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ON THE STABILITY OF THE SYSTEM WITH GENERALIZED POTENTIAL

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The paper [1] deals with the motion of mechanical systems subjected to the forces with generalized potential $V = V(q, \dot{q}, t)$. On the assumption of time-independence of the Lagrangian the stability of equilibrium and of steady motions of a system is investigated. In the paper [2] the following theorem was established.

If there exists a positive-definite function W of the generalized coordinates $q^1, q^2, \ldots, q^n$ and time t with the property that

$$\frac{\partial W}{\partial t} + \left(Q_\alpha + \frac{\partial W}{\partial q^\alpha}\right) \dot{q}^\alpha, \quad (\alpha = 1, \ldots, n), \tag{1}$$

is negative-valued or identically equal to zero, where the Q_α are the component of the generalized force, then the equilibrium of the holonomic system is stable

Knowing that the generalized forces obtained from a function

$$V = V_\alpha (q, t)\, \dot{q}^\alpha + \Pi (q, t) \tag{2}$$

by the prescription

$$Q_\alpha = \frac{d}{dt} \frac{\partial V}{\partial \dot{q}^\alpha} - \frac{\partial V}{\partial q^\alpha}, \tag{3}$$

where is $q = \{ q^1, \ldots, q^n\}$ and $\dot{q} = \{\dot{q}^1, \ldots, \dot{q}^n\}$, and using the expression (1), next stability conditions can be obtained:

1. If the natural potential $\Pi = \Pi(q, t)$ is a positive-definite function and $V(q,\dot{q},t) < V(q,\dot{q},t_0)$, the equilibrium of a system with the forces (3) is stable.

Proof: Let $W = \Pi(q^1, \ldots, q^n; t)$. Then from the expression (1) one can get

$$\frac{\partial \Pi}{\partial t} + \frac{\partial V_\alpha}{\partial q^\beta} \dot{q}^\alpha \dot{q}^\beta + \frac{\partial V_\alpha}{\partial t} \dot{q}^\alpha - \frac{\partial V_\beta}{\partial q^\alpha} \dot{q}^\alpha \dot{q}^\beta$$

and under the condition 1.,

$$\frac{\partial V}{\partial t} + \left(\frac{\partial V_\alpha}{\partial q^\beta} - \frac{\partial V_\beta}{\partial q^\alpha}\right) \dot{q}^\alpha \dot{q}^\beta \leqslant 0, \tag{4}$$

had is the assertion of the theorem.

If generalized potential $V = V(q,\dot{q})$ does not depend explicitly on time t, relation (4) become

$$\left(\frac{\partial V_\alpha}{\partial q^\beta} - \frac{\partial V_\beta}{\partial q^\alpha}\right) \dot{q}^\alpha \dot{q}^\beta = 0.$$

If Lagrangian of the mechanical system does not contain time t explicitely and if the potential $\Pi(q)$ is a positive — definite function of the coordinates $q^1, \ldots,$ q^n, the equilibrium of a system with generalized potential $V(q,\dot{q})$ is stable.

2. If the are $n - m$ $(m > n)$ cyclic coordinates $q^{m+1}, \ldots, q^n$ then expression (1) (see ref. [3]), can be written as

$$\frac{\partial V}{\partial t} + \left(\frac{\partial V_r}{\partial q^s} - \frac{\partial V_s}{\partial q^r}\right) \dot{q}^r \dot{q}^s \tag{5}$$

where q^s $(s,r = 1, \ldots, m)$ are the position (non cyclic) coordinates, or

$$\left(\frac{\partial V_r}{\partial q^s} - \frac{\partial V_s}{\partial q^r}\right) \dot{q}^r \dot{q}^s \tag{6}$$

when potential V does not contain time t explicitely.

In fact, the function W will be again the natural potential $\Pi = \Pi(q^1, \ldots, q^n,$ $t) > 0$. The generalized forces $Q\alpha$ $(\alpha = 1, \ldots, n)$ for a cyclic coordinates, reduce to

$$Q_\nu = \frac{d}{dt}\frac{\partial V}{\partial \dot{q}^\nu} = \frac{\partial V_\nu}{\partial q^r}\dot{q}^r \qquad (r = 1, \ldots, m;\ \nu + 1, \ldots, n)$$

and for positon coordinates, to

$$Q_r = \frac{d}{dt}\frac{\partial V}{\partial \dot{q}^r} - \frac{\partial V}{\partial q^r} = \frac{dV_r}{dt} - \frac{\partial V_\alpha}{\partial q^r}\dot{q}^\alpha - \frac{\partial \Pi}{\partial q^r},$$

where

$$\frac{dV_r}{dt} = \frac{\partial V_r}{\partial q^s}\dot{q}^s + \frac{\partial V_r}{\partial t},$$

and

$$\frac{\partial V_\alpha}{\partial q^r}\dot{q}^\alpha = \frac{\partial V_s}{\partial q^r}\dot{q}^s + \frac{\partial V_\nu}{\partial q^r}\dot{q}^\nu.$$

If the expressions for the coordinates Q_ν and Q_r of the generalized force mentioned above, and for $W = \Pi$ are substituted in expression (1), one can obtain (5) and (6) when function $V = V(q,\dot{q})$ does not explicitly depend on time t.

Under these conditions the steady motion of a mechanical system is stable.

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ОБ УСТОЙЧИВОСТИ СИСТЕМ С ОБОБЩЕННЫМ ПОТЕНЦИАЛОМ

Р е з ю м е

Рассматривется устоичивость равновесия и стационарных движений механических систем находящихся под действием сил, обладающим обощенным потенциалом $V = V_\alpha(q,t)\,\dot{q}^\alpha + \Pi(q,t)$

Исходя из критерия (1) показывается что если потенциал Π определенно-положительная функция и $V(q,\dot{q},t) \leqslant (q,\dot{q},t_0)$ то положение равновесия устойчиво.

Показывается также что если функция Лагранжа $L = T - V(q,\dot{q})$ не зависиот явно от времени и потенциал Π определенно — положительная функция, то положение равновесия и, при наличий циклических координат стационарные движения, рассматриваемой системы устойчиво.

O STABILNOSTI SISTEMA SA UOPŠTENIM POTENCIJALOM

I z v o d

Razmatra se stabilnost ravnotežnog stanja i stacionarnih kretanja mehaničkih sistema koji se nalaze pod dejstvom sila koje imaju uopšteni potencijal $V = V_\alpha(q,t)\,\dot{q}^\alpha + \Pi(q,t)$.

Polazeći od kriterijuma (1), pokazuje se kada je potencijal Π pozitivno defifinitna funkcija i kada $V(q,\dot{q},t) \leqslant V(q,\dot{q},t_0)$, da je položaj ravnoteže stabilan.

Takođe se pokazuje da kada funkcija Lagrange-a $L = T - V(q,\dot{q})$ eksplicitno ne zavisi od vremena i kada je potencijal pozitivno definitna funkcija, položaj ravnoteže, a takođe i stacionarno kretanje za koje postoje ciklične koordinate su stabilni.

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