{\varepsilon}_{kl}}-\frac{\partial\psi}{\partial\nu_{,k}}\nu_{,l}-\frac{\partial\psi}{\partial\Phi_{m,k}}\Phi_{m,l}-\frac{\partial\psi}{\partial\Phi^{P}_{m,k}}\Phi^{P}_{m,l}\right), \tag{3.13}$$

$$\tau_{kk}=3\rho\left(\frac{\partial\psi}{\partial\varepsilon_{kk}}-\frac{\partial\psi}{\partial\nu_{,l}}\nu_{,l}-\frac{\partial\psi}{\partial\Phi_{m,l}}\Phi_{m,l}-\frac{\partial\psi}{\partial\Phi^{P}_{m,l}}\Phi^{P}_{m,l}\right), \tag{3.14}$$

$$e_{ikl}\left(\rho\frac{\partial\psi}{\partial\nu_{,k}}\nu_{,l}+\rho\frac{\partial\psi}{\partial\Phi_{m,k}}\Phi_{m,l}+\rho\frac{\partial\psi}{\partial\Phi^{P}_{m,k}}\Phi^{P}_{m,l}\right)=0, \tag{3.15}$$

$$m_{kl} = \rho \frac{\partial \psi}{\partial \Phi_{k,l}}, \tag{3.16}$$

$$g = -\rho \frac{\partial \psi}{\partial \nu}, \tag{3.17}$$

$$h_k = \rho \frac{\partial \psi}{\partial \nu_{,k}}, \tag{3.18}$$

and inequality (3.8) reduces to

$$-\rho \frac{\partial \psi}{\partial \varepsilon^P_{k,l}} \dot{\varepsilon}^P_{kl} - \rho \frac{\partial \psi}{\partial \Phi^P_{m,k}} \dot{\Phi}^P_{m,k} - \rho \frac{\partial \psi}{\partial \kappa} \dot{\kappa} + \frac{1}{\theta} q_k \, \theta_{,k} \geqslant 0. \tag{3.19}$$

Considering the case of heat conduction in the undeformed state implies that

$$q_k \, \theta_{,k} \geqslant 0. \tag{3.20}$$

Similarly considering a state of the body with uniform temperature and utilizing (3.19) we find that

$$-\rho \frac{\partial \psi}{\partial \varepsilon^P_{kl}} \dot{\varepsilon}^P_{kl} \geqslant 0,$$

$$-\rho \frac{\partial \psi}{\partial \Phi^P_{k,l}} \dot{\Phi}_{k,l} \geqslant 0,$$

$$-\rho \frac{\partial \psi}{\partial \kappa} \dot{\kappa} \geqslant 0. \tag{3.21}$$

Employing (2.11) with (3.1) and (3.12)–(3.18) in the energy equation (2.8) yields

$$\rho \theta \dot{\eta} = q_{k,k} + \rho h - \rho \frac{\partial \psi}{\partial \varepsilon^P_{kl}} \dot{\varepsilon}^P_{kl} - \rho \frac{\partial \psi}{\partial \Phi^P_{k,l}} \dot{\Phi}^P_{k,l} - \frac{\partial \psi}{\partial k} \dot{k} \tag{3.22}$$

which shows that the work of plastic deformation is converted into heat and acts as a heat source distribution.

Furthermore, for incompressible granular materials the variation of $\dot{\nu}$ is restricted by equation (2.3), i.e.

$$\dot{\nu} = -\nu V_{j,j}. \tag{3.23}$$

For such cases, employing equation (3.23) the terms involving $\dot{\nu}$ in inequality (3.8) may be combined with coefficients of $V_{(l,k)}$.

Therefore, the stress tensor is then given by

$$\bar{\tau}_{kl} = \rho\left(\frac{\partial\psi}{\partial\bar{\varepsilon}_{kl}} - \frac{\partial\psi}{\partial\nu_{,k}}\nu_{,l} - \frac{\partial\psi}{\partial\Phi_{m,k}}\Phi_{m,l} - \frac{\partial\psi}{\partial\Phi^{P}_{m,k}}\Phi^{P}_{m,l}\right) + \left(\hat{g} + \rho\frac{\partial\psi}{\partial\nu}\right)\nu\,\delta_{kl}, \tag{3.24}$$

$$\tau_{kk} = 3\rho\left(\frac{\partial\psi}{\partial\varepsilon_{kk}} - \frac{\partial\psi}{\partial\nu_{,l}}\nu_{,l} - \frac{\partial\psi}{\partial\Phi_{m,l}}\Phi_{m,l} - \frac{\partial\psi}{\partial\Phi^{P}_{m,l}}\Phi^{P}_{m,l}\right) + 3\nu\left(\hat{g} + \rho\frac{\partial\psi}{\partial\nu}\right), \tag{3.25}$$

and $\hat{g}$ remains unrestricted.

4. The Yield Criterion

In order to describe the plastic deformation of the porous material defined by (3.4), the following flow surface is considered [3,4],

$$f = \hat{f}\,(\tau_{ij}, m_{ij}, \varepsilon^{P}_{ij}, \nu, \theta, k); \tag{4.1}$$

the plastic state is determined by the condition $f = 0$, while elastic states correspond to the conditions $f < 0$. Furthermore, to guarantee plastic flow, the condition

$$\frac{\partial f}{\partial\tau_{ij}}\dot{\tau}_{ij} = \frac{\partial f}{\partial m_{ij}}\dot{m}_{ij} + \frac{\partial f}{\partial\varepsilon^{P}_{ij}}\dot{\varepsilon}^{P}_{ij} + \frac{\partial f}{\partial\nu}\dot{\nu} + \frac{\partial f}{\partial\theta}\dot{\theta} + \frac{\partial\rho}{\partial k}\dot{k} = 0 \tag{4.2}$$

must be satisfied. Introducing

$$\xi = \frac{\partial f}{\partial\tau_{ij}}\dot{\tau}_{ij} + \frac{\partial f}{\partial m_{ij}}\dot{m}_{ij} + \frac{\partial f}{\partial\nu}\dot{\nu} + \frac{\partial f}{\partial\theta}\dot{\theta}, \tag{4.3}$$

the following "unloading", "neutral loading" and "loading" features are now defined.

Unloading Process: when

$$f = 0 \quad , \quad \xi < 0,$$

then

$$\dot{\varepsilon}^{P}_{ij} = 0 \text{ and } \dot{k} = 0. \tag{4.4}$$

Neutral loading process: when

$$f = 0 \quad , \quad \xi = 0,$$

then

$$\dot{\varepsilon}^{P}_{ij} = 0 \quad , \quad \dot{k} = 0. \tag{4.5}$$

Loading Process: when

$$f = 0 \quad , \quad \xi > 0,$$

then

$$\dot{\varepsilon}_{ij}^{P} \neq 0 \quad , \quad \dot{k} \neq 0. \tag{4.6}$$

Following Naghdi and Murch [4] and Perzyna [3], the following plastic flow relation it considered

$$\dot{\varepsilon}_{ij}^{P} = \Lambda \left(\frac{\partial f}{\partial \tau_{ij}} + \frac{\partial f}{\partial m_{ij}} \right). \tag{4.7}$$

Determinig Λ from equation (4.2), we obtain

$$\Lambda = -\left(\frac{\partial f}{\partial \tau_{ij}} \dot{\tau}_{ij} + \frac{\partial f}{\partial m_{ij}} \dot{m}_{ij} + \frac{\partial f}{\partial \nu} \dot{\nu} + \frac{\partial f}{\partial \theta} \dot{\theta} \right) \left[\frac{\partial f}{\partial \varepsilon_{ij}^{P}} \left(\frac{\partial f}{\partial \tau_{ij}} + \frac{\partial f}{\partial m_{ij}} \right) + \frac{\partial f}{\partial k} \bar{k} \left(\frac{\partial f}{\partial \tau_{pq}} + \frac{\partial f}{\partial m_{pq}} \right) \right]^{-1}, \tag{4.8}$$

where the work hardening parameter is assumed to be only a function of plastic strain and

$$\dot{k}\,(\dot{\varepsilon}_{kl}^{P}) = \Lambda \bar{k} \left(\frac{\partial f}{\partial \tau_{pq}} + \frac{\partial f}{\partial m_{pq}} \right). \tag{4.9}$$

On introducing the notation

$$h = -\left[\frac{\partial f}{\partial \varepsilon_{ij}^{P}} \left(\frac{\partial f}{\partial \tau_{ij}} + \frac{\partial f}{\partial m_{ij}} \right) + \frac{\partial f}{\partial k} \bar{k} \left(\frac{\partial f}{\partial \tau_{pq}} + \frac{\partial f}{\partial m_{pq}} \right) \right]^{-1} \tag{4.10}$$

and recalling the loading criteria, we obtain

$$\dot{\varepsilon}_{ij}^{P} = \begin{cases} 0 & \text{if } f < 0, \\ h < \xi > \left(\dfrac{\partial f}{\partial t_{ij}} + \dfrac{\partial f}{\partial m_{ij}} \right) & \text{if } f = 0, \end{cases} \tag{4.11}$$

where $< \xi >$ is defined such that

$$< \xi > = \begin{cases} 0 & \text{if } \xi \leqslant 0, \\ \xi & \text{if } \xi > 0. \end{cases} \tag{4.12}$$

In order to keep our considerations sufficienty general, in the absence of thermal effects, we introduce a static yield function in the form

$$F(\tau_{ij}, \varepsilon_{ij}^{P}, m_{ij}, \nu) = \frac{f(\tau_{ij}, \varepsilon_{ij}^{P}, m_{ij}, \nu)}{\kappa} - 1, \tag{4.13}$$

where the work hardening parameter is defined by the expression

$$\kappa = \bar{\kappa}(W_p) = \kappa(\textstyle\int \tau_{ij}\, d\varepsilon_{ij}^{P} + \int m_{ij}\, d\Phi_{i,j}), \tag{4.14}$$

with W_p being the work of plastic deformation. The constitutive equation (4.11) may now be rewritten as

$$\dot{\varepsilon}_{ij}^{P} = \gamma < \Phi(F) > \left(\frac{\partial f}{\partial \tau_{ij}} + \frac{\partial f}{\partial m_{ij}}\right), \tag{4.15}$$

where γ is a constant and symbol $< \Phi(F) >$ is defined as follows

$$< \Phi(F) > = \begin{cases} 0 & \text{if } F \leqslant 0 \\ \Phi(F) & \text{if } F > 0. \end{cases} \tag{4.16}$$

The function $\Phi(F)$ must be chosen to represent the behavior of material under dynamyc loading. Squaring both sides of (4.16), and denoting

$$I_2^P = {}^1/_2\, \dot{\varepsilon}_{ij}^P\, \dot{\varepsilon}_{ij}^P \tag{4.17}$$

the invariant of the inelastic strain rate tensor, it follows that

$$f(\tau_{ij}\, m_{ij}\, \varepsilon_{ij}^P, \nu) = K(W_p)\left\{1 + \Phi^{-1}\left[\frac{(I_2^P)^{1/2}}{\gamma}\left\{{}^1/_2\left(\frac{\partial f}{\partial \tau_{ij}} + \frac{\partial f}{\partial m_{ij}}\right)\left(\frac{\partial f}{\partial \tau_{ij}} + \frac{\partial f}{\partial m_{ij}}\right)\right\}^{-1/2}\right]\right\}. \tag{4.18}$$

This expression implicitly represents the dynamical yield condition for micropolar elastic-viscoplastic, work hardening porous material.

5. Application to soil dynamic

In the following a special static yield function which is appropriate for soil is considered, [3],

$$F = \frac{{}^1/_2\, D\,(J_1' + S_1') + J_2^{1/2} + S_2^{1/2}}{\kappa_0} - 1. \tag{5.1}$$

In (5.1) $D(\nu, \dot{\nu}, \varepsilon_{ij}^P)$ is a function describing the dilatation rate of soil, J_1' and S_1' denote, respectively, the first invariant of the stress tensor τ_{ij} and couple stress tensor m_{ij}, J_2 and S_2 are respectively the second invariant of the stress deviator $\bar{\tau}_{ij}$ and couple stress deviator $\bar{m}_{ij}$ and κ_0 is the plastic work hardening constant.

The constitutive equation (4.15) for the plastic strain rate now becomes

$$\dot{\varepsilon}_{ij}^P = \gamma < \Phi\left[\frac{{}^1/_2 D\,(J_1' + S_1') + J_2^{1/2} + S_2^{1/2}}{\kappa_0} - 1\right] > \left(D\delta_{ij} + \frac{\bar{\tau}_{ij}}{2J_2^{1/2}} + \frac{\bar{m}_{ij}}{2S_2^{1/2}}\right). \tag{5.2}$$

The dynamic yield condition (4.18) for the given static yield condition (5.1) has the form

$$ {}^1/_2 D\,(J_1' + S_1') + J_2^{1/2} + S_2^{1/2} = K\left\{1 + \Phi^{-1}\left[\frac{(I_2^P)^{1/2}}{\gamma}\left(\frac{3}{2}D^2 + 1\right)^{-1/}\right]\right\} \tag{5.3}$$

Contracting (5.2), the rate of cubical dilatation takes the following form

$$\dot{\varepsilon}^P_{ii} = 3D\gamma < \Phi \left[\frac{{}^1/_2 D (J_1' + S_1') + J_2^{1/2} + S_2^{1/2}}{\kappa_0} - 1 \right] >; \tag{5.4}$$

in the limiting case, $\gamma \to \infty$, we obtain from (5.2) the known constitutive equations for an elastic-perfectly plastic theory of soil according to Drucker and Prager [5] in which effect, of couple stresses are also included:

$$\dot{\varepsilon}^P_{ii} = \lambda \left[D\,\delta_{ij} + \frac{\bar{\tau}_{ij}}{2J_2^{1/2}} + \frac{\bar{m}_{ij}}{2S_2^{1/2}} \right], \tag{5.5}$$

where

$$\lambda = \left[I_2^P \Big/ \left(\frac{3}{2} D^2 + 1 \right) \right]^{1/2}; \tag{5.6}$$

the plastic rate of cubical dilatation is then expressed by the relation

$$\dot{\varepsilon}^P_{ii} = 3D\,\lambda. \tag{5.7}$$

To find an expression for the dilatation rate of soil, D, we consider the case of incompressible granules where from (2.3) allowing only plastic deformation we will get

$$\dot{\nu} = -\nu\,\dot{\varepsilon}^P_{ii}. \tag{5.8}$$

Upon substitution of (5.7) with (5.6) into (5.8) we arrive at the expression for D in the following form:

$$D = -2\dot{\nu}(18\nu^2 I_2^P - 3\dot{\nu}^2)^{-1/2}. \tag{5.9}$$

6. Concluding remarks

In the present paper, a micropolar theory of elastic-viscoplastic porous material is developed and its application to soil mechanics is discussed. The consideration of possible rotation of grains during the motion of granular material, is the main contribution of the present study. Rotation of granules appears to be quite compatible with the physics of the granular materials as the experimental observations. Its consideration, in the present theory, gives rise to the generation of couple stress tensor and anisotropic stress tensor.

It is conceived that the present theory of micropolar plasticity could be an interesting model of soil. Since soil is a granular plastic material and micropolar continuum theories have been proved to be a proper model for structured materials. Further investigations in that direction are left for furture studies.

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MIKROPOLARE THEORIE DER ELASTISCH-VISKOPLASTISCHEN PORÖSEN MEDIEN

Zusammenfassung

Eine allgemeine Kontinuum Theorie für die elastisch viskoplastischen porösen Medien wurde unter berückrichtigung der zulässigen Drehung vom Granulat formuliert. Die Grundgesetze des Gleichgewichtes wurden angegeben und aus den thermodynamischen Betrachtungen konstitutive Gleichungen hergeleited. Das Fliessverhalten der viskoplastischen porösen Medien wurde im einzelnen untersucht and die moglichkeit dessen Anwendung auf dem Bodenmechanik auch uberprüft.

MIKROPOLARNA TEORIJA ELASTO-VISKOPLASTIČNE POROZNE SREDINE

Izvod

U radu se proučava mikropolarna teorija elasto-viskoplastične sredine Glavni prilog ovog rada sastoji se u izvođenju jednačina balansa i sistema konstitutivnih jednačina na osnovu termodinamičke analize za slučaj kad granula ima rotaciju tokom kretanja. U radu se pokazuje da se na osnovu ove pretpostavke na prirodan način dolazi do pojma naponskog sprega, kao i anizotropnog tenzora napona. Proučeno je i viskoplastično tečenje i moguće aplikacije teorije na mehaniku tela. Takođe je pokazano da izložena teorija može biti interesantan model materijala tipa tla.

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