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UNSTEADY MASS TRANSFER AND FREE CONVECTION THROUGH A POROUS MEDIUM

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1. Introduction

The investigation of the free convection and mass transfer through a porous medium is useful in geophysics. Raptis et al. [1] have studied the steady free convection flow and mass transfer through a very porous medium bounded by an infinite vertical plate for the flow near the plate, by using the model of Yamamoto and Iwamura [2] for the generalized Darcy's law.

The purpose of the present work is to investigate the unsteady free convection flow and mass transfer through a very porous medium bounded by an infinite vertical porous plate with constant suction, when the temperature of the plate varies with the time about a non-zero constant and the temperature at the free-stream is constant. Also the species concentration on the plate and the free-stream are constant.

2. Analysis

We consider the steady two-dimensional motion of the incompressible viscous fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite plate. The x′-axis is taken along the vertical plate in the upward direction and the y′-axis taken normal to the plate. By using the usual Boussinesq's approximation and taking into account ref. [1] the problem is governed by the following equations:

$$\text{Continuity:} \quad \frac{\partial \upsilon'}{\partial y'} = 0, \tag{1}$$

$$\text{Momentum:} \quad \frac{\partial u'}{\partial t'} + \upsilon' \frac{\partial u'}{\partial y'} = g\beta (T' - T'_\infty) + g\beta^* (C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K'} u', \tag{2}$$

$$\text{Energy:} \quad \rho c_p \left(\frac{\partial T'}{\partial t'} + \upsilon' \frac{\partial T'}{\partial y'}\right) = k \frac{\partial^2 T'}{\partial y'^2}, \tag{3}$$

$$\text{Diffusion:} \quad \frac{\partial C'}{\partial t'} + \upsilon' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2}, \tag{4}$$

where u' and υ' are the velocity components in the x' and y'-directions, t' is the time T' the temperature of the fluid, C' the species concentration, T'_∞ the temperature away from the plate, C'_∞ the species concentration away from the plate, k the thermal conductivity, g the acceleration due to gravity, β the volumetric coefficient of thermal expansion, β^* the volumetric coefficient of expansion with concentration, ν the kinematic viscosity, K' the permeability of the porous medium, D the chemical molecular diffusivity, ρ the density and c_p the specific heat of the fluid at constant pressure.

The boundary conditions of the problem are:

$$\begin{aligned} &u' = 0, \quad \upsilon' = -\upsilon_0, \quad T' = T'_w + \varepsilon (T'_w - T'_\infty)\, e^{iw't'}, \quad C' = C'_w \quad \text{at } y' = 0, \\ &u' \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \qquad\qquad\qquad\qquad \text{at } y' \to \infty, \end{aligned} \tag{5}$$

where T'_w is the temperature of the plate, C'_w the species concentration of the plate, $\upsilon_0 > 0$ the suction velocity, ω' the frequency of oscillation of the plate temperature, ε a small quantity and $i = (\sqrt{-1}$ the imaginary quantity.

For constant suction, integrating Equation (1) we get in view of (5)

$$\upsilon' = -\upsilon_0 \tag{6}$$

where the negative sign indicates that the suction is towards the plate. Let us introduce the following dimensionless quantities

$$\begin{aligned} &y = \frac{y'\upsilon_0}{\nu}, \quad t = \frac{t'\upsilon_0^2}{\nu}, \quad \omega = \frac{\nu\omega'}{\upsilon_0^2}, \\ &u = \frac{u'}{\upsilon_0}, \quad T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\ &P = \frac{\rho\nu c_p}{k}, \quad Sc = \frac{\nu}{D}, \quad K = \frac{\upsilon_0^2}{\nu^2} K', \\ &G = \frac{\nu g \beta (T'_w - T'_\infty)}{\upsilon_0^3}, \quad G^* = \frac{\nu g \beta^* (C'_w - C'_\infty)}{\upsilon_0^3}, \end{aligned} \tag{7}$$

where P is the Prandtl number, Sc the Schmidt number, K the permeability parameter, G the Grashof number and G^* is the modified Grashof number. In terms of these variables Equations (2), (3) and (4), on taking into account (6), become

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = GT + G^* C + \frac{\partial^2 u}{\partial y^2} - \frac{1}{K} u, \tag{8}$$

$$P\left(\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial y^2}, \tag{9}$$

$$Sc\left(\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y}\right) = \frac{\partial^2 C}{\partial y^2}, \tag{10}$$

and

$$u = 0, \quad T = 1 + \varepsilon e^{i\omega t}, \quad C = 1 \quad \text{at} \quad y = 0,$$
$$u \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as} \quad y \to \infty. \tag{11}$$

To solve the Equations (8), (9) and (10) we represent the velocity, the temperatyre and the concentration in the neighbourhood of the plate in powers of small ε ($\ll$) as

$$u(y,t) = u_0(y) + \varepsilon\, e^{i\omega t}\, u_1(y) + \ldots,$$
$$T(y,t) = T_0(y) + \varepsilon\, e^{i\omega t}\, T_1(y) + \ldots, \tag{12}$$
$$C(y,t) = C_0(y) + \varepsilon\, e^{i\omega t}\, C_1(y) + \ldots.$$

Substituting (12) in Equations (8), (9) and (10) and equating harmonic and nonharmonic terms we take

$$u_0'' + u_0' - \frac{1}{K} u_0 = - G T_0 - G^* C_0, \tag{13}$$

$$u_1'' + u_1' - i\,\omega\, u_1 - \frac{1}{K} u_1 = - G T_1 - G^* C_1, \tag{14}$$

$$T_0'' + P T_0' = 0, \tag{15}$$

$$T_1'' + P T_1' - i\,\omega\, P T_1 = 0, \tag{16}$$

$$C_0'' + Sc C_0' = 0, \tag{17}$$

$$C_1'' + Sc C_1' - i\,\omega\, Sc C_1 = 0, \tag{18}$$

where the prime denote differentiation with respect to y. The appropriate boundary conditions are

$$u_0(0) = 0, \quad T_0(0) = 1, \quad C_0(0) = 1, \quad u_1(0) = 0, \quad T_1(0) = 1, \quad C_1(0) = 0,$$
$$u_0(\infty) = 0, \; T_0(\infty) = 0, \; C_0(\infty) = 0, \; u_1(\infty) = 0, \; T_1(\infty) = 0, \; C_1(\infty) = 0. \tag{19}$$

The solutions of Equations (13)-(18) subject to the boundary conditions (19) are the following

$$u_0 = L_1\, e^{-Py} + L_2\, e^{-Scy} + L_3\, e^{R_3 y}, \tag{20}$$

$$u_1 = L_4\, e^{R_1 y} + L_5\, e^{R_5 y}, \tag{21}$$

$$T_0 = e^{-Py}, \tag{22}$$

$$T_1 = e^{R_1 y}, \tag{23}$$

$$C_0 = e^{-Scy}, \tag{24}$$

$$C_1 = 0,$$

where

$$R_2 = \frac{-1 + \sqrt{1 + 4\frac{1}{K}}}{2}, \qquad R_1 = \frac{-P - \sqrt{P^2 + 4i\omega P}}{2},$$

$$R_4 = \frac{-1 + \sqrt{1 + 4\left(i\omega + \frac{1}{K}\right)}}{2}, \quad R_3 = \frac{-1 - \sqrt{1 + 4\frac{1}{K}}}{2}$$

$$L_1 = -\frac{G}{(P + R_2)(P + R_3)}, \qquad R_5 = \frac{-1 - \sqrt{1 + 4\left(i\omega + \frac{1}{K}\right)}}{2}.$$

$$L_3 = -(L_1 + L_2), \qquad L_2 = -\frac{G^*}{(Sc + R_2)(Sc + R_3)}$$

$$L_5 = -L_4, \qquad L_4 = -\frac{G}{(R_1 - R_4)(R_1 - R_5)}.$$

We can now express the velocity field in terms of the fluctuating parts as follows

$$u(y,t) = u_0(y) + \varepsilon(M_r \cos\omega t - M_i \sin\omega t), \tag{26}$$

where

$$M_r + iM_i = u_1. \tag{27}$$

Hence, the expression for the transient velocity for $\omega t = \frac{\pi}{2}$ is given by

$$u = u_0 - \varepsilon M_i. \tag{28}$$

The dimensionless velocity profiles for different values of Grashof number G, modified Grashof number G^* and the permeability parameter K are plotted in Fig. 1. where Prandtl number $P = 0.71$ (air), $\omega = 5$, $\omega t = \frac{\pi}{2}$ $\varepsilon = 0.2$, $Sc = 0.6$.

The value of Schmidt number, $Sc = 0.6$, was chosen in such a way to represent water-vapour at approximately 25°C and 1 atmosphere. It is evident (Fig. 1) that the velocity is increasing with an increase of the permeability parameter or the modified Grashof number. Also from the Table 1 we conclude that when the dimensionless frequency increases, the velocity decreases near the plate.

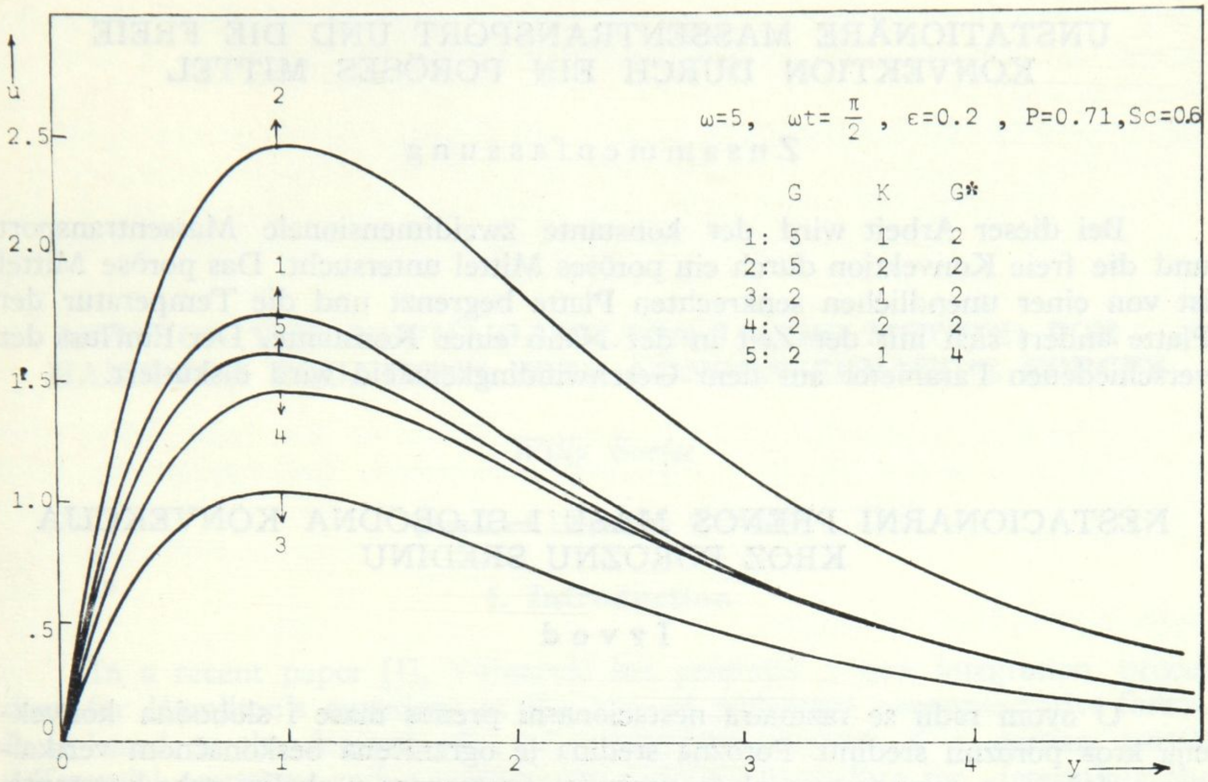

Fig. 1. Velocity profiles.

Table 1: Rates of transient velocity when $G = 5$, $\omega t = \frac{\pi}{2}$, $P = 0.71$, $Sc = 0.6$, $G^* = 2$, $\varepsilon = 0.2$

y	$\omega = 5$		$\omega = 10$	
	$K = 1$	$K = 2$	$K = 1$	$K = 2$
0.00	0.000	0.000	0.000	0.000
0.25	1.050	1.336	1.038	1.324
0.50	1.582	2.072	1.561	2.050
0.75	1.792	2.411	1.766	2.384
1.00	1.811	2.498	1.786	2.474
1.25	1.723	2.435	1.706	2.417
1.50	1.583	2.285	1.573	2.276
1.75	1.422	2.093	1.418	2.090
2.00	1.258	1.855	1.259	1.886
2.25	1.102	1.676	1.105	1.680
2.50	0.958	1.477	0.961	1.481
2.75	0.827	1.292	0.830	1.295
3.00	0.712	1.123	0.714	1.126

REFERENCES

[1] A. Raptis, G. Tzivanidis and N. Kafousias, *Free convection and mass transfer flow through a porous medium bounded by an infinite vertical limiting surface with constant suction,* Let. in Heat and Mass Transfer 8, 417-424 (1981).

[2] K. Yamamoto and N. Iwamura, *Flow with convective acceleration through a porous medium,* Journal of Eng. Mathematics 10, 41-54 (1976).

UNSTATIONÄRE MASSENTRANSPORT UND DIE FREIE KONVEKTION DURCH EIN PORÖSES MITTEL

Zusammenfassung

Bei dieser Arbeit wird der konstante zweidimensionale Massentransport und die freie Konvektion durch ein poröses Mittel untersucht. Das poröse Mittel ist von einer unendlichen senkrechten Platte begrenzt und die Temperatur der Platte ändert sich mit der Zeit in der Nähb einer Konstante. Der Einfluss der verschiedenen Parameter auf dem Geschwindingkeitsfeld wird diskutiert.

NESTACIONARNI PRENOS MASE I SLOBODNA KONVEKCIJA KROZ POROZNU SREDINU

Izvod

U ovom radu se razmatra nestacionarni prenos mase i slobodna konvekcija kroz poroznu sredinu. Porozna sredina je ograničena beskonačnom vertkalnom pločom, a temperatura ploče varira sa vremenom u okolini neke konstante. Diskutuje se uticaj radnih parametara na polje brzine.

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