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NAIVE MODEL OF THE SHIP SCREW PROPELLER ROTATIONAL NOISE SPECTRUM

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List of symbols: $a = 1{,}8\,\pi\ \mathrm{rad}^{-1}$; c — speed of sound in water; $C_L(\alpha)$ — lift coefficient of the hydrofoil of unit width and unit length; $C_D(\alpha)$ — drag coefficient of the hydrofoil of unit width and unit length; D — propeller diameter; $f_m = mZn$ — m-th harmonic frequency; i — imaginary unit; $J = v_0/(nD)$ — propeller advance ratio; k, l, m — integers; $k_{Tm} = T_m/(\rho\, n^2 D^4)$ — normalized RMS value of the m-th harmonic component of the fluctuating thrust; n — number of propeller revolutions per second; $p_m(r', \chi)$ — RMS value of acoustic pressure component at the frequency f_m in the field point (r', χ); p_{ref} — reference value of the acoustic pressure; r — radial coordinate on the blade; r' — distance between the propeller and the receiver; r_0 — unit length; R — propeller radius; R_0 — hub radius; $s(r)$ — blade profile length at the radius r; t — time; T_m — RMS value of the m-th harmonic component of the fluctuating thrust; $u(r, \theta)$ — magnitude of the water inflow velocity to the blade at the point (r, θ); $v(r, \theta)$ — axial component of the velocity disturbance at the point (r, θ) on the blade; v_0 — ship's speed; $V_l(r) = |\mathcal{V}_l(r)|/2$ — amplitude of l-th angular harmonic of $v(r, \theta)$; $\mathcal{V}_l(r)$ — complex amplitude of l-th harmonic of $v(r, \theta)$; Z — number of blades; α — angle of water inflow to the profile measured from zero-lift direction; $\alpha(r, \theta)$ — angle of water inflow to the blade at the radius r when the blade is in the angular position θ, measured from zero-lift direction; $\alpha(r)$ — component of $\alpha(r, \theta)$ determined by v_0 only; $\vartheta(r)$ — angle describing blade skewness: angle between the reference direction at the blade and the straight line connecting the propeller centre and the point of application of lift and drag of the blade element at the radius r; the direction of propeller rotation corresponds to the increasing values of $\vartheta(r)$; θ — reference blade angular position; the direction of propeller rotation corresponds to the increasing values of θ; θ_k — k-th blade angular position; $\upsilon(r) = v(r, \theta)/v(R, \theta)$ — $v(r, \theta)$ of the radially similar inflow field normalized to the value at the propeller radius; ρ — water density; $\varphi_l(r) = \arg \mathcal{V}_l(r)$ — phase angle of the l-th harmonic of $v(r, \theta)$; $\phi(r, \theta)$ — angle of water inflow to the blade at the radius r when the blade is in the angular position θ, measured from the tangent to the rotation circle; χ — angle between propeller axis and the straight line connecting the propeller and the receiver;

$$\psi(l, Z) = \begin{cases} 1, & \text{if } l/Z \text{ is an integer,} \\ 0, & \text{otherwise;} \end{cases}$$

$Re(z)$ — real part of z; RMS value of $z(t)$ equals

$$\left[\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} z^2(t)\, dt\right]^{1/2}.$$

Introduction

A non-cavitating screw propeller with Z identical blades of finite thickness revolving n times per unit time in a non-uniform water inflow velocity field generates periodic sound at the frequencies $f_m = mZn$, $m = 1, 2, \ldots$. This mechanism exists even if the propeller produces no thrust and can be modelled by rotating water sources and sinks of constant strength. The infinitesimally thin blade propeller producing thrust in a uniform water inflow velocity field is also a source of sound at frequencies f_m, $m = 1, 2, \ldots$. In this case an appropriate model would be a system of constant rotating forces. The acoustic efficiency of the two described sound generating mechanisms is low if the sources or forces rotate at deep subsonic speeds.

The circumstances are drastically changed if the propeller operates in a circumferentially non-uniform water inflow field. This generates thrust fluctuations as fluctuating inertial forces caused by the varying water inflow to a blade of finite thickness. Thus in a circumferentially non-uniform water velocity field both generating mechanisms, the one related to the thrust and the other caused by the water displacement, give rise to varying forces. This is modelled by a time varying force fixed spatially relative to the propeller axis, correspoding to the acoustic dipole in the propeller axis.

Described mechanisms of periodic low-frequency sound generation related to the propeller rotation, so called rotational noise, can be summarized as follows:

(a) water displacement in a uniform inflow field,

(b) thrust in a uniform inflow field,

(c) water displacement in circumferentially non-uniform inflow field,

(d) thrust in the circumferentially non-uniform inflow field.

In an even slightly non-uniform velocity field the mechanism (d) is dominant [1]. The smoothest real inflow field behind the ship hull appears to be non-uniform enough for the last mechanism to prevail. Thus, in contrary to the case of an airplane propeller where the natural turbulence in the air is the dominant source of the inflow non-uniformity, in the case of a ship propeller the appropriate rotational noise generating mechanism is the time varying dipole in the propeller axis caused by the thrust fluctuations.

Based on the considerations presented above Miniovich [1] and Ross [2] constructed a simple dipole model of the far-field rotational noise of the acoustically free screw propeller. Ross formulated the model as follows:

$$p_m(r', \chi) \simeq p_m(r_0, 0)\, \frac{r_0}{r'}\, |\cos \chi|, \tag{1}$$

$$p_m(r_0\ 0) = \frac{\rho^{1/2}}{2c^{3/2}r_0} m Z n^3 D^4 k_{Tm}, \tag{2}$$

$$k_{Tm} = \frac{T_m}{\rho n^2 D^4}, \quad m = 1, 2, \ldots. \tag{3}$$

The input data describing a propeller in (1) — (3) are: Z, D, n and the normalized amplitude spectrum k_{Tm}, $m = 1, 2, \ldots$, of the fluctuating thrust. It is the aim of this paper to construct a simple model yielding an estimate of k_{Tm}, $m = 1, 2, \ldots$, assuming that sufficient data on the propeller and water inflow velocity field are available. The simplicity of the model makes a direct analysis of the influence of various parameters possible. However this simplicity is due to the use of various assumptions and neglecting of various details that makes it a naive model, which could yield only rough estimate of the rotational noise amplitude spectrum. The main purpose of this model is to give a direct insight into the parameters influence, which is required in preliminary stages of propeller design.

Model construction

The notation used in course of model construction is given in Figs. 1 and 2. The direction of propeller rotation in Fig. 1 has been given for reference only. It will be shown that the direction of rotation has no influence on the obtained results.

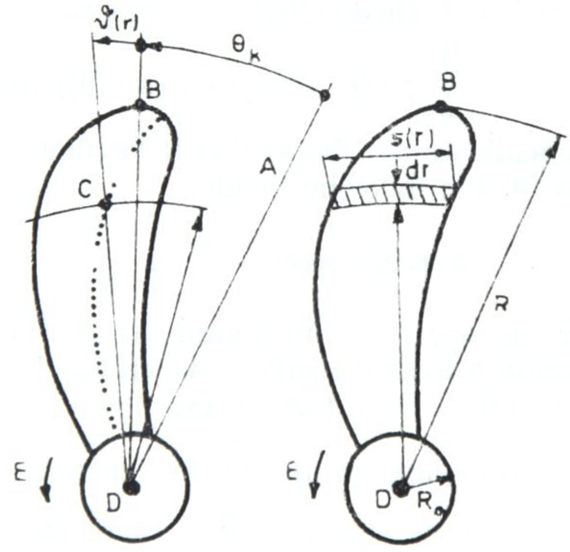

Fig. 1

Blade geometry: A — angular reference, B — outermost point of the blade, C — point of application of the forces experienced by the profile element at the radius r, D — centre of the rotation, E — direction of rotation. Dotted line curve connects points C at various radii.

The propeller blades are denoted by $k = 1, 2, \ldots, Z$; k-th blade angular position is θ_k. The reference blade used for defining the propeller angular position θ is denoted by $k = 1$ $(\theta = \theta_1)$; blades $k = 2, 3, \ldots$, are shifted from $k = 1$ in the direction of rotation. The blade element between r and $r + dr$ is the profile element experiencing lift $(\rho u^2(r, \theta)/2)\, C_L(\alpha)\, s(r)\, dr$ and drag $(\rho u^2(r, \theta)/2)\, C_D(\alpha) \times s(r)\, dr$, where $\alpha = \alpha(r, \theta)$ is the angle between the local zero-lift direction and the direction of the water inflow velocity at (r, θ), and $u(r, \theta)$ is the magnitude of this velocity. The above refers to the reference blade. However the circumstances are the same on the other blades.

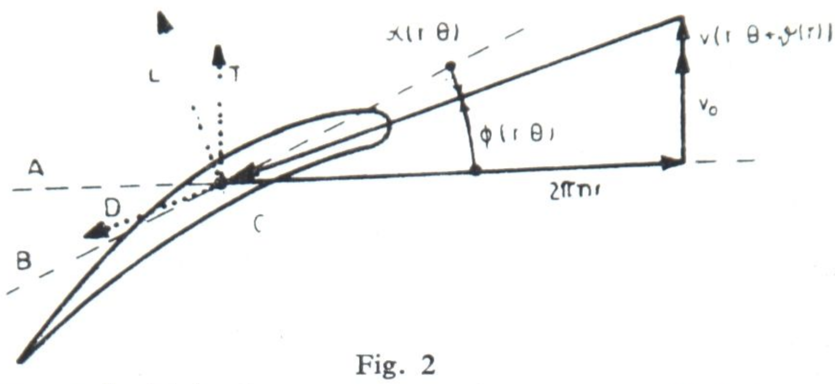

Fig. 2

Circumstances at the blade element at the radius r with the blade in the angular position θ: A — circumferential direction, B — zero-lift direction, C — point of application of the forces experienced by the element, L — lift, D — drag, T — thrust.

Assuming

$$v_0 \ll 2\pi nr, \qquad |v(r, \theta)| \ll 2\pi nr, \tag{4}$$

so that $u(r, \theta) \simeq 2\pi nr$, resulting thrust produced by the blade element is:

$$\frac{1}{2} \rho\, s(r) (2\pi nr)^2 \left[C_L(\alpha(r, \theta)) \cos \phi(r, \theta) - C_D(\alpha(r, \theta)) \sin \phi(r, \theta)\right] \mathrm{d}r,$$

or since $\cos \phi(r, \theta) \simeq 1$ and $|\sin \phi(r, \theta)| << 1$, due to (4)*,

$$\frac{1}{2} \rho\, s(r) (2\pi nr)^2\, C_L(\alpha(r, \theta))\, \mathrm{d}r. \tag{5}$$

The inflow angle depends on $v(r, \theta)$, so that it fluctuates around some value $\alpha(r)$. Taking into account (4) the following holds:

$$\alpha(r, \theta) \simeq \alpha(r) + \frac{v(r, \theta + \vartheta(r))}{2\pi nr}. \tag{6}$$

Here the propeller skewness is introduced by deriving the inflow angle to the blade in the position θ from the inflow velocity at the point $C\ (r, \theta + \vartheta(r))$ of the blade element at radius r (see Figs. 1 and 2).

In small inflow angle approximation the function $C_L(\alpha)$ reads [2]:

$$C_L(\alpha) \simeq a\,\alpha, \qquad a = 1{,}8\,\pi\ \mathrm{rad}^{-1}. \tag{7}$$

Here the definition of α as given in Fig. 2 is assumed: α is referenced to zero-lift direction of the blade.

From (5) — (7) the blade element thrust is obtained:

$$\frac{a}{2} \rho s(r)\ (2\pi nr)^2 \left[\alpha(r) + \frac{v(r, \theta + \vartheta(r))}{2\pi nr}\right] \mathrm{d}r.$$

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* Here drag vanishes from the calculations. The reason for this is in the geometrical relations. Now the question arises: why to consider only thrust and not momentum of drag as well? That is because the acoustic efficiency of the momentum of drag is negligible compared to thrust, since momentum of drag, i.e. its time-varying component is equivalent to a pair of closely spaced dipoles, a quadrupole.

Here only the second term,

$$a\pi\rho nrs(r)\, v(r, \theta + \vartheta(r))\, dr \tag{8}$$

depends on θ, so it is the only source of propeller thrust fluctuations.

Let us fix the reference blade ($k = 1$) kinematics stating $\theta = 2\pi nt$, so that k-th blade kinematics is

$$\theta_k = 2\pi nt + 2\pi(k-1)/Z, \qquad k = 1, 2, \ldots, Z \tag{9}$$

($\theta = \theta_1$).

Replacing θ in (8) by θ_k the waveform of k-th blade element thrust is obtained. Actually this is only one component of the thrust, the one in which the fluctuating thrust is included. The waveform reads as follows:

$$a\pi\rho nrs(r)\, v(r, 2\pi nt + \vartheta(r) + 2\pi(k-1)/Z)\, dr, \qquad k = 1, 2, \ldots, Z. \tag{10}$$

This model is based on the assumption that the blade thrust reacts to the inflow velocity with no lag.

Neglecting the random components in the inflow velocity field, $v(r, \theta)$ can be regarded as periodic in θ, so that it can be represented by Fourier series:

$$v(r,\theta) = \sum_{l=-\infty}^{\infty} \mathcal{V}_l(r)\, e^{il\theta}, \ \theta \in [0, 2\pi], \tag{11}$$

$$\mathcal{V}_l(r) = \frac{1}{2\pi} \int_0^{2\pi} v(r,\theta)\, e^{-il\theta}\, d\theta, \ l = 0, \pm 1, \pm 2, \ldots. \tag{12}$$

Substitution of (11) into (10) and summation over all blades yields

$$a\pi\rho nrs(r) \sum_{l=-\infty}^{\infty} \mathcal{V}_l(r)\, e^{il\vartheta(r)}\, e^{i2\pi lnt} \sum_{k=1}^{Z} e^{i2\pi l(k-1)/Z}\, \mathrm{dr}. \tag{13}$$

The sum over k equals $Z\psi(l, Z)$ where

$$\psi(l, Z) = \begin{cases} 1, & \text{if } l/Z \text{ is an integer,} \\ 0, & \text{otherwise,} \end{cases}$$

so that the contribution of propeller element between r and $r + dr$ to the fluctuating thrust component follows from (13):

$$a\pi\rho nZrs(r) \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \mathcal{V}_l(r)\, e^{il\vartheta(r)}\, \psi(l, Z)\, e^{i2\pi lnt}\, \mathrm{dr}. \tag{14}$$

The constant term was here omitted.

Eq. (14) is the complex notation of the thrust component waveform consisting of waves at the frequencies ln, $l = 1, 2, \ldots$. The function $\psi(l, Z)$ filters out all the frequencies but those which are integral multiples of nZ:

$$f_m = mZn, \qquad m = 1, 2, \ldots . \tag{15}$$

The component of (14) at the frequency f_m is

$a\pi\rho nZrs\,(r)\,[\mathcal{V}_{-mZ}\,(r)\,e^{-imZ\vartheta(r)}\,e^{-i2\pi mZnt} + \mathcal{V}_{mZ}\,(r)\,e^{imZ\vartheta(r)}\;e^{i2mZnt}]\,\mathrm{dr},\; m = 1, 2, \ldots,$

or

$$a\pi\rho nZrs\,(r)\;\mathcal{V}_{mZ}\,(r)\;Re\;\{e^{i[2\pi mZnt + mZ\vartheta(r) + \varphi_{mZ}(r)}\}\;\mathrm{dr},\; m = 1, 2, \ldots . \tag{16}$$

Here we used

$$\mathcal{V}_l\,(r) = \frac{1}{2}\;V_l\,(r)\,e^{i\varphi_l(r)}, \tag{17}$$

with the parity rules $V_{-l}\,(r) = V_l\,(r)$ and $\varphi_{-l}\,(r) = -\varphi_l\,(r)$, $l = 0, \pm 1, \pm 2, \ldots,$ these rules being the consequence of the fact that $v\,(r, \theta)$ is a real valued function.

By integrating (16) over the blade from the hub to the propeller tip the waveform of the total propeller thrust component at the frequency f_m is obtained:

$$\frac{a\pi}{8} ZJRe\left\{e^{i2\pi mZnt}\,\frac{1}{v_0\,R^3}\int_{R_0}^{R} rs\,(r)\;V_{mZ}\;(r)\;\;e^{i[mZ\vartheta(r)+\varphi_{mZ}(r)]}\;\mathrm{dr}\right\}\; m = 1, 2, \ldots . \tag{18}$$

Here the propeller advance ratio

$$J = \frac{v_0}{nD} \tag{19}$$

was introduced and the result was normalized by ρn^2D^4 as in (3). k_{Tm} defined by (3) is the RMS value of the wave (18):

$$k_{Tm} \simeq \frac{a\pi}{8\sqrt{2}}\,ZJ\,\frac{1}{v_0\,R^3}\left|\int_{R_0}^{R} rs\,(r)\;V_{mZ}\,(r)\;\;e^{i[mZ\vartheta(r)+\varphi_{mZ}(r)]}\;\;\mathrm{dr}\right|,\; m = 1, 2, \ldots . \tag{20}$$

The result (20) holds for both directions of propeller rotation, if $\vartheta\,(r)$ describing the skewness of blade is measured correctly (cf. the list of symbols). To prove this it is sufficient to note that all the quantities determining the phase angle of the blade element thrust contribution, i.e. θ, $\vartheta\,(r)$, and the blade number k, increase in the direction of rotation. Thus since $\theta = 2\pi nt$, the waveforms do not depend on the direction of rotation so that all the results hold for both directions of rotation.

Formulae (1), (2) and (20) form the naive model of the acoustically free screw propeller rotational noise far field.

Non-skewed propeller in the radially similar inflow field

The special case of the non-skewed propeller,

$$\vartheta\,(r) \equiv 0, \tag{21}$$

operating in the radially similar inflow field,

$$v(r, \theta) = \upsilon(r)\, v(R, \theta), \quad r \in [R_0, R], \tag{22}$$

is both analytically attractive and practically possible.

Reducing (22) to $V_l(r) = \upsilon(r)\, V_l(R)$, $\varphi_l(r) = \varphi_l(R)$, $l = 0, \pm 1, \pm 2, \ldots$, $r \in [R_0, R]$, and using this and (21) in (20) one obtains:

$$k_{Tm} \simeq \frac{a\pi}{8\sqrt{2}} ZJ \frac{V_{mZ}(R)}{v_0} \left| \frac{1}{R^3} \int_{R_0}^{R} \nu(r)\, rs(r)\, \mathrm{d}r \right|, \quad m = 1, 2, \ldots. \tag{23}$$

The spectrum of k_{Tm} has the same form as the inflow velocity field spectrum, $V_{mZ}(R)/v_0$, $m = 1, 2, \ldots$, taking from it only the multiples of the number of blades, while the blade elements of different radii contribution to k_{Tm} is proportional to the profile length moment $rs(r)$ and the magnitude of the inflow velocity at the given radius. As can be seen, in the special case (21) / (22) the influence of the various propeller and water inflow field parameters to the rotational noise is rather evident.

Comparison to the quasi-stationary model

The naive model was compared to the quasi-stationary model in one case of the nonskewed propeller operating in an approximately radially similar inflow field. A quasi-stationary model developed in [3] was used. The values of k_{Tm} obtained by the procedure from [3] and those calculated according to (23) were used in (2) to produce the source level

$$20 \log (p_m(r_0, 0)/p_{ref}), \quad m = 1, 2, \ldots \quad \text{(decibels, dB)},$$

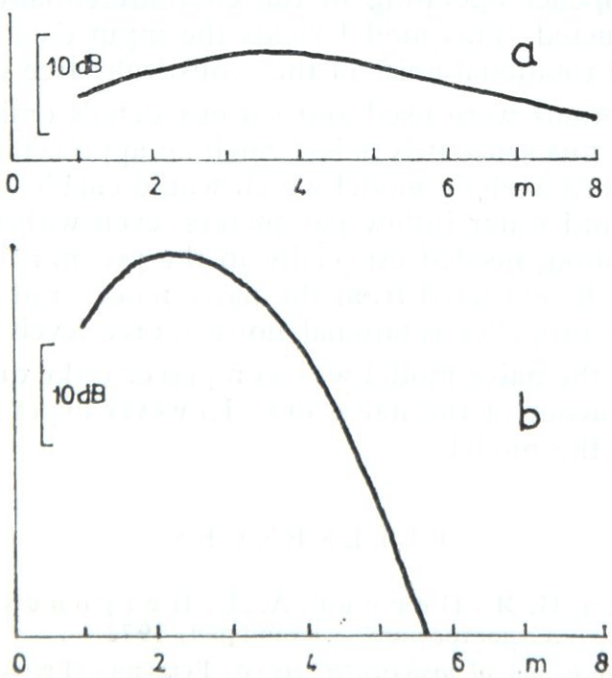

Fig. 3

Comparison of the rotational noise spectra patterns obtained by means of two methods: *a* — quasi-stationary model, *b* — naive model.

$r_0 = 1$ m. The envelope of the dependence of this level on m are drawn in Fig. 3 showing an appreciable discrepancy bettween the two models when $m > 3$ or 4.

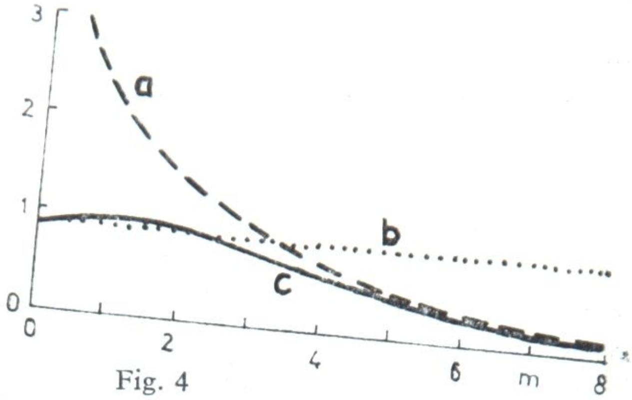

Fig. 4

Comparison of three methods for calculation of k_{Tm}, $m = 1, 2, \ldots$, after Ross [2]: a — two-dimensional non-stationary, b — quasi-stationary, c — three-dimensional non-stationary.

The quasi-stationary model accuracy was discussed in [2]. The comparison of three metheds for calculation of k_{Tm} is recapitulated in Fig. 4, where relative values of k_{Tm} are indicated. Taking the three-dimensional nonstationary model as the most accurate one it could be observed that the quasi-stationary model yields quite accurate values if $m < 3$ or 4 and that this model over-estimates k_{Tm} if $m > 3$ or 4. Thus the result of the comparison of the naive model and the quasi-stationary one as given in Fig. 3 could be judged as advantageous to the naive model.

Conclusions

The simple model of the fluctuating thrust component spectrum for the non--cavitating screw propeller operating in the circumferentially non-uniform water inflow field is constructed. This model yields the input data needed in the dipole model of the far-field rotational noise of the acoustically free screw propeller.

Various assumptions were used and various details omitted in the construction of the model. In this sense it is naive. Such an approach aims at the construction of the simple closed analytic model which would enable us to see the influence of various propeller and water inflow parameters, even without numerical realization. Such an application, needed especially in the preliminary stages of propeller design, is what could be expected from the naive model, rather than accurate predictions of the screw propeller rotational noise source levels.

In one example the naive model was compared to the quasi-stationary model. The results were in favour of the naive one. However experiments are needed for the final checking of the model.

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НАИВНАЯ МОДЕЛЪ СПЕКТРА ЗВУКА ВРАЩЕНИЯ СУДОВОГО ГРЕБНОГО ВИНТА

Р е з ю м е

В работе конструирована простая замкнутая аналитическая моделъ звука вращения некавитирующего гребного винта работающего в периферически неоднородном поле скоростей.

NAIVNI MODEL SPEKTRA ROTACIONOG ŠUMA BRODSKOG VIJČANOG PROPELERA

I z v o d

Konstruiran je jednostavan zatvoreni analitički model (20) spektra fluktuacione komponente poriva brodskog vijčanog propelera koji ne kavitira a radi u obodno nejednolikom polju brzina pritjecanja vode. Model daje potrebne podatke za ocjenu spektra dalekog polja rotacionog šuma akustički slobodnog vijka modeliranog dipolom, prema (1) i (2).

U model (20) ugrađen je niz pretpostavki a pri konstruiranju je ispušteno više detalja; to ga čini *naivnim*. Cilj ovakvog pristupa bio je matematički model, koji će, i bez numeričke realizacije, omogućiti izvođenje ocjene utjecaja pojedinih parametara vijka i polja brzine pritjecanja. Otud i namjena modela: korištenje u ranim fazama projektiranja vijka, a ne točni računi rotacionog šuma.

U važnom specijalnom slučaju radijalno sličnog polja brzine pritjecanja, model poprima oblik (23), prema kojem je spektar fluktuacija poriva istog oblika kao i spektar polja brzine pritjecanja, s tim da u prvome dolaze samo harmonici čiji su redovi višekratnici broja krila. U tom je slučaju doprinos amplitudi s elemenata krila na raznim radijusima proporcionalan radijusu, te duljini profila i amplitudi brzine pritjecanja na tom radijusu.

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