

TO THE DESCRIPTION OF NON-ISOTHERMIC ELASTIC-PLASTIC DEFORMATION

Natalija Naerlović-Veljković

(Received April 20, 1981)

1. Introduction

In the most of papers dealing with the analysis of elastic-plastic behaviour of material, the contribution of heat flux to the evolution of the anelastic process remained out of view. On the other side, in a number of recent papers from the domain of thermoelasticity, there was pointed at the role of the heat flux in describing finite wave speeds thermoelastic materials. It is also well known that the thermal state must be taken into consideration in describing the behaviour of material in the neighbourhood of a crack. Hence, there exists a reason to investigate the contribution of the heat flux in the evolution of mechanically dissipative processes.

The thermodynamic process in the body B is determined by equations of motion of its particles X_K :

$$x_i = x_i(\underline{X}, t), \quad (1.1)$$

by the field of temperature $T = T(\underline{X}, t)$, mass density $\rho = \rho(\underline{X}, t)$, heat flux $\underline{q} = \underline{q}(\underline{X}, t)$, Piola-Kirchhoff stress tensor $t_{IJ} = t_{JI}$, specific internal energy u and two sets of internal state variables represented by the m -vector α_μ ($\mu = 1, \dots, m$) and n -vector β_ν ($\nu = 1, \dots, n$). The internal state variables are describing the viscous (α) and plastic (β) aspects of material behaviour respectively. They thus determine the dependence of the actual process in the material on its thermo-mechanical history through the present values of variables $\underline{\alpha}$ and $\underline{\beta}$. The internal variables $\underline{\alpha}$ and $\underline{\beta}$ may be scalars or components of tensors, hence being unchanged under rigid-body motion [1]. Thereby, the evolution equation for the set $\underline{\alpha}$ is supposed to be continuous in t , for all $t \geq t_0$ (t_0 belongs to the reference state), while $\underline{\beta}$, being dependent on the sign of the loading index Φ , disappears for $\Phi \leq 0$.

2. Kinematical consideration

The deformation of a volume element of B at time t is determined by deformation gradients $x_{i,K}$ with corresponding deformation tensors:

$$C_{KL} = x_{i,K} x_{i,L}; \quad E_{KL} = \frac{1}{2} (C_{KL} - \delta_{KL}). \quad (2.1)$$

Whenever the process of deformation is not purely elastic, it becomes possible to think about the decomposition of the total strain on its elastic and additional plastic parts, introducing the notion of a fictitious relaxed intermediate configuration of the material. The deformation of volume elements in this intermediate configuration is in general incompatible and can be expressed in terms of elastic or additional plastic distortions, respectively. It is by means of these distortions that we can establish the local correspondence between the reference and relaxed and relaxed and current deformed configuration at the time t . Let us denote by $u^{(\lambda)}$, the local Cartesian system of reference at the particle X_K . The local correspondence between reference and relaxed or relaxed and current configurations may be expressed through the double relation [2]:

$$\theta_K^{(\lambda)} dX_K = du^{(\lambda)} = \Psi_i^{(\lambda)} dx_i, \quad (2.2)$$

where $\theta_K^{(\lambda)}$ and $\Psi_i^{(\lambda)}$ stand for elastic and additional plastic distortions respectively. In terms of these distortions, the elastic and additional plastic deformation tensors may be represented in the form:

$$2 E_{KL}^{(e)} = \theta_K^{(\lambda)} \theta_L^{(\lambda)} - \delta_{KL}, \quad 2 E_{KL}^{(p)} = C_{KL} - \theta_L^{(\lambda)} \theta_P^{(\lambda)}. \quad (2.3)$$

3. Thermodynamic analysis

The equilibrium state of the body may be described through the coordinates of deformation tensor (2.1) and temperature, as independent variables, in accordance with law of balance of mass, momentum and energy. For the description of a thermomechanical process, additional variables are necessary, consisting of the heat flux vector, gradients of temperature and internal vectors $\underline{\alpha}$ and $\underline{\beta}$:

$$\begin{aligned} E_{KL} &= \frac{1}{2} (x_{i,K} x_{i,L} - \delta_{KL}), \quad T, \\ G_K &= T_{,i} x_{i,K}, \\ Q_K &= q_i x_{i,K} \\ \alpha_\mu &= \{\alpha_1, \dots, \alpha_m\}, \quad \beta_\nu = \{\beta_1, \dots, \beta_n\}. \end{aligned} \quad (3.1)$$

The rate of change of the heat flux and the internal m -vector $\underline{\alpha}$ occur in accordance with the following equations of evolution:

$$\dot{Q}_K = F_K (\underline{E}, T, \underline{G}, \underline{Q}, \underline{\alpha}, \underline{\beta}), \quad \dot{\alpha}_\mu = f_\mu (\underline{E}, T, \underline{G}, \underline{Q}, \underline{\alpha}, \underline{\beta}). \quad (3.2)$$

The process of the plastic deformation is governed by the loading index Φ , vanishing for $\Phi \leq 0$. The loading index represents a linear function of the rate of change of independent variables of the preceding process:

$$\Phi = \underline{A} \dot{\underline{E}} + a_1 \dot{T} + \underline{a}_2 \dot{\underline{G}} + \underline{a}_3 \dot{\underline{Q}} + \underline{a}_4 \dot{\underline{\alpha}}, \quad (3.3)$$

with tensor \underline{A} , scalar a_1 , vectors \underline{a}_2 and \underline{a}_3 and m -vector \underline{a}_4 , which all depend on the set of variables (3.1). The evolution law for $\underline{\beta}$ may be written as:

$$\dot{\beta}_\nu = r_\nu \langle \Phi \rangle \quad (3.4)$$

where $\underline{r} = \underline{r} (\underline{E}, T, \underline{G}, \underline{Q}, \underline{\alpha}, \underline{\beta})$, $\langle \Phi \rangle = \Phi H(\Phi)$ and $H(\Phi)$ being the Heaviside step function of the loading index.

Elementary processes in an elastic-plastic body may be considered as a sequence of equilibrium states, until the rate of deformation remained moderate. Under this restriction, we assume the existence of the entropy η per unit mass of the body and the specific free energy $\psi = u - \eta T$. The thermodynamic admissibility of processes is connected with their consistency with the Second law, expressing the statement that in every neighbourhood of a given state there exist states which cannot be reached from the given state by an adiabatic or isentropic transition. Going out from the local form of the Second law:

$$-\rho(\dot{\psi} + \eta \dot{T}) + \underline{t} \cdot \underline{\dot{E}} + \frac{\underline{Q} \cdot \underline{G}}{T} \geq 0, \quad (3.5)$$

with $\psi = \psi(\underline{E}, T, \underline{G}, \underline{Q}, \underline{\alpha}, \underline{\beta})$, following restrictions may be obtained:

$$\underline{t} = \rho \frac{\partial \psi}{\partial \underline{E}}, \quad \eta = -\frac{\partial \psi}{\partial T}, \quad 0 = \frac{\partial \psi}{\partial \underline{G}} \quad (3.6)$$

and

$$-\rho \left(\frac{\partial \psi}{\partial \underline{Q}} F + \frac{\partial \psi}{\partial \underline{\alpha}} \underline{f} + \frac{\partial \psi}{\partial \underline{\beta}} \underline{r} \langle \Phi \rangle \right) + \frac{\underline{Q} \cdot \underline{G}}{T} \geq 0. \quad (3.7)$$

If we replace the free energy by the free enthalpy $\zeta = \zeta(\underline{t}, T, \underline{Q}, \underline{\alpha}, \underline{\beta})$

$$\rho \zeta = \underline{t} \cdot \underline{E} - \rho \psi, \quad (3.8)$$

alternative relations, corresponding to Equ. (3.6)–(3.7) may be obtained:

$$\underline{E} = \rho \frac{\partial \zeta}{\partial \underline{t}}, \quad \eta = \frac{\partial \zeta}{\partial T} \quad (3.9)$$

and

$$\rho \left(\frac{\partial \zeta}{\partial \underline{Q}} F + \frac{\partial \zeta}{\partial \underline{\alpha}} \underline{f} + \frac{\partial \zeta}{\partial \underline{\beta}} \underline{r} \langle \Phi \rangle \right) + \frac{\underline{Q} \cdot \underline{G}}{T} \geq 0. \quad (3.10)$$

4. Discussion

In the most papers dealing with elastic-plastic deformation, f.i. [3], [4], the assumption, that thermoelastic moduli are independent of plastic deformation, is built in the theory. Here we introduce an analogous assumption about independence of thermoelastic moduli on dissipative variables \underline{Q} , $\underline{\alpha}$ and $\underline{\beta}$. Hence, the validity of following relations is here assumed:

$$\frac{\partial^2 \underline{E}}{\partial \underline{t} \partial \underline{Q}} = 0, \quad \frac{\partial^2 \underline{E}}{\partial \underline{t} \partial \underline{\alpha}} = 0, \quad \frac{\partial^2 \underline{E}}{\partial \underline{t} \partial \underline{\beta}} = 0 \quad (4.1)$$

$$\frac{\partial^2 \underline{E}}{\partial T \partial \underline{Q}} = 0, \quad \frac{\partial^2 \underline{E}}{\partial T \partial \underline{\alpha}} = 0, \quad \frac{\partial^2 \underline{E}}{\partial T \partial \underline{\beta}} = 0. \quad (4.2)$$

The effect of these assumptions consists in splitting of the total strain, Equ. (2.3) in the following way:

$$\underline{E} = \underline{E}^{(e)}(\underline{t}, T) + \underline{E}^{(p)}(\underline{Q}, \underline{\alpha}, \underline{\beta}), \quad (4.3)$$

with corresponding expression for the free enthalpy:

$$\zeta(\underline{t}, T, \underline{Q}, \underline{\alpha}, \underline{\beta}) = \zeta_1(\underline{t}, T) + \underline{t} \cdot \underline{E}^{(p)} + \zeta_2(T, \underline{Q}, \underline{\alpha}, \underline{\beta}). \quad (4.4)$$

In addition to (4.1)–(4.2) we require that the specific heat at constant stress, defined by:

$$c = T \frac{\partial^2 \zeta}{\partial T^2} \quad (4.5)$$

would be independent of \underline{Q} , $\underline{\alpha}$ and $\underline{\beta}$. In such a case the third term in Equ. (4.4) may be, at the most, linear in T :

$$\zeta_2(T, \underline{Q}, \underline{\alpha}, \underline{\beta}) \rightarrow T\eta_D(\underline{Q}, \underline{\alpha}, \underline{\beta}) - u_D(\underline{Q}, \underline{\alpha}, \underline{\beta}). \quad (4.6)$$

Following Lubliner [4], who was, on the other hand, referring to Cottrell, we identify η_D as the configurational entropy. In the present case this quantity is connected with the phenomenon of the plastic deformation, its history and with the process of heat conduction. The free enthalpy takes, hence, the form:

$$\zeta(\underline{t}, T, \underline{Q}, \underline{\alpha}, \underline{\beta}) = \zeta_1(\underline{t}, T) + \underline{t} \cdot \underline{E}^{(p)} + T\eta_D - u_D, \quad (4.7)$$

undergoing the restriction (3.10):

$$\underline{t} \cdot \dot{\underline{E}}^{(p)} + T\dot{\eta}_D - \dot{u}_D + \frac{\underline{Q} \cdot \underline{G}}{T} \geq 0. \quad (4.8)$$

In the absence of temperature gradients inequality (4.8) must still be satisfied, hence:

$$\underline{t} \cdot \dot{\underline{E}}^{(p)} + T\dot{\eta}_D - \dot{u}_D \geq 0, \quad (4.9)$$

where

$$\dot{\underline{E}}^{(p)} = \frac{\partial \underline{E}^{(p)}}{\partial \underline{Q}} \underline{F} + \frac{\partial \underline{E}^{(p)}}{\partial \underline{\alpha}} \underline{f} + \frac{\partial \underline{E}^{(p)}}{\partial \underline{\beta}} \underline{r} < \Phi >, \quad (4.10)$$

etc. But, when $\Phi \leq 0$, inequality (4.9) reduces to:

$$\left(\underline{t} \cdot \frac{\partial \underline{E}^{(p)}}{\partial \underline{Q}} + T \frac{\partial \eta_D}{\partial \underline{Q}} - \frac{\partial u_D}{\partial \underline{Q}} \right) \cdot \underline{F} + \left(\underline{t} \cdot \frac{\partial \underline{E}^{(p)}}{\partial \underline{\alpha}} + T \frac{\partial \eta_D}{\partial \underline{\alpha}} - \frac{\partial u_D}{\partial \underline{\alpha}} \right) \cdot \underline{f} \geq 0. \quad (4.11)$$

Now we resolve the rate of plastic deformations in two parts:

$$\dot{\underline{E}}^{(p)} = \dot{\underline{E}}^{(p)'} + \dot{\underline{E}}^{(p)''} \quad (4.12)$$

$$\dot{\underline{E}}^{(p)'} \equiv \frac{\partial \underline{E}^{(p)}}{\partial \underline{Q}} \underline{F} + \frac{\partial \underline{E}^{(p)}}{\partial \underline{\alpha}} \underline{f}, \quad \dot{\underline{E}}^{(p)''} \equiv \frac{\partial \underline{E}^{(p)}}{\partial \underline{\beta}} \underline{r} < \Phi >.$$

The inequality (4.9) takes now the form:

$$\underline{t} \cdot \dot{\underline{E}}^{(p)'} + \underline{t} \cdot \dot{\underline{E}}^{(p)''} + T\dot{\eta}_D - \dot{u}_D \geq 0. \quad (4.13)$$

This inequality may be satisfied even if,

$$\dot{u}_D - T\dot{\eta}_D - \underline{t} \cdot \dot{\underline{E}}^{(p)'} \leq \underline{t} \cdot \dot{\underline{E}}^{(p)''} < 0. \quad (4.14)$$

This last expression represents the thermodynamic ground for the explanation of large Bauschinger effect during a dissipative process followed by memory effect and heat conduction, in absence of temperature gradients.

It may be noted that the preceding discussion does not exclude the existence of an additional relation determining, in the space of variables (3.1), a domain which cannot be left [5], [6]:

$$Y(\underline{E}, T, \underline{G}, \underline{Q}, \alpha, \beta) \leq 0. \quad (4.15)$$

The function Y represents the yield condition. In most theories, the loading index is given through the yield condition in the form:

$$\Phi = \frac{\partial Y}{\partial \underline{E}} \dot{\underline{E}} + \frac{\partial Y}{\partial T} \dot{T} + \frac{\partial Y}{\partial \underline{G}} \dot{\underline{G}} + \frac{\partial Y}{\partial \underline{Q}} \dot{\underline{Q}} + \frac{\partial Y}{\partial \alpha} \dot{\alpha}, \quad (4.16)$$

being a special case of the expression (3.3).

Acknowledgement. This paper is a part of the Research project which has been supported by the Office for Scientific Research of Serbia.

R E F E R E N C E S

- [1] B. D. Coleman, M. E. Gurtin: *Thermodynamics with Internal state variables*. The Journal of Chemical Physics, **47**, 2, 597—613 (1967).
- [2] N. Naerlović-Veljković, R. Stojanović, L. Vujošević *Primena opšte teorije inkompatibilnih deformacija na termoelastičnost*, Tehnika, 1, 9—12, Beograd 1969.
- [3] Th. Lehmann *On the theory of large, non-isothermic, elastic-plastic and elastic-viscoplastic deformations*, Arch. Mech. Stosow. 29, 3, 393—409, 1977.
- [4] L. Lubliner *A simple theory of plasticity*, Int. J. Solids Structures, 10, 3, 313—319, 1974.
- [5] M. A. Eisenberg, Chong-Won Lee and A. Phillips *Observations on the theoretical and experimental foundations of thermoplasticity*. Int. J. Solids Structures, 13, 12, 1239—1255, 1977.
- [6] F. Sidoroff *On the formulation of plasticity and viscoplasticity with internal variables*, Arch. Mech. Stosow. 27, 5—6, 807—819, 1975.

PRILOG OPISU NEIZOTERMIČKE ELASTOPLASTIČNE DEFORMACIJE

I z v o d

U radu se posmatra elastoplastični proces deformacije u kome ulogu nezavisno promenljivih igraju, pored tenzora deformacije i temperature, još i toplotni fluks i dve grupe unutrašnjih promenljivih stanja. Brzina promene ove druge grupe unutrašnjih promenljivih stanja uslovljena je znakom tzv. indeksa opterećenja i iščezava, kad god ovaj poslednji nije pozitivan. Inače, brzine promene toplotnog fluksa, kao i obe grupe unutrašnjih promenljivih stanja, upravljaju se u skladu sa pretpostavljenim zakonima evolucije. Sa tako usvojenim modelom materijala izvedena su termodinamička ograničenja koja slede iz Drugog zakona termodinamike i sprovedena diskusija. Tom prilikom je uvedena, umesto specifične slobodne energije, funkcija specifične slobodne entalpije, čiji je oblik usklađen sa pretpostavkom da su termoelastični moduli materijala nezavisni od disipativnih promenljivih u koje su ubrojani: toplotni fluks, skup viskoznih i skup plastičnih unutraš-

njih promenljivih. Pokazana je termodinamička zasnovanost pojave Bauschinger-ovog efekta, pri čemu negativan efekt rada na plastičnoj deformaciji pokriva pozitivan doprinos vezan za uticaj konfiguracione entropije, veličine koja potiče od plastične deformacije kao i njene istorije i od procesa provođenja toplote i u odsustvu temperaturnih gradijenata.

ПРИЛОЖЕНИЕ ОПИСАНИЮ НЕИЗОТЕРМИЧЕСКОЙ УПРУГО-ПЛАСТИЧЕСКОЙ ДЕФОРМАЦИИ

Р е з ю м е

Работа заключается в исследовании следствий термодинамических ограничений относящихся к процессу упруго-пластической деформации. К числу внутренних переменных вязкого и пластического типа добавляется, в качестве внутреннего вектора, вектор теплового потока, с соответствующим эволюционным уравнением. Из диссипационного неравенства, выраженного через функцию удельной свободной энтальпии, выводится заключение о термодинамической обоснованности эффекта Баушингера.

Natalija Naerlović - Veljković
Faculty of Civil Engineering, University of Belgrade
11000 Belgrade, Bul. Revolucije 73, Yugoslavia