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THE ELASTIC — PLASTIC CONSTITUTIVE RELATION

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Introduction

If a body[1] deformed undeı stress is released from that stress, it might or might not recover its original shape. If it does recover, we call the deformation o the body elastic, if it does not, we call it elastic-plastic deformation. In the latte case, therefore, part of the deformation is not recoverable on releasing the stress but is left as a permanent change in the shape of the body. That part of the defor mation is called plastic deformation. The recoverable (reversible) part of deformation is called elastic deformation. They combine to give the total, elastic-plastic deformation of the body.

This work deals with the analysis of such deformation. There are two difficulties to be faced. Each of them comes from a different source: one is associated with the plastic part of the deformation, the other with the elastic part; and they are different in nature. The plastic part of the deformation introduces difficulty frcm the physical point of view: it constitutes an extremely complicated phenomenon which is happening on the microscale inside the material. The theory which attempts to include all its aspects is likely to be prohibitively difficult to either formulate or apply. The elastic part of the deformation introduces difficulty from the kinematical point of view: its proper combination with the plastic part of the deformation in giving the total deformation is still a subject of controversy.

This work is oriented toward the second difficulty. The kinematical model introduced by Lee [1] has been selected as coping with the difficulty. The selection was fruitful, indeed. The model led to a kinematically rigorous derivation of the rate-type constitutive law which appears to be correctly done for the first time in this *analysis*. The model and its consequences raise the theory of plasticity, at least for isotropic bodies, to the level of modern continuum mechanics.

1. Rate-type (Incremental) Time Independent Theory

Consider the body in its initial (stress free) configuration B_0. Let it be deformed under the action of some external agency into the configuration B_t such that the motion (deformation) from B_0 to B_t is given by a single valued mapping

$$\underset{\sim}{x}=\underset{\sim}{\chi}(\underset{\sim}{X}, t) \tag{1.1}$$

[1]) In this analysis, by body we mean a solid whose mechanical properties do not depend explicitely on time (Solid without a natural time, [5]).

which carries the material particle from its initial position X into its current position $\underset{\sim}{x}$ at time t (Fig. 1.1). The function (1.1), defined in Euclidian space E_3 with the rectangular Cartesian coordinate system (X_1, X_2, X_3), is assumed to be continuously differentiable of class C_3. Let the motion χ involve elastic-plastic deformation. Observations have shown such materials to be „simple" in the continuum sence, i.e. the deformation will be present in the constitutive equations only through the deformation gradient matrix

$$F = \partial \underset{\sim}{x} / \partial \underset{\sim}{X}. \tag{1.2}$$

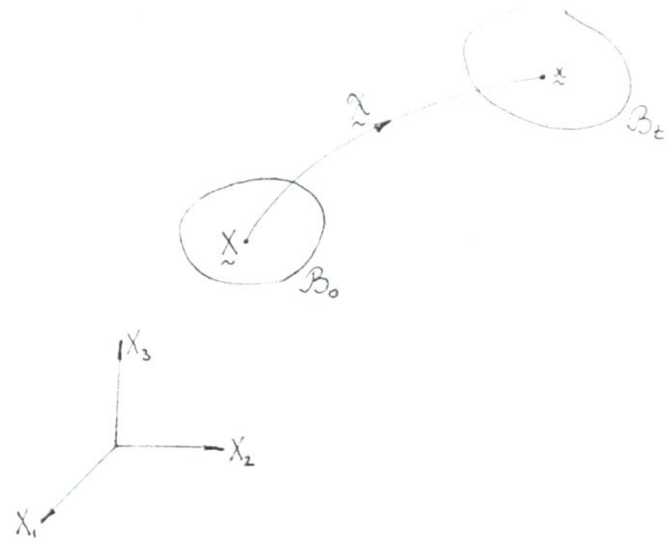

Fig. 1.1

Following Lee [1], we now introduce the intermediate configuration P_t by distressing the whole body from its current configuration B_t and by reducing the temperature to the initial value (Fig. 1.2).

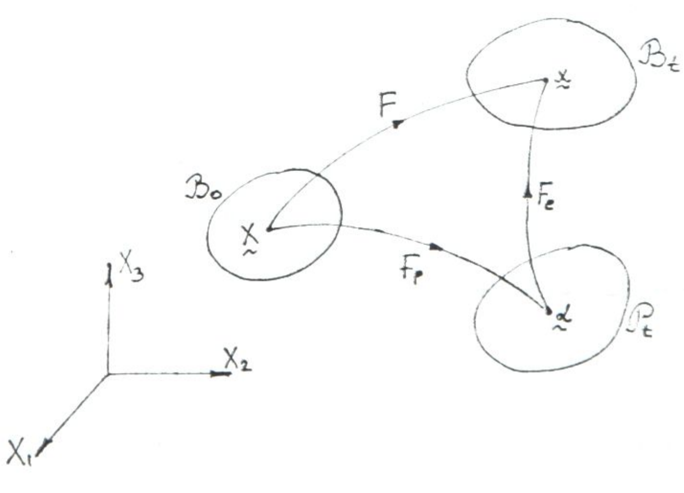

Fig. 1.2

The configuration P_t then comprises a pure plastic deformation, for thermal expansion and elastic strain components are both zero. Since for inhomogeneous deformation, unloading the body leaves a nonzero field of residual stresses, complete distresing makes the configuration P_t incompatible. Hence, the correspondence between material points in configuration B_t and P_t is not one to one. Yet, by confining attention to a small enough neighborhood of the particle so that deformation there can be approximated by a homogeneous one, we can always establish the relationships:

$$\begin{aligned} d\underset{\sim}{\alpha} &= F_p\, d\underset{\sim}{X} \\ d\underset{\sim}{x} &= F_e\, d\underset{\sim}{\alpha}, \end{aligned} \tag{1.3}$$

where $d\underset{\sim}{X}$, $d\,\underset{\sim}{\alpha}$ and $d\underset{\sim}{x}$ are infinitesimal vectors from corresponding homogeneously deformed neighborhoods, and matrices F_e and F_p correspond to the pure (thermo-) elastic and pure plastic parts of the deformation, respectively. Of course, the components of F_e and F_p are not the partial derivatives as are the components of F.[1] From relations (1.3) we now can establish at each point of the deformed body the following decomposition

$$F = F_e\, F_p. \tag{1.4}$$

The decomposition (1.4) is of central importance in all kinematics of finite elastic--plastic deformation processes. It decomposes the total deformation gradient F into the pure (thermo-) elastic part F_e and pure plastic part F_p.[2]

The elastic deformation F is governed by the classical finite elasticity law [4, p. 309]

$$T = 2\,\frac{\rho}{\rho_0}\,F_e\,\frac{\partial\psi_e}{\partial C_e}\,F_e^T \tag{1.5}$$

where T is the Cauchy stress tensor, $\psi_e = \psi_e(C\,,\,\theta)$ is the Helmholtz free energy per unit initial volume, $C_e = F_e^T F_e$ is the right Cauchy-Green deformation tensor, θ is temperature, and ρ and ρ_0 are the densities in configurations B_t and P_t, respectively.[3] However, the „deformation gradient" matrix F_e is not uniquely defined because we can always superpose an arbitrary rotation on the intermediate configuration and still have that configuration unstressed. Therefore, if the material is anisotropic, the stress response in (1.5) would be dependent on the rigid-body rotation of the intermediate configuration. For further analysis, we shall restrict ourselves to the case of isotropy of the body in its virgin configuration B_0. According to experimental evidence [3, p. 24], isotropy remains approximately preserved until quite large deformations. We shall accordinagly assume that the body in the

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1) Because, although we have the global mapping $\underset{\sim}{x} = \underset{\sim}{x}(\underset{\sim}{X}, t)$, we in general cannot define the global mappings $\underset{\sim}{x} = \underset{\sim}{x}(\underset{\sim}{\alpha}, t)$ and $\underset{\sim}{\alpha} = \underset{\sim}{\alpha}(\underset{\sim}{X}, t)$ between the corresponding configurations.

2) The decomposition (1.4) is defined at an arbitrary material point $\underset{\sim}{X}$ of the deforming medium, i.e.

$$F(\underset{\sim}{X}, t) = F_e(\underset{\sim}{X}, t)\, F_p(\underset{\sim}{X}, t)$$

so that in that sense all three matrices (operators) F, F_e and F_p can be viewed as fields globally defined over the space $\underset{\sim}{X}$ or $\underset{\sim}{x}$. However, we cannot talk gloоally about the field $F_e(\underset{\sim}{\alpha}, t)$, except, of course, in the case of a homogeneous deformation, and possibly some other cases.

3) We assume the incompressibility of plastic flow, hence the density in configuration $\mathcal{P}_t$ is the same as the initial density ρ_0 in the configuration $\mathcal{B}_0$. We also assume [1, p. 3] that the elastic properties of material are not influenced by the previous plastic flow.

intermediate configuration is still isotropic. Then the function ψ_e is an isotropic function of C_e and rigid-body rotation of the intermediate configuration doesn't influence the stress response (1.5). Alternatively, the decomposition (1.4) is not unique, because we can always have

$$F = F_e F_p = (F_e Q)(Q^T F_p) = \bar{F}_e \bar{F}_p \tag{1.6}$$

for an arbitrary orthogonal Q. Since for isotropic bodies, our theory is independent of the above choice of F_e, we can choose that F_e which is the most convenient for us, i.e. which most simplifies our expressions. Therefore, we choose F_e corresponding to distressing without rotation, i.e. $R_e = I$ and

$$F_e = V_e, \tag{1.7}$$

where V_e is the symmetric left stretch tensor and R_e the orthogonal rotation tensor from the polar decomposition theorem $F_e = V_e R_e$. Substituting (1.7) into (1.5), this becomes

$$\tau = 2V_e \frac{\partial \psi_e}{\partial C_e} V_e, \tag{1.8}$$

where the Kirchhoff stress

$$\tau = \frac{\rho_0}{\rho} T \tag{1.9}$$

is introduced. But the matrix product on the right hand side in (1.8) is commutative since all matrices are parallel to the stress matrix, and therefore

$$\tau = 2C_e \frac{\partial \psi_e}{\partial C_e} \tag{1.10}$$

as, in view (1.7), $V_e^2 = C_e$. The law (1.10) is the constitutive law for the elastic part of the deformation.

The structure of the constitutive law for the plastic part of the deformation is quite different. Plasticity is a fluid type phenomenon which is governed by a rate (incremental, flow) type relation which involves the strain rate rather than strain in its structure. Restricting ourselves to the case of time-independent plasticity, the law governing the plastic part of deformation in our theory, takes the form [1, p. 5]

$$D_p = \frac{1}{\mathcal{G}} \left(\frac{\partial f}{\partial \tau} : \dot{\tau} + \frac{\partial f}{\partial \theta} \dot{\theta} \right) \frac{\partial f}{\partial \tau} \tag{1.11}$$

where $f = g(\tau) - c = 0$ is the yield function, c representing the hardening, „:" stands for the „trace", „." is the material derivative, $D_p = \mathrm{sym}(\dot{F}_p F_p^{-1})$ is the plastic streaching tensor, and $\mathcal{G}$ is a scalar which contains information about the history of defofmation and which for the time being is not of interest to us in this analysis.

For isothermal deformation, (1.11) reduces to

$$D_p = \frac{1}{\mathcal{G}} \left(\frac{\partial f}{\partial \tau} : \dot{\tau} \right) \frac{\partial f}{\partial \tau}, \tag{1.12}$$

which parallels the classical relation for isothermal work hardening given by Hill [3, p. 38], and which is seen to be not complicated by the inclusion of finite strain and nonlinear elasticity.

Our objective is now to combine the elastic law (1.10), which is in finite (not rate-type) form, with the plastic law (1.12), which is in the rate-type form, into a single relation — the constitutive law of elastic-plastic material.

First, we establish the kinematical relation which will serve as a basis for assembling the elastic and plastic parts of the deformation. Consider the velocity gradient of the total deformation [1]

$$L = \dot{F}\,F^{-1}. \tag{1.13}$$

Introducing the decomposition (1.4), this becomes

$$L = L_e + F_e L_p F_e^{-1}, \tag{1.14}$$

where $L_e = \dot{F}_e\,F_e^{-1}$ and $L_p = \dot{F}_p\,F_p^{-1}$ are the „velocity gradients" corresponding to the elastic and plastic part of deformation, respectively.[1] Since $L_p = D_p + W_p$, (1.14) can be rewritten as

$$L = L_e + F_e D_p F_e^{-1} + F_e W_p F_e^{-1}, \tag{1.15}$$

where D_p is the plastic stretching and W_p the plastic spin tensor. Further, we decided to do the distressing without rotation (1.7), and (1.15) bec. mes

$$L = L_e + V_e D_p V_e^{-1} + V_e W_p V_e^{-1} \tag{1.16}$$

with

$$L_e = \dot{V}_e V_e^{-1}. \tag{1.17}$$

But for isotropic material, the principal axes of stress and stretch V_e are coincident, and since D_p has also the principal axes coincident with those of stress, the matrices V_e and D_p are commutative and V_e and V_e^{-1} cancel each other. Hence, (1.16) reduces to

$$L = L_e + D_p + V_e W_p V_e^{-1}. \tag{1.18}$$

By taking the symmetric and antisymmetric parts, (1.18) gives:

$$D = D_e + D_p + (V_e W_p V_e^{-1})_S \tag{1.19}$$

$$W = W_e + (V_e W_p V_e^{-1})_A. \tag{1.20}$$

But

$$D_e = \frac{1}{2}\,V_e^{-1}\dot{C}_e V_e^{-1}, \tag{1.21}$$

where

$$C_e = V_e V_e\,, \tag{1.22}$$

[1]) We keep the term „velocity gradient" for both L_e and L_p, although it is not quite appropriate since, in general, neither of them is derivable from a global velocity field. (Recall the remarks in footnotes 1) and 2), page 5.)

and the first and third terms on the right hand side of (1.19) combine to give

$$D_e + (V_e W_p V_e^{-1})_S = \frac{1}{2} V_e^{-1} \overset{\nabla}{C}_e V_e^{-1}, \tag{1.23}$$

where

$$\overset{\nabla}{C}_e = \dot{C}_e - W_p C_e + C_e W_p \tag{1.24}$$

is the Jaumann derivative of C_e with respect to the plastic spin W_p. Thereore (1.19) can be written in the form

$$D = \frac{1}{2} V_e^{-1} \overset{\nabla}{C}_e V_e^{-1} + D_p \tag{1.25}$$

or, for brevity

$$D = \mathcal{D}_e + D_p, \tag{1.26}$$

where

$$\mathcal{D}_e = \frac{1}{2} V_e^{-1} \overset{\nabla}{C}_e V_e^{-1}. \tag{1.27}$$

The relation (1.25) with (1.27) is the desired kinematical relation for combining the elastic and plastic parts of deformation. It is of fundamental importance in all rate-type theory: it decomposes the stretching tensor D into the elastic part $\mathcal{D}_e$ and plastic part D_p. We observe that $\mathcal{D}_e \neq D_e$ and that one has to be extremely careful and precise in making the decomposition of total stretching into the elastic and plastic parts. It is true that $D \cong \mathcal{D}_e$, but in developing the rate type theory and in arriving at the correct form of the constitutive law, it is essential to distinguish between D_e and $\mathcal{D}_e$, and therefore to carefully use the exact kinematical relation (1.26), rather than the approximation

$$D = D_e + D_p. \tag{1.28}$$

Before we proceed with the construction of the constitutive law, we first prove the objectivity of (1.26). Consider the rigid body rotation of the current deformed state by the proper orthogonal transformation $Q(t)$

$$\underset{\sim}{x}^* = Q \underset{\sim}{x}. \tag{1.29}$$

The deformation gradient changes to

$$F^* = QF. \tag{1.30.1}$$

Since elastic distressing is considered to occur without rotation, (1.7), the intermediate configuration $\mathcal{P}_t$ must be subjected to the same rotation, and this constraint must be introduced into the objectivity requirements. We thus find the following transformation laws for the elastic and plastic „deformation gradients", V_e and F_p, respectively:

$$V_e^* = QV_eQ^T \tag{1.30.2}$$

$$F_p^* = QF_p. \tag{1.30.3}$$

It is clear that the composition of (1.30.2) and (1.30.3) leads to the transformation law (1.30.1).

In view of (1.30.1) — (1.30.3), it can be further easily shown that the following transformations arise:

$$D^* = QDQ^T \tag{1.30.4}$$

$$D_p^* = QD_pQ^T \tag{1.30.5}$$

$$C_e^* = QC_eQ^T \tag{1.30.6}$$

$$\dot{C}_e^* = Q\dot{C}_eQ^T + \dot{Q}C_eQ^T + QC_e\dot{Q}^T \tag{1.30.7}$$

$$\overset{\nabla}{C}{}_e^* = Q\overset{\nabla}{C}_eQ^T \tag{1.30.8}$$

$$D_e^* = Q\mathcal{D}_eQ^T + \frac{1}{2}\,Q(V_e\dot{Q}^TQV_e^{-1} + V_e^{-1}Q^T\dot{Q}V_e)\,Q^T \tag{1.30.9}$$

$$\mathcal{D}_e^* = Q\mathcal{D}_eQ^T. \tag{1.30.10}$$

It is now clear that (1.26) is materially objective since all three quantities D, D_p and $\mathcal{D}_e$ behave, under change of frame, according to the same transformation law, as is seen from (1.30.4), (1.30.5) and (1.30.10). However, the relation (1.28) is not materially objective since $\mathcal{D}_e$ is not objective, as is seen from (1.30.9).

We now proceed to establish the constitutive law by using (1.26). We first establish the constitutive law for $\mathcal{D}_e$. By taking the Jaumann derivative of (1.10)[1] with respect to W_p

$$(\;)^{\nabla} = (\dot{\;}) - W_p(\;) + (\;)W_p, \tag{1.31}$$

we have

$$\tau = 2\overset{\nabla}{C}_e\frac{\partial\psi_e}{\partial C_e} + 2C_e\left(\frac{\partial^2\psi_e}{\partial C^2} : \overset{\nabla}{C}_e\right) \tag{1.32}$$

or, in component form (dropping the index „e" for the moment)

$$\tau_{ij} = \left[2\delta_{i\alpha}\left(\frac{\partial\psi}{\partial C}\right)_{\beta j} + 2C_{ik}\left(\frac{\partial^2\psi}{\partial C^2}\right)_{kj\alpha\beta}\right]\overset{\nabla}{C}_{\alpha\beta}. \tag{133}$$

Substituting C_e from (1.27) into (1.33), this becomes

$$\overset{\nabla}{\tau}_{ij} = 4\left[V_{im}V_{n\beta}\left(\frac{\partial\psi}{\partial C}\right)_{\beta j} + C_{ik}V_{\alpha m}V_{n\beta}\left(\frac{\partial^2\psi}{\partial C^2}\right)_{kj\alpha\beta}\right]\mathcal{D}_{mn} \tag{1.34}$$

or for short

$$\overset{\nabla}{\tau}_{ij} = \tilde{\Pi}_{ijmn}\;\mathcal{D}_{mn} \tag{1.35}$$

i.e.

$$\overset{\nabla}{\tau} = \tilde{\Pi}\;[\mathcal{D}]. \tag{1.36}$$

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[1]) Restricting ourselves to the isothermal case when the free energy ψ_e becomes the strain energy $\psi_e = \psi_e(C_e)$.

Inverting (1.36) for $\mathcal{D}_e$, we obtain

$$\mathcal{D}_e = \tilde{\Lambda}_e [\overset{\nabla}{\tau}], \tag{1.37}$$

which is the desired form of the rate-type constitutive law for the elastic part of deformation.

With regard to the plastic part of the deformation, we recall the law (1.12). Under the change of frame (1.29), this law behaves, for the case of isotropic hardening, i.e. isotropic f, according to

$$D_p^* = \frac{1}{\mathcal{G}} \left(\frac{\partial f}{\partial \tau^*} : \dot{\tau}^* \right) \frac{\partial f}{\partial \tau^*} \tag{1.38}$$

i.e.

$$D_p^* = Q \left[\frac{1}{\mathcal{G}} \left(Q \frac{\partial f}{\partial \tau} Q^T : \dot{\tau}^* \right) \frac{\partial f}{\partial \tau} \right] Q^T. \tag{1.39}$$

In order that (1.39) satisfies (1.30.4), it is clear that we must have such a rate τ in (1.12), that in (1.39)

$$Q \frac{\partial f}{\partial \tau} Q^T : \dot{\tau}^* \equiv \frac{\partial f}{\partial \tau} : \dot{\tau}. \tag{1.40}$$

But equality (1.40) holds irrespectively of whether the material $(\ )^{\cdot}$ or the Jaumann derivative $(\ )^{\nabla}$ is used, as is to be expected due to the isotropy of the yield criterion, f then being a scalar quantity.[1] Hence, we can use either of them in the constitutive law (1.12), and we choose the Jaumann derivative $(\ )^{\nabla}$, so that (1.12) becomes

$$D_p = \frac{1}{\mathcal{G}} \left(\frac{\partial f}{\partial \tau} : \overset{\nabla}{\tau} \right) \frac{\partial f}{\partial \tau}, \tag{1.41}$$

In the component form this reads

$$D_{ij}^p = \left(\frac{1}{\mathcal{G}} \frac{\partial f}{\partial \tau_{ij}} \frac{\partial f}{\partial \tau_{mn}} \right) \overset{\nabla}{\tau}_{mn} \tag{1.42}$$

i.e.

$$D_{ij}^p = \Lambda_{ijmn}^p \overset{\nabla}{\tau}_{mn} \tag{1.43}$$

or

$$D_p = \Lambda_p [\overset{\nabla}{\tau}]. \tag{144}$$

Now, substitution of (1.37) and (1.44) into (1.26) gives

$$D = (\tilde{\Lambda}_e + \Lambda_p) [\overset{\nabla}{\tau}] \tag{1.45}$$

i.e.

$$D = \tilde{\Lambda} [\overset{\nabla}{\tau}]. \tag{1.46}$$

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[1]) We could have concluded this immediately since $\frac{\partial f}{\partial \tau} : \tau \equiv f$ is a scalar quantity which remains invariant under the change of frame.

This is the rate-type law for the elastic-plastic material. It gives the velocity strain D as a function of the stress rate τ and the tensor (operator) $\tilde{\Lambda}$ which is function of the current state (i.e. stress, and other quantities which define the state). Inverting (1.46) for τ, we obtain the final form of the rate-type constitutive law

$$\overset{\nabla}{\tau} = \tilde{\mathcal{S}}\,[D]. \tag{1.47}$$

We, however, observe that in the laws (1.46) and (1.47), the Jaumann derivative $(\ )^{\nabla}$ is with respect to the plastic spin W_p. Although such a structure of the constitutive law is very prescient in revealing the nature of the kinematics of the elastic-plastic deformation process, in the application of the theory it would be awkward and computationally expensive to work with this structure containing the Jaumann derivative $(\ )^{\nabla}$, since the spin W_p is not simply expressed in terms of the velocity field as is W. Therefore, we now formulate the constitutive laws (1.46) and (1.47), in terms of the Jaumann derivative with respect to the total spin W, rather than the plastic spin W_p.

By taking the Jaumann derivative of (1.10) with respect to W

$$(\ )^{0} = (\ )^{\cdot} - W(\ ) + (\ )\,W, \tag{1.48}$$

we have

$$\overset{0}{\tau}_{ij} = \mathcal{G}_{ij\alpha\beta}\,\overset{0}{C}_{\alpha\beta}, \tag{149}$$

or

$$\overset{0}{\tau} = \mathcal{G}_e\,[\overset{0}{C}_e], \tag{1.50}$$

where, as in (1.33),

$$\mathcal{G}_{ij\alpha\beta} = 2\left[\delta_{i\alpha}\,\frac{\partial\psi}{\partial C_{\beta j}} + C_{ik}\left(\frac{\partial^2\psi}{\partial C^2}\right)_{kj\alpha\beta}\right]. \tag{1.51}$$

But

$$\overset{0}{C}_e = \dot{C}_e - W C_e + C_e W \tag{1.52}$$

and substitution of (1.20) and (1.22), after the appropriate collecting of terms, leads to

$$\overset{0}{C}_e = \mathcal{D}_e\,C_e + C_e\,\mathcal{D}_e \tag{1.53}$$

which can be rewritten as

$$\overset{0}{C}_e = \Sigma_e\,]\mathcal{D}_e] \tag{[1.54)}$$

with

$$\Sigma_{\alpha\beta mn} = \delta_{\alpha m}\,C_{n\beta} + \delta_{\beta n}\,C_{\alpha m}. \tag{1.55}$$

Substituiton of (1.54) into (1.50) now gives

$$\overset{0}{\tau} = \Pi_e\,[\mathcal{D}_e] \tag{1.56}$$

where

$$\Pi_{ijmn} = \mathcal{S}_{ij\alpha\beta}\, \Sigma_{\alpha\beta mn}. \tag{1.57}$$

Inverting (1.56) for D_e , we then get

$$\mathcal{D}_e = \Lambda_e\, [\overset{0}{\tau}]. \tag{1.58}$$

With regard to the plastic part of deformat'on we only observe that the constitutive law (1.44) will be equally valid if the Jaumann derivative ( )$^\nabla$ is replaced by the Jaumann derivative ( )$^\circ$, i.e.

$$D_p = \Lambda_p\, [\overset{0}{\tau}]. \tag{1.59}$$

Substitution of (1.58) and (1.59) into (1.26) gives

$$D = (\Lambda_e + \Lambda_p)\, [\overset{0}{\tau}] \tag{1.60}$$

i.e.

$$D = \Lambda\, [\overset{0}{\tau}]. \tag{1.61}$$

Inversion of (1.61) for $\overset{\circ}{\tau}$ leads to the final form of the rate-type constitutive law

$$\overset{0}{\tau} = \mathcal{S}\, [D]. \tag{1.62}$$

The laws (1.61) and (1.62) are equivalent to laws (1.46) and (1.47). Since in (1.61) and (1.62), the Jaumann derivative is with respect to the total spin W which is directly expressible in terms of the velocity distribution, the forms (1.61) and (1.62) will be used in the application of the theory (for example, in the construction of the variational integral).

It can be shown that tensors $\tilde{\Lambda}$, $\tilde{\mathcal{S}}$ and Λ, $\mathcal{S}$ possess the important symmetry properties. For example:

$$\mathcal{S}_{ijkl} = \mathcal{S}_{jikl} = \mathcal{S}_{ijlk}$$

$$\mathcal{S}_{ijkl} = \mathcal{S}_{klij}. \tag{1.63}$$

This directly leads to the existence of rate potential functions and symmetry of the stiffness matrix in the bnite element formulations of boundary-value problems [6].

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THE ELASTIC — PLASTIC CONSTITUTIVE RELATION

Abstract

Almost all current elastic-plastic theory is based on the total strain-rate being expressed as the sum of elastic and plastic strain-rates. This assumption and the kinematic model on which it is based are invalid and lead to erroneous conclusions.

By defining plastic deformation as the residual deformation after unloading to zero macroscopic stress, the careful separation of elastic and plastic strain-rates is correctly achieved. On this basis the rate (incremental) type constitutive law for time independent material is rigorously established. The law is not restricted to infinitesimal elastic strains, and Prandtl-Reuss equations follow as a special case of this general theory.

The presentation and the development of the whole theory is given at the level of modern continuum mechanics, and thus appears now to be formulated in a satisfactory manner.

ELASTO — PLASTIČNA KONSTITUTIVNA RELACIJA

Izvod

U analizi elasto-plastične deformacije se gotovo uvijek pretpostavlja da je brzina deformacije jednaka sumi elastičnog i plastičnog djela. Ova pretpostavka sa kinematičkim modelom na kome je bazirana je netačna i vodi pogrešnim zaključcima.

Definišući plastičnu deformaciju kao zaostalu deformaciju nakon što se deformisano tjelo rastereti do napona makroskopski jednakog nuli, izvršena je pražljiva i tačna separacija brzine deformacije na elastičan i plastičan dio. Na osnovu toga, izveden je po prvi put na rigorozan način konstitutivni zakon izvodnog tipa za materijal u uslovima elasto-plastične deformacije. Ovaj zakon nije ograničen na male elastične deformacije, pa se Prandtl-Reuss-ove jednačine pojavljuju kao specijalan slučaj ove opšte teorije.

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