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EFFECTS OF SUCTION AND FREE CONVECTION CURRENT ON OSCILLATORY FLOW OF A RAREFIED GAS PAST AN INFINITE VERTICAL POROUS PLATE

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1. Introduction

The oscillatory flow past an infinite vertical porous plate with constant suction was studied by Soundalgekar (1973 a, b) under no-slip conditions, and under slip-flow boundary conditions it was studied by Soundalgekar (1977). In these papers, the suction velocity was not considered in an explicit manner. Hence Soundalgekar, Vighnesam and Murty (1980) re-studied the problems of Refs (1973 a, b) by considering in an explicit way the suction velocity parameter under no-slip conditions. Also, the mean velocity was shown to be affected by the frequency of the free stream oscillations. It is now proposed that the effects of rarefaction of the medium on the flow past an infinite vertical porous plate under first-order velocity-slip and temperature-jump boundary conditions be studied. In Sec. 2. the mathematical analysis is presented and in Sec. 3. the conclusions are set out.

2. Mathematical Analysis

We consider a two-dimensional flow of an incompressible viscous rarefied gas past an infinite vertical porous plate with constant suction. The x' axis is taken along the plate in the vertically upward direction and y' axis is taken normal to the plate. Then following Refs. [1—3], under usual Boussinesq's approximation, the upward motion of the rarefied gas under first-order velocity slip and temperature-jump boundary conditions can be shown to be governed by the following coupled non-linear equations:

$$\frac{1}{4}\frac{\partial u}{\partial t}-\gamma\frac{\partial u}{\partial y}=\frac{1}{4}\frac{\partial U}{\partial t}+G\theta+\frac{\partial^2 u}{\partial y^2}, \tag{1}$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t}-P\gamma\frac{\partial^2\theta}{\partial y^2}=\frac{\partial^2\theta}{\partial y^2}+PE\left(\frac{\partial u}{\partial y}\right)^2. \tag{2}$$

The nondimensional quantities are defined as follows:

$$u = u'/U_0,\ \gamma = v_0/U_0,\ U = U'/U_0,\ G = \frac{\nu g \beta (T'_w - T'_\infty)}{U_0^3},$$

$$\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} \cdot P = \mu c_p/k,\ E = \frac{U_0^2}{c_p(T'_w - T'_\infty)},\ y = y'\ U_0/\nu,$$

$$t = t'\ U_0^2/4\nu,\quad \omega = 4\,\omega'\nu/U_0^2. \tag{3}$$

Here, u' is the velocity of gas in the direction of x' axis, v_0 is the suction velocity, U_0 is the mean velocity of the free stream velocity, $U'(t')$ the free stream velocity, ν the kinematic viscosity, β the coefficient of volume expansion, g the acceleration due to gravity, t' the time, ω the frequency of the free stream oscillations, c_p the specific heat at constant pressure, T' the temperature of the gas near the plate, T'_w the temperature of the plate, T'_∞ the temperature of the fluid in the free stream, μ the viscosity, γ the suction parameter, P the Prandtl number, G the Grashof number, and E the Eckert number.

The first-order velocity-slip and temperature-jump boundary conditions are

$$u = h_1 \frac{du}{dy},\ \theta = 1 + h^2 \frac{d\theta}{dy} \text{ at } y = 0, \tag{4}$$

$$u = U(t) = 1 + \frac{\varepsilon}{2}(e^{i\omega t} + e^{-i\omega t}),\ \theta = 0 \text{ as } y \to \infty,$$

where $h_1 = L_1\ U_0/\nu$, $h_2 = L_2\ U_0/\nu$ are the velocity-slip and temperature-jump coefficients respectively.

To solve these coupled non-linear equations, we assume in the neighbourhood of the plate the following:

$$u(y,t) = u_0(y) + \frac{\varepsilon}{2}(u_1\ e^{i\omega t} + \bar{u}_1\ e^{-i\omega t}), \tag{5}$$

$$\theta(y,t) = \theta_9(y) + \frac{\varepsilon}{2}(\theta_1\ e^{i\omega t} + \bar{\theta}_1\ e^{i\omega t}) + \frac{\varepsilon^2}{2}(\theta_2\ e^{i\omega t} + \bar{\theta}_2\ e^{-2i\omega t}). \tag{6}$$

Here, $^-$ denotes the complex conjugate of the corresponding quantity.

We substitute Eqs. (5) and (6) in Eqs. (1), (2) and (4), equate the harmonic and nonharmonic terms, neglect the coefficient of ε^2 in the momentum equation and ε^3 in the energy equation, and we get the following:

$$u''_0 + \gamma u'_0 = -G\theta_0, \tag{7}$$

$$u''_1 + \gamma u'_1 - \frac{i\omega}{4} u_1 = -G\theta_1 - \frac{i\omega}{4}, \tag{8}$$

$$\theta_0'' + \gamma P \theta_1' = - PE\left(u_0'^2 + \frac{\varepsilon^2}{2} u_1' \bar{u}_1'\right), \tag{9}$$

$$\theta_1'' + \gamma P \theta_1' - \frac{i \omega P}{4} \theta_1 = - 2 PE u_0' u_1', \tag{10}$$

$$\theta_2'' + \gamma P \theta_2' - \frac{i \omega P}{2} \theta_2 = - \frac{1}{2} PE u_1'^2, \tag{11}$$

and two more equations for $\bar{\theta}_1$, $\bar{\theta}_2$ similar to Eqs. (10) and (11).

The corresponding boundary conditions are

$$u_0 = h_1 u_0', \quad u_1 = h_1 u_1', \quad \theta_9 = 1 + h_2 \theta_0', \quad \theta_1 = h_2 \theta_1',$$

$$\theta_2 = h_2 \theta_2', \qquad \text{at } y = 0, \tag{12}$$

$$u_9 = 1, \quad u_1 = 1, \quad \theta_1 = 0, \quad \theta_2 = 0 \quad \text{as } y \to \infty.$$

Equations (7) — (11) are still coupled and non-linear equations whose closed form solutions are not available. Hence, we again find the approximate solutions by expanding u_0, θ_0, . . . in powers of E, Eckert number, as E is always small (< 1) for incompressible fluids

$$u_0(y) = u_{01}(y) + E u_{02}(y),$$

$$\theta_0(y) = \theta_{01}(y) + E \theta_{02}(y), \text{ etc.} \tag{13}$$

We substitute Eq. (13) in Eqs. (7) — (12), equate the coefficients of different powers of E, neglect those of E^2, and get a set of coupled linear equations that are solved in closed form. To save space, they are not mentioned here. We have obtained the expressions for the mean velocity u_0, the mean temperature θ_0, and numerical values of u_0 and θ_0 are calculated for all values of the parameters for $P = 0.71$ (air). In considering the values of G, the Grashof number, we have taken all the real values of G, as it has been shown in Refs. [1] and [2] that $G < 0$ corresponds to heating of the plate by free-convection currents and $G > 0$ then corresponds to cooling of the plate by free convection currents.

It is interesting to see from Eqs. (7) and (9) that the mean velocity and the mean temperature are affected by the frequency of the free stream velocity. The mean velocity profiles are shown in Fig. 1. We observe from this figure that for $G > 0$, the mean velocity increases with increasing ω. An increase in the velocity slip parameter h_1 leads to an increase in the mean velocity whereas an increase in the temperature-jump coefficient leads to a decrease in the mean velocity. Also, greater cooling of the plate by free convection currents, or greater viscous dissipative heat, causes a rise in the mean velocity, but an increase in suction velocity leads to a decrease in the mean velocity.

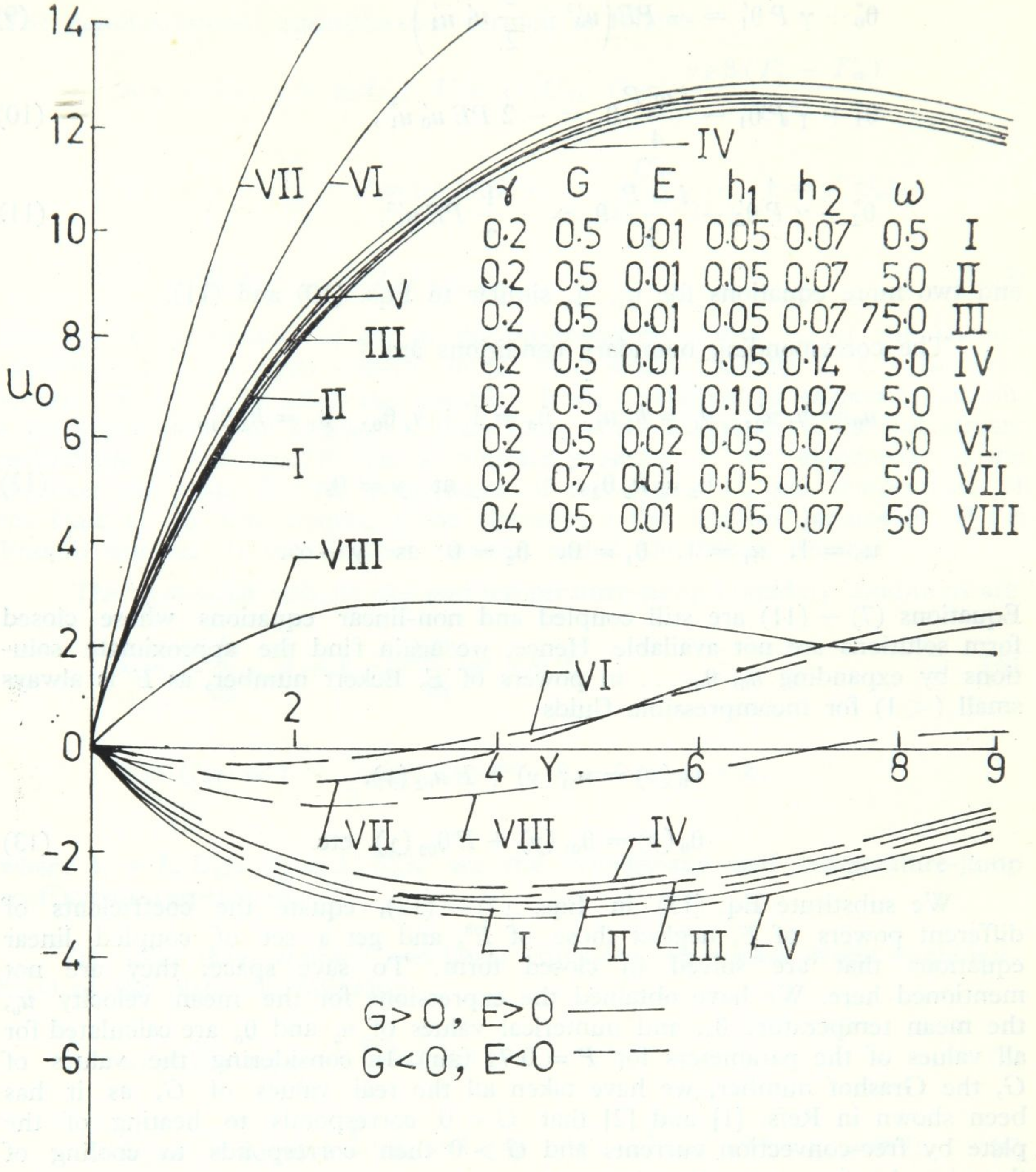

Fig. 1. Mean velocity profiles $P = 0.71$ $\varepsilon = 0.2$

When $G < 0$, i.e., when the plate is being heated by free convection currents, the mean velocity is found to increase with increasing ω. The mean velocity, increases with increasing h_1 and h_2. But, greater heating of the plate, or greater viscous dissipative heat or an increase in γ leads to an increase in the mean velocity. The mean temperature profiles are shown on Fig. 2, for air. It is interesting to see the effect of the frequency of oscillation on the mean temperature. When $G > 0$, i.e., in the presence of the plate being cooled by free convection currents, we observe that an increase in ω leads to an increase in the mean temperature of air. Again, an increase in h_1, h_2 or γ

leads to a decrease in the mean temperature. But the mean temperature increases with increasing G, i.e., due to greater cooling of the plate or E, due to greater viscous dissipative heat.

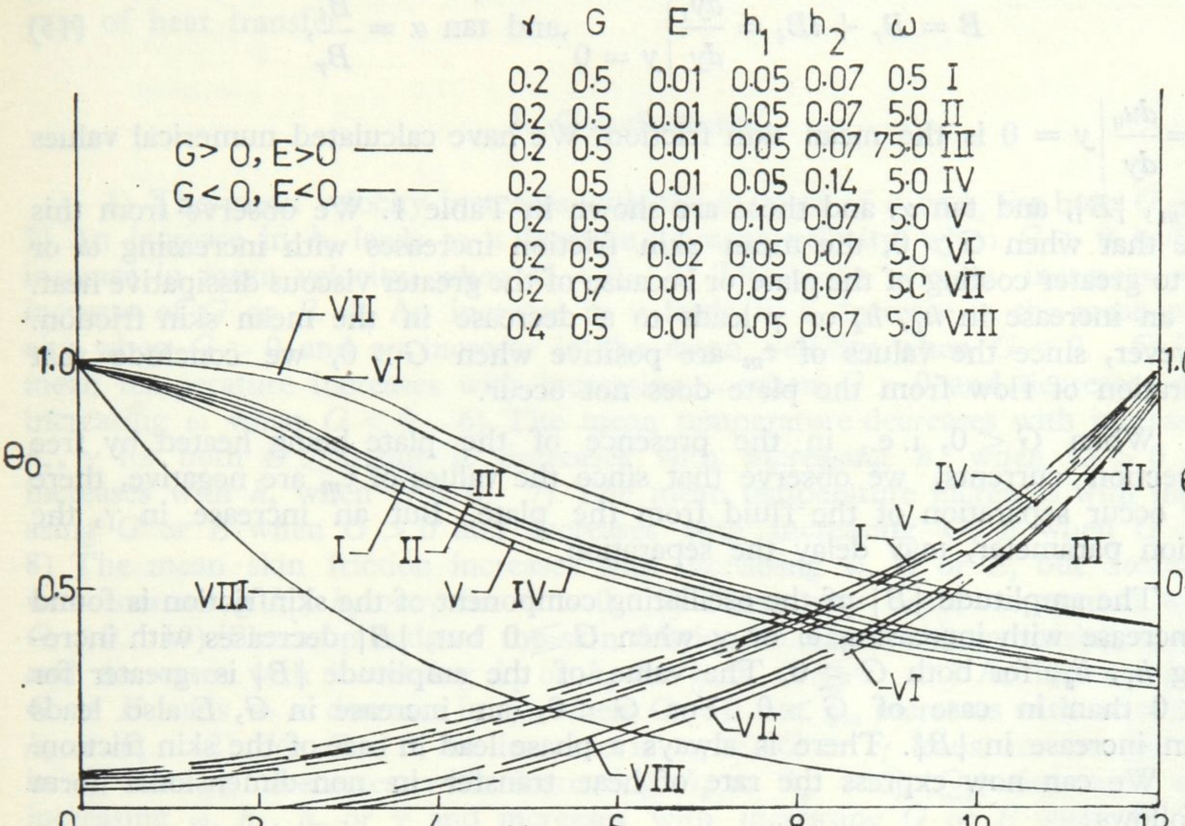

Fig. 2. Mean temperatur profiles, $P = 0.71$, $\varepsilon = 0.2$

When $G < 0$, in the presence of the plate being heated by free convection currents, we observe that the mean temperature decreases due to increasing ω. An increase in h_2 leads to a fall in the mean temperature, whereas an increase in h_1 leads to a rise in the mean temperature. Greater heating of the plate or greater viscous dissipative heat or an increase in γ leads to a decrease in the mean temperature.

We can now calculate the unsteady part of the velocity and temperature field by solving equations for u_{11}, u_{12}, θ_{11}, θ_{12} and substituting in Eqs. (13). The expression for the transient velocity and transient temperature can be derived following Refs. [1 — 3]. Since the effects of these parameters on the transient field are the same as that of the mean field, we have not discussed it here.

We now study the effects of these parameters on the skin friction. It is given in nondimensional form by

$$\tau = \left.\frac{\partial u_0}{\partial y}\right|_{y=0} + \varepsilon\, e^{i\omega t} \left.\frac{\partial u_1}{\partial y}\right|_{y=0}, \tag{14}$$

where $\tau = \tau'/\rho U_0{}^2$.

This can be expressed in terms of the amplitude and phase as

$$\tau = \tau_m + \varepsilon |B| \cos(\omega t + \alpha),$$

where

$$B = B_r + iB_i = \left.\frac{dy_1}{dy}\right|_{y=0}, \text{and } \tan\alpha = \frac{B_i}{B_r}, \tag{15}$$

$\tau_m = \left.\frac{du_0}{dy}\right|_{y=0}$ is the mean skin friction. We have calculated numerical values of τ_m, $|B|$, and $\tan\alpha$, and these are shown in Table 1. We observe from this table that when $G > 0$, the mean skin friction increases with increasing ω or due to greater cooling of the plate or because of the greater viscous dissipative heat. But an increase in h_1, h_2 or γ leads to a decrease in the mean skin friction. However, since the values of τ_m are positive when $G > 0$, we conclude that separation of flow from the plate does not occur.

When $G < 0$, i. e., in the presence of the plate being heated by free convection currents, we observe that since the values of τ_m are negative, there may occur separation of the fluid from the plate. But an increase in γ, the suction parameter, may delay the separation.

The amplitude $|B|$ of the oscillating component of the skinfriction is found to increase with increasing ω or γ when $G \gtrless 0$ but, $|B|$ decreases with increasing h_1, h_2, for both $G \gtrless 0$. The value of the amplitude $|B|$ is greater for $G < 0$ than in case of $G > 0$. For $G < 0$, an increase in G, E also leads to an increase in $|B|$. There is always a phase lead in case of the skin friction.

We can now express the rate of heat transfer in non-dimensional form as follows:

$$q = q_m + \varepsilon\,|Q_1| \cos(\omega t + \alpha_1) + \varepsilon^2\,|Q_2| \cos(2\,\omega t + \alpha_2), \tag{16}$$

where $q_m = -\left.\frac{d\theta_0}{dy}\right|_{y=0}$ is the mean rate of heat transfer and

$$\begin{aligned} Q_1 &= Q_{1r} + iQ_{1i} = \left.\frac{d\theta_1}{dy}\right|_{y=0}, \quad \tan\alpha_1 = \frac{Q_{1i}}{Q_{1r}}, \\ Q_2 &= Q_{2r} + iQ_{2i} = \left.\frac{d\theta_2}{dy}\right|_{y=0}, \quad \tan\alpha_2 = \frac{Q_{2i}}{Q_{2r}}. \end{aligned} \tag{17}$$

Here q_m is the mean rate of heat transfer, $|Q_1|$, $|Q_2|$ are amplitudes of first and second harmonics of the rate of heat transfer, respectively, and $\tan\alpha_1$, $\tan\alpha_2$ are the phases of the first and second harmonics of the rate of heat transfer. The numerical values of q_m are given in Table 2. We observe from this table that an increase in ω leads to a decrease in q_m when $G > 0$, q_m increases with increasing h_1 or h_2 or γ. However, an increase in G or E leads to a decrease in q_m. When $G < 0$, an increase in ω, E, G or γ leads to an increase in q_m whereas an increase in h_1 or h_2 leads to a decrease in q_m.

In Table 3, the numerical values of $|Q_1|$, $|Q_2|$, $\tan\alpha_1$ and $\tan\alpha_2$ are entered. We observe from this table that when $G \gtrless 0$, an increase in ω, h_1, h_2 or γ leads to a decrease in $|Q_1|$ whereas an increase in G or E leads to an increase in $|Q_1|$. With increasing ω for both $G \gtrless 0$, $|Q_2|$ increases significantly. But, it is not significantly affected by other parameters for both $G \gtrless 0$.

There is always a phase lag in case of the first harmonic of the rate of heat transfer, and there is a phase lead in case of the second harmonic of the rate of heat transfer.

3. Conclusions

1) The mean velocity increases with the increase of ω or h_1 for both $G \gtrless 0$. 2) An increase in h_2 leads to a decrease in mean velocity, when $G > 0$ and an increase in mean velocity, when $G < 0$. 3) The mean velocity increases with increase of G or E. 4) An increase in γ leads to a decrease in the mean velocity when $G > 0$ and an increase in the mean velocity when $G < 0$. 5) The mean temperature increases with increasing ω when $G > 0$ and decreases with increasing ω when $G < 0$. 6) The mean temperature decreases with increasing h_2, γ for both $G \gtrless 0$ but it decreases with increasing h_1 when $G > 0$ and increases with h_2 when $G < 0$. 7) The mean temperature increases with increasing G or E when $G > 0$ and decreases with increasing G, E when $G < 0$. 8) The mean skin friction increases with increasing ω, G or E, but decreases with increasing h_1, h_2 or γ for $G > 0$. 9) There may occur separation when $G < 0$. 10) The amplitude of the skin friction increases with increasing ω or γ and decreases with increasing h_1 or h_2 for both $G \gtrless 0$. 11) An increase in ω, G or E leads to a decrease in q_m when $G > 0$ but q_m increases with increasing h_1 or h_2. 12) For $G < 0$, an increase in ω, E, G or γ leads to an increase in q_m but q_m decreases with increasing h_1 or h_2. 13) $|Q_1|$ decreases with increasing ω, h_1, h_2 or γ and increases with increasing G or E when $G \gtrless 0$. 14) $|Q_2|$ increases with increasing ω for both $G \gtrless 0$. 15) There is always a phase-lag in case of the first harmonic of the rate of heat transfer and phase-lead in case of the second harmonic of the rate of heat transfer.

Table 1 — Values of τ_m, $|B|$, $\tan\alpha$, for $P = 0.71$, $\varepsilon = 0.2$

γ	G	E	h_1	h_2	ω	τ_m	$\|B\|$	$\tan\alpha$
0.2	0.5	0.01	0.05	0.07	5.0	4.9914	1.1380	0.8127
0.2	0.5	0.01	0.05	0.07	75.0	4.9915	3.7675	0.4708
0.2	0.5	0.01	0.05	0.14	5.0	4.9643	1.1380	0.8132
0.2	0.5	0.01	0.10	0.07	5.0	4.9364	1.0892	0.7572
0.2	0.5	0.02	0.05	0.07	5.0	6.3330	1.1367	0.8093
0.2	0.7	0.01	0.05	0.07	5.0	8.6855	1.1368	0.8095
0.4	0.5	0.01	0.05	0.07	5.0	2.1312	1.2077	0.7161
0.2	—0.5	—0.01	0.05	0.07	5.0	—1.9573	1.1505	0.8193
0.2	—0.5	—0.01	0.05	0.07	5.0	—1.9573	3.7677	0.7409
0.2	—0.5	—0.01	0.05	0.14	75.0	—1.2933	1.1405	0.8197
0.2	—0.5	—0.01	0.10	0.07	5.0	—1.9396	1.0916	0.7630
0.2	—0.5	—0.02	0.05	0.07	5.0	—1.6079	1.1416	0.8223
0.2	—0.7	—0.01	0.05	0.07	5.0	—1.0679	1.1416	0.8223
0.4	—0.5	—0.01	0.05	0.07	5.0	—1.2593	1.2083	0.7187

Table 2 — Values of q_m, for $P = 0.71$, $\varepsilon = 0.2$

γ	G	E	h_1	h_2	ω	$q_m \times 10^{-3}$
0.2	0,5	0.01	0.05	0.07	5.0	3.37931698
0.2	0.5	0.01	0.05	0.07	75.0	3.16588804
0.2	0.5	0.01	0,05	0.14	5.0	5.83368932
0.2	0.5	0.01	0.10	0.07	5.0	6.13558398
0.2	0,5	0.02	0.05	0.07	5.0	—0.133843777
0.2	0.7	0.01	0.05	0.07	5.0	—0.119782489
0.4	0.5	0.01	0.05	0.07	5.0	0.255785881
0.2	0.5	0.01	0.05	0.07	5.0	0.249922853
0.2	—0.5	—0.01	0.05	0.07	75.0	0.250136282
0.2	—0.5	—0.01	0.05	0.07	5.0	0.245269032
0.2	—0,5	—0.01	0.10	0.14	5.0	0.247906648
0.2	—0.5	—0 02	0.05	0.07	5.0	0.359243294
0.2	—0.7	—0.01	0.05	0.07	5.0	0.361923598
0.4	—0.5	—0.01	0.05	0.07	5.0	0.287824532

Table 3 — Values of $|Q_1|$, $|Q_2|$ $\tan\alpha_1$, $\tan\alpha_2$, for $P = 0.71$, $\varepsilon = 0.2$

γ	G	E	h_1	h_2	ω	$\|Q_1\|$	$\|Q_2\| \times 10^{-3}$	$\tan\alpha_1$	$\tan\alpha_2$
0.2	0.5	0.01	0.05	0.07	5.0	0.0240	1.1731	—0.0258	0.6133
0.2	0.5	0,01	0.05	0.07	75.0	0.0197	2.8167	—0.2734	0.3058
0.2	0.5	0.01	0.05	0.14	5.0	0.0226	1.0949	—0.0658	0.5421
0.2	0.5	0.01	0.10	0.07	5.0	0.0227	1.0747	—0.0599	0.5221
0.2	0.5	0.02	0.05	0.07	5.0	0.0482	2.3462	—0.0258	0.6133
0.2	0.7	0.01	0.05	0.07	5.0	0.0330	1.1731	—0.0251	0.6133
0.4	0.5	0.01	0.05	0.07	5.0	0.0129	1.2788	—0.0085	0.4942
0.2	—0.5	—0.01	0.05	0.07	5.0	0.0212	1.1731	—0.0202	0·6133
0.2	—0.5	—0.01	0.05	0.07	75.0	0.0175	2.8167	—0.2719	0.3058
0.2	—0.5	—0.01	0.05	0.14	5.0	0.0200	1.0949	—0.0602	0.5421
0,2	—0.5	—0.01	0.10	0.07	5.0	0.0201	1.0747	—0.0543	0.5221
0.2	—0.5	—0.02	0.05	0.07	5.0	0.0425	2.3462	—0.0202	0·6133
0.2	—0.7	—0.01	0.05	0.07	5.0	0.0303	1.1731	—0.0211	0.6133
0.4	—0.5	—0.01	0.05	0.07	5.0	0.0076	1.278	0.0554	0.4942

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ЭФФЕКТЫ ВСАСЫВАНИЯ И СВОБОДНЫХ КОНВЕКЦИОННЫХ ПОТОКАХ НА КОЛЕБАНИЕ ТЕЧЕНИЯ РАЗРЕЖЕННОГО ГАЗА МИМО ВЕРТИКАЛЬНОЙ ПОРИСТОЙ ПЛАСТИНЫ

Резюме

Дано приблизительное решение спареных нелинейных уравнений, управляющих потоком разреженного газа мимо бесконечной вертикальной пористой пластины при следующих условиях: 1) при постоянной скорости всасывания, 2) при скорости первого порядка и пограничных условиях резкого изменения температуры, 3) при присутствии свободных конвекционных потоках и 4) при колебаниях свободного потока с постоянным ненулевым средним.

Профили средней скорости и температуры приведены на графиках, а численные значения T_m (среднее приповерхностное трение), q_m (средняя скорость теплопередачи) B (амплитуда приповерхностного трения), тан α (фаза приповерхностного трения), $|Q_1|$, $|Q_2|$ (амплитуды первой гармоники скорости теплопередачи), тан α_1 и тан α_2 (фазы первой и второй гармоники), приведены в таблицах. Средняя скорость возрастает с ω при $G \gtrless 0$ ($G > 0$ — при охлаждении пластины; $G < 0$ — при нагревании пластины). Средняя температура увеличивается с увеличением ω при $G > 0$ и уменьшается с увеличением ω при $G < 0$.

USISNI EFEKTI I SLOBODNA KONVEKTIVNA STRUJA U OSCILATORNOM TEČENJU RAZREĐENOG GASA KOJI PROLAZI KROZ BESKONAČNU VERTIKALNU POROZNU PLOČU

Izvod

U radu je dato približno rešenje spregnutih nelinearnih jednačina tečenja razređenog gasa koje se odvija pored vertikalne beskonačne porozne ploče. Rešenje ovog problema provedeno je pod uslovima konstantne brzine usisavanja, pri brzini prvog reda i graničnim uslovima nagle promene temperature, u prisustvu slobodne struje sa konstantnim nenultim usrednjenjem.

Profili srednje brzine i temperature prikazani su na grafiku, a brojne vrednosti: T_m (— srednje površinsko trenje), q_m (— srednja brzina toplotne provodnjivosti), B (— amplituda površinskog trenja), $\tan \alpha$ (— faza površinskog trenja), $|Q_1|$, $|Q_2|$ (— amplitude prvog i drugog harmonika brzine toplotne provodljivosti), $\tan \alpha_1$ i $\tan \alpha_2$ (— faze prvog i drugog harmonika) su date u tablicama. Srednja brzina raste sa ω pri $G \gtrless 0$ ($G > 0$ — pri hlađenju ploče; $G < 0$ — pri zagrevanju ploče). Srednja temperatura se povećava sa povećanjem ω za $G > 0$, a umanjuje se sa uvećanjem pri $G < 0$.

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