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HALL EFFECTS ON THE HYDROMAGNETIC FLOW PAST AN ACCELERATED PLATE

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1. Introduction

The problem considered in this paper is that of the effects of Hall currents on the unsteady hydromagnetic flow past an infinite flat plate when a uniform magnetic field acts in a plane which makes an angle θ with the plane transverse along to the plate. Recent studies [1 — 4] on the hydromagnetic flows with Hall currents are mainly focused upon those where the magnetic field is imposed normal to the plate.

In an ionised gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiralling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both electric and magnetic fields. This phenomenon, well-known in the literature, is called the Hall effect. The study of magnetohydronamic viscous flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics.

2. Equations of motion

We use rectangular Cartesian coordinates. The x-axis is chosen along the plate and the plate executes the accelerate motion in the direction of x-axis. Ths y-axis is perpendicular to the plate and the fluid in contact with the plate occupiee the region $y \geqslant 0$. The leading edge of the plate should be taken as coincident with the z-axis. The uniform magnetic field $\vec{B}_0$ is imposed in a direction parallel to the plane which makes an angle θ with the xy-plane (i.e. the plane $z = 0$). The flow configuration, together with the coordinate system used, is shown in Fig. 1. The effect of Hall currents give rise to a force whose y-component is balanced pressure gradient, but the z-component causes the flow in that direction. If the magnetic Reynolds number is small, the induced magnetic field can be neglected in comparison with the applied magnetic field $\vec{B}_0$, which is assumed to be constant in space and time. In addition, we assume that the fluid is isotropic and homogeneous and

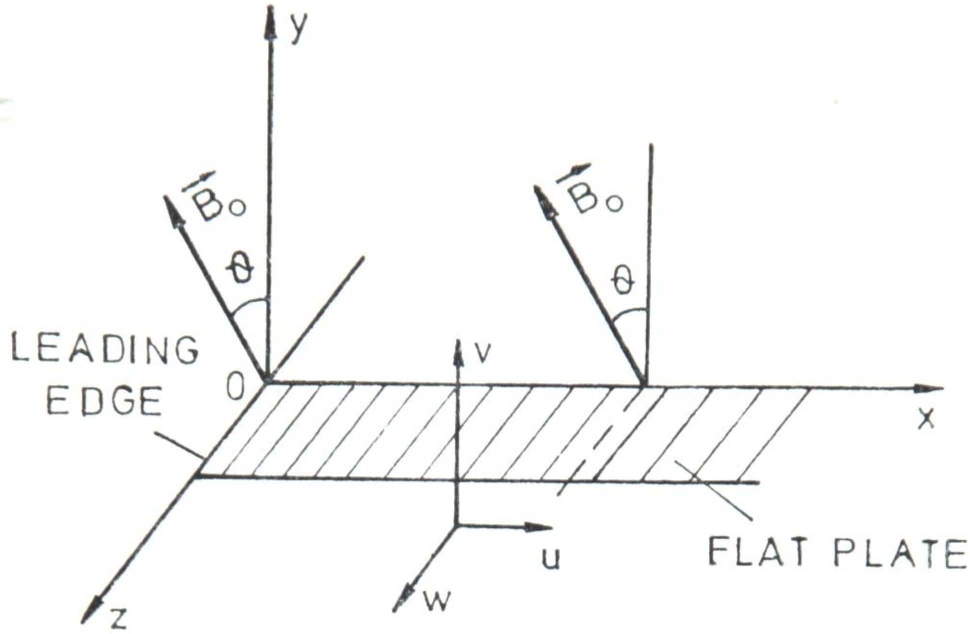

Fig. 1. The flow configuration with the coordinate system used.

has the scalar constant viscosity and electric conductivity. The equations of continuity and momentum along with Maxwell's equations for the unsteady flow of a viscous incompressible and electrically conducting fluid are

$$\nabla \cdot \vec{V} = 0, \tag{1}$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\, \vec{V} = -\frac{1}{\rho}\, \nabla q + \nu \nabla^2 \vec{V} + (1/\rho)\, \vec{J} \times \vec{B} \tag{2}$$

$$\nabla \times \vec{H} = \vec{J}, \quad \nabla \times \vec{E} = 0, \quad \nabla \cdot \vec{B} = 0, \tag{3)—(5}$$

where ρ, ν, t, p, $\vec{V}$, $\vec{B}$, $\vec{H}$, $\vec{J}$ and $\vec{E}$ are the density, the kinematic viscosity, the time, the fluid pressure, the fluid velocity, the magnetic induction, the magnetic field, the electric current density and the electric field, respectively.

Further, the generalized Ohm's law, taking Hall effects into account is

$$\vec{J} = \underline{\sigma} (\vec{E} + \vec{V} \times \vec{B} - \frac{1}{e\, n_e}\, \vec{J} \times \vec{B} + \frac{1}{e\, n_e}\, \underline{\nabla}\, p_e. \tag{6}$$

where $\sigma = e^2 n_e \tau / m_e$ and e, n_e, τ, m_e and p_e are the electric charge, the number density of electrons, the electron collision time, the mass of an electron and the electron pressure, respectively.

Since the plate is infinite in length, all the variables are functions of y and t only. Then we assume $\vec{V} = (u(y, t), o, w(y, t))$, $\vec{B}_0 = (o, B_0 \lambda, B_0 \sqrt{1 - \lambda^2})$, $\vec{E} = (E_x, E_y, E_z)$, $J = (J_x, J_y, J_z)$ where $\lambda = \cos \theta$. By applying the usual boundary layer approximation and considering the fluid to be partially ionized, in which the electron pressure is negligible, the basic equations (1) to (6) lead to

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0}{\rho (1+\omega^2 \tau^2)} \left[u B_0 - E_y \sqrt{1-\lambda^2} + E_z \lambda - \omega \tau (E_x - w B_0 \lambda) \right],$$

$$0 = -\frac{1}{\rho} \frac{dp}{dy} - \frac{\sigma B_0}{\rho (1+\omega^2 \tau^2)} \sqrt{1-\lambda^2} \left[E_x - w B_0 \lambda + \omega \tau (u B_0 - \right.$$

$$\left. - E_y \sqrt{1-\lambda^2} + E_z \lambda) \right], \tag{7}$$

$$\frac{\partial w}{\partial t} = \nu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0}{\rho (1+\omega^2 \tau^2)} \lambda \left[E_x - w B_0 \lambda + \omega \tau (u B_0 - Ey \sqrt{1-\lambda^2} + E_z \lambda) \right],$$

where ω is the cyclotron frequency of electrons. The second equation of (7) shows that the magnetic force in y-direction is balanced by the pressure gradient, hence there is no flow in that direction.

The boundary conditions of the problem are

$t \leqslant 0$: $u = w = 0$ everywhere,

$t > 0$:

$$u = A t^\alpha, \quad w = 0 \qquad \text{at } y = 0,$$

$$u \to 0, \quad w \to 0 \qquad \text{as } y \to \infty,$$

where $\alpha \geqslant 0$ and A is a constant.

We now consider further the case of a short circuit problem in which the applied electric field $\vec{E} = 0$, i.e. the magnetic field is fixed relative to the fluid. Also the induced electric field is negligible. Under these assumptions, the equations (7) become

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{(1+\omega^2 \tau^2)} (u + \omega \tau \lambda w),$$

$$\frac{\partial w}{\partial t} = \nu \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0^2}{\rho (1+\omega^2 \tau^2)} \lambda (w \lambda - \omega \tau u), \tag{9}$$

subject to the boundary conditions (8).

Solutions of equations (9) are sought by expanding $u(y, t)$ and $w(y, t)$ in series of small powers of (mt) as

$$u = A t^\alpha \sum_{i=1}^{\infty} (mt)^i u_i (\eta), \quad w = A t^\alpha \sum_{i=1}^{\infty} (mt)^i w_i (\eta), \tag{10}$$

where $\eta = y/2\sqrt{\nu t}$ and $m = \sigma B_0^2/\rho$. Substituting (10) in (9) and equating the coefficients of like powers of (mt), we obtain a set of linear differential eγuations of the form

$$u_0'' + 2\eta u_0' - 4 \alpha u_0 = 0,$$

$$u_1'' + 2\eta\ u_1' - 4(\alpha+1)\ u_1 = \frac{4}{1+\omega^2\ \tau^2}\ u_0,$$

$$w_1'' + 2\eta\ w_1' - 4(\alpha+1)\ w_1 = -\frac{4\lambda\ \omega\ \tau}{1+\omega^2\ \tau^2} u_0, \tag{11}$$

$$u_2'' + 2\eta\ u_2' - 4\ (\alpha+2)\ u_2 = \frac{4}{1+\omega^2\ \tau^2}\ (u_1 + \lambda\omega\ \tau\ w_1),$$

$$w_2'' + 2\eta\ w_2' - 4(\alpha+2)\ w_2 = \frac{4\lambda}{1+\omega^2\ \tau^2}\ (\lambda\ w_1 - \omega\ \tau\ u_1),$$

etc. anticipating the result $w_0 = 0$. The dashes denote the differentiation with respect to η. The boundary conditions (8) reduce to

$$u_0 = 1,\quad u_i = w_i = 0 \quad \text{at}\ \eta = 0,$$

$$u_0 \to 0,\quad u_i,\quad w_i \to 0 \quad \text{as}\ \eta \to \infty,\quad \text{for}\ i \geqslant 1. \tag{12}$$

The solution of (11) satisfying the boundary conditions (12) is given by

$$u_0 = 2^{2\alpha}\ \Gamma(\alpha+1)\ g_\alpha(\eta),$$

$$u_1 = \frac{1}{1+\omega^3\ \tau^2}\left[2^2(\alpha+1)\ \Gamma(\alpha+2)\ g_{\alpha+1}(\eta) - 2^{2\alpha}\ \Gamma(\alpha+1)\ g_\alpha\ (\eta)\right],$$

$$w_1 = \frac{\lambda\omega\tau}{1+\omega^2\ \tau^2}\left[2^{2\alpha}\ \Gamma(\alpha+1)\ g_\alpha(\eta) - 2^{2(\alpha+1)}\ \Gamma(\alpha+2)\ g_{\alpha+1}\ (\eta)\right],$$

$$u_2 = \frac{1-\lambda^2\ \omega^2\ \tau^2}{(1+\omega^2\ \tau^2)^2}\left[2^{2\alpha-1}\ \Gamma(\alpha+1)\ g_\alpha\ (\eta) - 2^{2(\alpha+1}\ \Gamma(\alpha+2)\ g_{\alpha+1}(\eta) + \right.$$

$$\left. + 2^{2\alpha+3}\ \Gamma(\alpha+3)\ g_{\alpha+2}\ (\eta)\right],$$

$$w_2 = \frac{\lambda\omega\ \tau\ (1+\lambda^2)}{(1+\omega^2\ \tau^2)^2}\left[2^{2(\alpha+1)}\ \Gamma(\alpha+2)\ g_{\alpha+1}(\eta) - 2^{2\alpha-1}\ \Gamma(\alpha+1)\ g_\alpha(\eta) - \right.$$

$$\left. - 2^{2\ \alpha+3}\ \Gamma(\alpha+3)\ g_{\alpha+2}\ (\eta)\right],$$

where

$$g_\alpha\ (\alpha) - \frac{2}{\sqrt{\pi}\ \Gamma(2\ \alpha+1)} \int_\eta^\infty (s-\eta)^2\ \exp\ (-s^2)\ ds,$$

is the Gauss' error function.

3. Results

We have presented the non-dimensional velocity components $u/A\,t^{\alpha}$ and $w/A\,t^{\alpha}$ for an impulsive flow ($\alpha = 0$) in Figs. 2 and 3 for several values of the Hall parameter $\omega\tau$ with $(mt) = 0.5$. In Fig. 2 the magnetic field is applied transverse to the plate ($\lambda = 1$) whereas in Fig. 3 it makes an angle of 30^0 ($\lambda = 1/2$) with the plate. We observe from these figures that the primary velocity u increases with increase in $\omega\tau$. As the distance from the plate increases, w first increases, reaches a maximum and then asymptotically decays to zero at a certain distance from the plate. Further, the cross-flow w forms a maximum profile for a particular value of $\omega\tau$. It is interesting to note from (13) and Figs. 2 and 3 that for constant values of (mt) and $\omega\tau$, the primary flow u reaches a maximum value and the secondary flow w decays to zero as $\theta \to 90°$, i.e. when the ma-

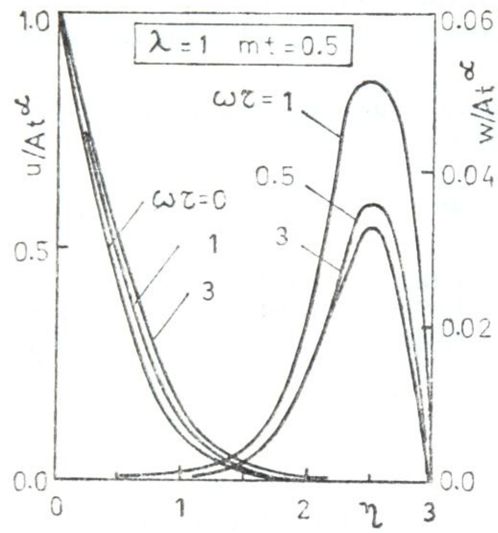

Fig. 2. Profiles of the primary velocity $u/A\,t^{\alpha}$ and secondary velocity $w/A\,t^{\alpha}$ for $\theta = 0°$ and $(mt) = 0.5$.

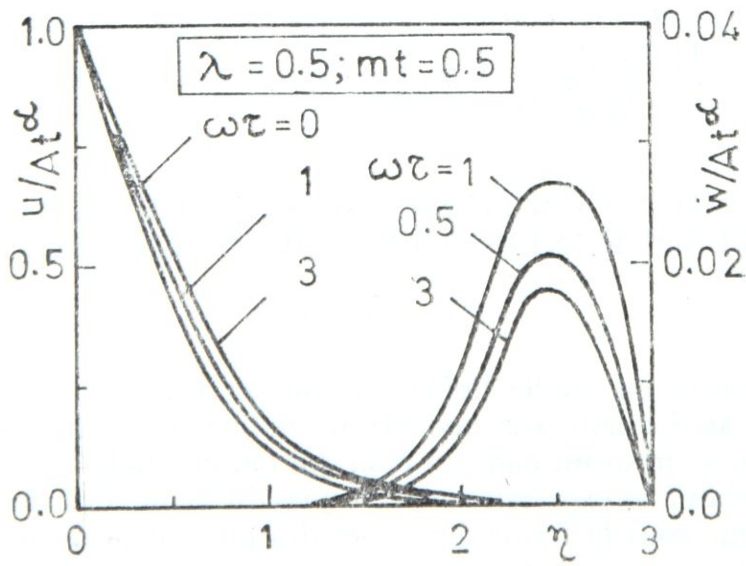

Fig. 3. Profiles of the primary velocity $u/A\,t^{\alpha}$ and secondary velocity $w/A\,t^{\alpha}$ for $\theta = 60°$ and $(mt) = 0.5$.

gnetic field acts transverse to the flow direction. However, for a fixed value of (mt), the increase of u with increase in θ is quite small and it is almost imperceptible from Figs. 2 and 3.

The non-dimensional skin-friction components on the plate along the x and z directions can be written as

$$\tau_x = \frac{1}{2\sqrt{mt}}\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}, \quad \tau_z = \frac{1}{2\sqrt{mt}}\left(\frac{\partial w}{\partial \eta}\right)_{\eta=0}. \tag{14}$$

Values of $-\tau_x$ and τ_z are given in Table I for some values of θ and $\omega\tau$ with $(mt) = 0.5$. It is seen from this Table that for fixed values of $\omega\tau$ the components $-\tau_x$ and τ_z decrease with an increase in θ. Further for fixed θ, the skin-friction for the primary flow decreases when $\omega\tau$ increases. On the other hand, τ_z first increases from zero, reaches a maximum and then decreases with increase in $\omega\tau$. This shows that Hall currents may exert a profound influence on the flow.

Table I. Values of $-\tau_x$ and τ_z for $(mt) = 0.5$.

	$-\tau_x$			τ_z		
$\omega\tau$ θ	0°	60°	90°	0°	60°	90°
0	1.162	1.162	1.162	0.	0.	0.
0.5	1.099	1.095	1.094	0.138	0.073	0.
1	0.996	0.989	0.987	0.183	0.094	0.
3	0.839	0.837	0.836	0.117	0.059	0.

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L'EFFET HALL DANS UN ÉCOULEMENT HYDROMAGNÉTIQUE SUR UNE PLAQUE INFINIE EN MOUVEMENT ACCÉLERÉ

Resume

Dans ce travail on étudie l'effet du courant Hall sur l'écoulement hydromagnétique non-stationnaire sur une plaque infinie dans un champ magnétique appliqué constant se trouvant dans un plan qui fait un angle θ avec le plan normal à la plaque. On déduit les expressions analytiques du champ des vitesses et du frottement sur la plaque sous la forme des séries des puissances du temps. On discute en detail le cas des mouvements brusques de la plaque et on analyse les éffets des paramétres Hall $\omega\tau$, de l'angle θ du champ magnétique (mt) sur l'écoulement considéré.

HALL-ov EFEKT KOD HIDROMAGNETNOG TEČENJA PO BESKONAČNOJ PLOČI U UBRZANOM KRETANJU

Izvod

U radu se ispituje efekt Hall-ove struje kod nestacionarnog hidromagnetnog tečenja po beskonačnoj ploči u stalnom ravanskom magnetnom polju čija ravan gradi ugao θ sa normalom na ravan ploče. Izvode se analitički izrazi za polje brzina i za komponente smičućih napona trenja na dodiru sa pločom, u vidu stepenog reda po vremenu. Detaljno se diskutuje slučaj ubrzanog kretanja ploče uz analizu efekta Hall-ovih parametara $\omega\tau$, ugla θ i magnetnog polja (mt) na posmatrano tečenje.

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