TEORIJSKA I PRIMENJENA MEHANIKA 6, pp. 81–93, 1980, UDK 539

ABOUT SOME PROPERTIES OF THE SOLUTIONS THE BOUNDARY VALUE PROBLEMS OF THE BENDING OF A PLATE BY THE IMPROVED THEORY

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(Received Mart, 1980)

1. Let an infinite plate be cut along x-axis by a finite number of cuts $[a_k, b_k]$, where

(1.1) $$-\infty = a_0 < b_0 < a_1 < b_1 < \cdots < a_n < b_n < \infty$$

The cut $[a_0, b_0]$ is thus infinite The boundary conditions for the first and the second boundary value problems of bending will be taken in the form

$$G[\Phi(z), \Psi(z)] =$$

(1.2) $$= [\Phi(z) + \overline{\Phi(z)} + \overline{\Phi'(z)} + \overline{\Psi(z)}]^{\pm} = g^{\pm}(x)$$

$$F[\Phi(z), \Psi(z)] =$$

(1.3) $$= [-\varkappa\Phi(z) + \overline{\Phi(z)} + z\overline{\Phi'(z)} + \overline{\Psi(z)} + \alpha_0 \overline{\Phi''(z)}]^{\pm} = f^{\pm}(x)$$

where $2h$ is the plate thickness and

(1.4) $$\alpha_0 = \frac{2(8+\gamma)}{5(1-\gamma)} h^2 > 0$$

If the plate is not loaded in its region continuously, it is

(1.5) $$g(x) = K_1 \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial z}\right)$$

(1.6) $$f(x) = K_2 \left[G(w) + i \int^{x} H(w)\, dx\right] + ic$$

$$x \in L = \bigcup_{k=0}^{k=n} [a_k, b_k]$$

where all the designations refer to paper [1]. Firstly, the boundary value problem (1—3) will be treated where besides the given boundary load $f(x)$ also point loads in a finite number of given points on the plate will be taken. An analogous assumption will be made for the boundary value problem (1—2). In both cases the solution in the neighbourhood of the end points of cuts will be completely different, as a consequence of Kirchhoff's bending theory.

2. Let us first write the part of the functions $\Phi(z)$ and $\Psi(z)$ characterizing the load resultant of the cut $[a_k, b_k]$. Here it is possible to eliminate the singularity of the behaviour of these functions in end points a_k and b_k if instead of the known forms the following enes are introduced

$$\Phi^{(k)}(z)=\int_{a_k}^{b_k}\left[A_k(\xi-a_k)^2\, b_k-\xi)^2\ln(z-\xi)+\right.$$

$$\left.+\frac{B_k(\xi-a_k)^3(b_k-\xi)^3}{z-\xi}\right]d\,\xi \tag{2.1}$$

$$\Psi^{(k)}(z)=\int_{a_k}^{b_k}\left[\frac{\overline{B}_k(\xi-a_k)^3(b_k-\xi)^3-A_k\,\xi\,(\xi-a_k)^2(b_k-\xi)^2}{z-\xi}+\right.$$

$$\left.+\frac{B_k\,\xi\,(\xi-a_k)^3(b_k-\xi)^3}{(z-\xi)^2}\right]d\,\xi \tag{2.2}$$

$$1\leqslant k\leqslant n$$

These two functons as well as the function $\Phi^{(k)''}(z)$ are bounded in points a_k and b_k.

The constants A_k and B_k can be calculated by means of the corresponding formulae [2] and they are proportional to the resultant of force and couple on the cut $[a_k, b_k]$. The above expressions do not suit an infinite cut, therefore let us assume that $b_0<0$, $0\notin L$

and

$$\Phi^{(0)}(z)=\int_{2b_0}^{b_0}\left[A_0(\xi-2\,b_0)^2\,(b_0-\xi)^2\,ln\,(z-\xi)+\right.$$

$$\left.+\frac{B_0(\xi-2\,b_0)^3(b_0-\xi)^3}{z-\xi}\right]d\,\xi \tag{2.3}$$

$$\Psi^{(0)}(z)=\int_{2b_0}^{b_0}\left[\frac{\overline{B}_0(\xi-2\,b_0)^3(b_0-\xi)^3-A_0\,\xi\,(\xi-2\,b_0)^2(b_0-\xi)^2}{z-\xi}+\right.$$

$$\left.+\frac{B_0\,\xi\,(\xi-2\,b_0)^3(b_0-\xi)^3}{(z-\xi)^2}\right]d\,\xi \tag{2.4}$$

The loads in the given points T_i, $1 \leqslant i \leqslant m$ can be described by the functions

$$\Phi^{(i)}(z) = \alpha_i \ln(z - z_i) + \frac{\beta_i}{z - z_i} \tag{2.5}$$

$$\Psi^{(i)}(z) = \frac{\bar{\beta}_i - \alpha_i \bar{z}_i}{z - z_i} + \frac{\beta_i \bar{z}_i}{(z - z_i)^2} \tag{2.6}$$

Constants α_i and β_i are proportional again to the given loads in points T_i. Thus the functions are obtained

$$\tilde{\Phi}(z) = \sum_{k=0}^{k=n} \bar{\Phi}^{(k)}(z) + \sum_{i=1}^{i=m} \Phi^{(i)}(z) \tag{2.7}$$

$$\tilde{\Psi}(z) = \sum_{k=0}^{k=n} \Psi^{(k)}(z) + \sum_{i=1}^{i=m} \Psi^{(i)}(z) \tag{2.8}$$

which desribe the local behaviour of the solution in the solution in the neighbourhood of points t_i and cuts $[a_k, b_k]$ in the sense of the load resultant. The whole solution of the boundary value problem will be expressed by the functions

$$\Phi(z) = \tilde{\Phi}(z) + \Gamma_1 + \Phi_0(z) \tag{2.9}$$

$$\Psi(z) = \tilde{\Psi}(z) + \Gamma_2 + \Psi_0(z) \tag{2.10}$$

where Γ_1 and Γ_2 are two given constants with a known physical meanings, while in the intersected plane $\Phi_0(z)$ and $\Psi_0(z)$ are two holomorphic functions corresponding to the boundary load where the load resultant is separately equal to zero on each cut. From (2—9) and (2—10) it follows

$$\tilde{\Phi}(z) = R \ln z + \frac{M}{z} + O\left(\frac{1}{z^2}\right) \tag{2.11}$$

$$\tilde{\Psi}(z) = \frac{\bar{M}}{z} + O\left(\frac{1}{z^2}\right) \tag{2.12}$$

where

$$R = \sum_{k=0}^{k=n} A_k \mathcal{S}_k + \sum_{i=1}^{i=m} \alpha_i \tag{2.13}$$

$$M = \sum_{k=0}^{k=n} (A_k U_k + B_k K_k) + \sum_{i=1}^{i=m} (-\alpha_i z_i + \beta_i) \tag{2.14}$$

$$\mathcal{S}_k = \int_{a_k}^{b_k} (\xi - a_k)^2 (b_k - \xi)^2 \, d\xi = \frac{1}{30}(b_k - a_k)^5, \quad 1 \leqslant k \leqslant n \tag{2.15}$$

(2.16)
$$\mathscr{S}_0 = \int_{2b_0}^{b_0} (\xi - 2\,b_0)^2 (b_0 - \xi)^2\, d\,\xi = -\frac{1}{30} b_0^5$$

(2.17)
$$U_k = -\int_{a_k}^{b_k} \xi - a_k)^2 (b_k - \xi)^2\, d\,\xi = -\frac{1}{60} (b_k - a_k)^5 (b_k + a_k),\ 1 \leqslant k \leqslant n$$

(2.18)
$$U_0 = -\int_{2b_0}^{b_0} \xi\,(\xi - 2\,b_0)^2 (b_0 - \xi)^2\, d\,\xi = \frac{1}{20} b_0^6$$

(2.19)
$$V_k = \int_{a_k}^{b_k} (\xi - \xi_k)^3 (b_k - \xi)^3\, d\,\xi = \frac{1}{140} (b_k - a_k)^7,\ 1 \leqslant k \leqslant n$$

(2.20)
$$V_0 = \int_{2b_0}^{b_0} (\xi - 2\,b_0)^3 (b_0 - \xi)^3\, d\,\xi = -\frac{1}{140} b_0^7$$

Thus

(2.21)
$$\Phi_0(z) = o\left(\frac{1}{z}\right)$$

(2.21′)
$$\Psi_0(z) = o\left(\frac{1}{z}\right)$$

(2.22)
$$\Phi(z) = R\,ln\,z + \Gamma_1 \frac{M}{z} + o\left(\frac{1}{z}\right)$$

(2.22′)
$$\Psi(z) = \Gamma_2 + \frac{M}{z} + o\left(\frac{1}{z}\right)$$

In addition, it will be required

(2.23)
$$\Phi_0'(z) = o\left(\frac{1}{z^2}\right)$$

(22.3′)
$$\Phi_0''(z) = o\left(\frac{1}{z^3}\right)$$

The following form will be assumed for the load function $f(x)$:

(2.24)
$$f^{\pm}(x) = \gamma_0\,ln\,|\,x\,| + \gamma_1^{\pm} + \frac{\gamma_2}{x} + o\left(\frac{1}{x^{1+\varepsilon}}\right),\ \varepsilon > 0$$

For $ln\, z$ let the complex plane be cut along the negative part of the real axis and let

$$\Phi^{\pm}(x) = R\, ln\,|x| + R\pi i + \Gamma_1 + \frac{M}{x} + o\left(\frac{1}{x}\right)$$

If in calculating the function $f(x)$, we proceed from point a_k in the upper part of the cut and then move in the positive direction towards the same point a_k in the lower part of the cut, then it follows

(2.25) $$\lim_{x \to a+0} [f^{-}(x) - f^{+}(x)] = 2\pi i (\varkappa + 1) A_k \mathcal{S}_k \qquad 0 \leqslant k \leqslant n$$

and especially still

(2.26) $$2\pi i (\varkappa + 1) A_0 \mathcal{S}_0 = \gamma_1^{-} - \gamma_1^{+}$$

and hence

(2.27) $$Re\left[\gamma_1^{-} - \gamma_1^{+}\right] = 0$$

Further there is still

(2.28) $$(1 - \varkappa) R = \gamma_0$$

(2.29) $$Im\,[\gamma_0] = 0$$

(2.30) $$(1 - \varkappa) M = \gamma_2$$

(2.31) $$ic_0 + \gamma_1^{+} = \left[(1-\varkappa)\,\Gamma_1 + \overline{\Gamma}_2\right] + R - \pi i (1 + \varkappa) R$$

and

(2.32) $$Re\,[(1-\varkappa)\,\Gamma_1 + \Gamma_2] = \frac{1}{2} Re\left[\gamma_1^{-} + \gamma_1^{+}\right] - \frac{\gamma_0}{1-\varkappa}$$

If $Im\left[\gamma_1^{+}\right]$ is given, then the real constant c_0 can be calculated from (2.31).

Finaly the equations

(2.33) $$\int_{a_k}^{b_k} [f^{+}(\xi) - f^{-}(\xi)]\, d\xi = 2\pi i (\varkappa + 1)(a_k \mathcal{S}_k + U_k) A_k + 2\pi i (\varkappa + 1) B_k, \quad 1 \leqslant k \leqslant n$$

(2.34) $$\int_{-\infty}^{b_0} \left\{[f^{+}(\xi) - f^{-}(\xi)] - \left[\gamma_1^{+} - \gamma_1^{-}\right]\right\} d\xi = 2\pi i (\varkappa + 1)(b_0 \mathcal{S}_0 + U_0) A_0 + 2\pi i (\varkappa + 1) B_0 V_0$$

are obtained.

If the following designation

$$f_0^{\pm}(x)=f^{\pm}(x)-F^{\pm}[\tilde{\Phi}(z),\ \tilde{\Psi}(z)]-F[\Gamma_1,\ \Gamma_2]+ic_k$$

(2.35) $$x\in[a_k,\ b_k]=L_k$$

is introduced then boundary value problem (1.3) is reduced to the boundary value problem

(2.36) $$F^{\pm}[\Phi_0(z),\ \Psi_0(z)]=f_0^{\pm}(x)$$

with requirements (2.21), (2.21′), and (2.23)

From the ubove expressions it follows

(2.37) $$f_0^{\pm}(x)=o\left(\frac{1}{x^{1+\varepsilon}}\right),\ \varepsilon>0$$

(2.37′) $$f_0^{+}(a_k+0)=f_0^{-}(a_k+0)$$

(2.37″) $$\int_{ak}^{bk}[f_0^{+}(\xi)-f_0^{-}(\xi)]\,d\xi=0,\ 0\leqslant k\leqslant n$$

The solution of the boundary value problem (2.36) can be found in the easiest way by introducing the function [3]

(2.38) $$\Omega_0(z)=\overline{\Phi}_0(z)+z\,\overline{\Phi}_0'(z)+\overline{\Psi}_0(z)+\alpha_0\overline{\Phi}_0''(z)$$

From (2.36) there follows:

(2.39) $$-\varkappa\Phi_0^{+}(x)+\Omega_0^{-}(x)=f_0^{+}(x),$$

(2.39′) $$-\varkappa\Omega_0^{-}(x)+\Omega_0^{+}(x)=f_0^{-}(x),$$

wherefrom

(2.40) $$\varkappa\Phi_0(z)+\Omega_0(z)=\frac{1}{2\pi i}\int_L\frac{[f_0^{-}(\xi)-f_0^{+}(\xi)]}{\xi-z}\,d\xi$$

(2.41) $$-\varkappa\Phi_0(z)+\Omega_0=\frac{X_0(z)}{2\pi i}\left[\int_L\frac{[f_0^{+}(\xi)+f_0^{-}(\xi)]}{X_0^{+}(\xi)\cdot(\xi-z)}\,d\xi+P_n\left(\frac{1}{\xi}\right)\right],$$

where

(2.42) $$X_0(z)=\frac{z^n}{\sqrt{\prod_{k=1}^{k=n}(z-a_k)\cdot\prod_{k=0}^{k=n}(z-b_k)}}$$

And for polynom $P_n(z)$, it is

(2.43) $$P_n(0)=0.$$

Taking into account the equation (2.37) and additional requirements about the existence of the continuous $f''(x)$ on L and

(2.44) $$f_0'^{\pm}(x)=o\left(\frac{1}{x^{2+\varepsilon}}\right)$$

(2.44) $$f_0''^{\pm}(x)=o\left(\frac{1}{x^{3+\varepsilon}}\right)$$

from (2.40) and (2.41) there immediately follow (2.21), (2.23 and (2.21′).

For the determination of the real constants $3n$, that is, the real constants c_k, $1\leqslant k\leqslant n$ and the coefficients of the polynom $P_n\left(\frac{1}{z}\right)$ there exist equations assuring the uniformity of the bending w:

(2.45) $$\oint_{\lambda_k} d\left(\frac{\partial w}{\partial x}+i\frac{\partial w}{\partial y}\right)=0$$

(2.45′) $$\oint_{\lambda_k} dw=0,\quad 1\leqslant k\leqslant n$$

where λ_k is a curvee round the cut L_k. If this curve is pressed to the cut, then, taking into account (2.37′) and (2.37″) it follows

(2.46) $$\int_{a_k}^{b_k}[\Phi_0^-(\xi)-\Phi_0^+(\xi)]\,d\xi=0$$

$$\int_{a_k}^{b_k}\xi\,Re\{[f_0^-(\xi)-f_0^+(\xi)]+$$

(2.46) $$+(\varkappa+1)[\Phi_0^-(\xi)-\Phi_0^+(\xi)]\}\,d\xi=0$$

$$1\leqslant k\leqslant n$$

Thus, the present problem can be solved if the equations (2.13), (2.14), (2.27) — (2.32) ad all the requirements about the behaviour of the function $f(x)$ on L are fulfilled.

3. In solving the boundary value problem (1.2) the same expressions for $\Phi(z)$ and $\Psi(z)$ are used as before, and on'v constants A_k and B_k are yet to be defined. Here it is taken

(3.1) $$g^{\pm}(x)=\vartheta_0\,ln|x|+\vartheta_1+\frac{\vartheta_2}{x}+o\left(\frac{1}{x^{1+\varepsilon}}\right),\quad \varepsilon>0$$

wherefrom there follows

$$2R=\vartheta_0 \tag{3.2}$$

$$I_m[\vartheta_0]=0 \tag{3.3}$$

$$2M=\vartheta_2 \tag{3.4}$$

and

$$2\Gamma_1+\overline{\Gamma}_2=\vartheta_1-\frac{1}{2}\vartheta_0 \tag{3.5}$$

If the following designation

$$g_0^{\pm}(x)=g^{\pm}(x)-G^{\pm}[\tilde{\Phi}(z),\ \tilde{\Psi}(z)]-G[\Gamma_1,\ \Gamma_2] \qquad x\in L \tag{3.6}$$

is introduced then boundary value problem (1.2) obtains the form

$$G^{\pm}[\Phi_0(z),\ \Psi_0(z)]=g_0^{\pm}(x) \tag{3.7}$$

The functions $\Phi_0(z)$ and $\Psi_0(z)$ are obtained in an analogous way as in the previous case. It is only necesary that in equations (2.39) and (2,39′) we take $\varkappa=-1$ and $\alpha_0=0$.

In the function $g_0(x)$ there are still unknown constants A_k and B_k, $0\leqslant k\leqslant n$. However at arbitrary values of these constants the uniqueness of the function w is valid. For the determination of $5n+3$ real constants firlstly the equations (3.2) and (3.4) and (2.46) from the previous chapter can be used. The rest of the equations follow from the connection of the solution in points b_k and a_{k+1}, $0\leqslant k\leqslant n-1$. These take the form

$$K_1\int_{b_k}^{a_{k+1}} d\left(\frac{\partial w}{\partial x}+i\,\frac{\partial w}{\partial y}\right)=K_1\left\{\left[\frac{\partial w}{\partial x}+i\,\frac{\partial w}{\partial y}\right](x=a_{k+1})-\right.$$

$$\left.-\left[\frac{\partial w}{\partial x}+i\,\frac{\partial w}{\partial y}\right](x=b_k)\right\}=\int_{b_k}^{a_{k+1}}[\Phi(\xi)+\Omega(\xi)]\,d\xi \tag{3.8}$$

$$K_1\int_{b_k}^{a_{k+1}} dw=K_1\,[w(x=a_{k+1})-w(x=b,)]=$$

$$=-\int_{b_k}^{a_{k+1}}\xi\,Re\,[\Phi(\xi)+\Omega(\xi)]\,d\xi+$$

$$+K_1\left\{a_{k+1}\,\frac{\partial w}{\partial x}(x=a_{k+1})-b_k\,\frac{\partial w}{\partial x}(x=b_k)\right\} \tag{3.9}$$

If for $g_0(x)$ the analogous requirements are set as for $f_0(x)$, then the properties (2—21), (2—23) and (2—21′) result. The problem can be solved if the equations (3—2) — (3—5) as well as all the properties required from $g(x)$ are fulfilled.

4. Let $z=c$ be an arbitrary end point of a cut. From the solution of the first or the second boundary value problem there follows that the behaviour of the solving functions $\Phi(z)$ and $\Psi(z)$ in the neighbourhood of each c depends only upon the behaviour of the functions $\Phi_0(z)$ and $\Psi_0(z)$ and $\Omega_0(z)$. From equations (2—40) and (2—41) it follows that in general

(4—1) $$\Phi_0(z)=O\left((z-c)^{-\frac{1}{2}}\right)$$

(4—2) $$\Omega_0(z)=O\left((z-c)^{-\frac{1}{2}}\right)$$

but the functions $\varphi_0(z)\,dz$ and $\omega_0(z)=\int\Omega_0(z)\,dz$ are bounded. While from (2—38) using the designation $\chi_0''(z)=\psi_0'(z)=\Psi_0(z)$ it results:

(4—3) $$\chi_0(z)=\int\left[\overline{\omega}_0(z)+\varphi_0(z)\right]dz-z\,\varphi_0(z)-\alpha_0\,\Phi_0(z)$$

and

(4—4) $$\chi_0(z)=O\left((z-c)^{-\frac{1}{2}}\right)$$

But it is

(4—5) $$K_1 w_0=\overline{z}\,\varphi_0(z)+z\,\overline{\varphi_0(z)}+\chi_0(z)+\overline{\chi_0(z)}$$

Wherefrom it follows that the solution of the boundary value problem (1—3), in all enumerated conditions possesses an unpleasant property in general

(4—6) $$w=O\left((z-c)^{-\frac{1}{2}}\right)$$

The solution w of the boundary value problem (1—2) is bounded in the neighbourhood of the end point c. But if by this solution the load function the cut $f(x)$ is calculated, then the following behaviour of the latter is obtained in the neighbourhood of end points c:

(4—7) $$f(x)=O\left((x-c)^{-\frac{5}{2}}\right)$$

The singularities (4—6) and (4—7) in the classical theory do not appear.

From equations (2—40) and (2—41) it follows that the unbondedness (4—6) in the potnt c does not appear if the following condition is fulfilled

4—8) $$\int_L\frac{\left[f_0^+(\xi)+f_0^-(\xi)\right]}{\chi_0^+(\xi)\cdot(\xi-c)}\,d\xi+P_n\left(\frac{1}{c}\right)=0$$

There are are $2n+1$ of these equatinns and their fulfillment implies a strong reduction of the cllas of functions $f(x)$ which can represent a certain loading of the cut. The number of conditions is still considerably increased if in addition to the boundednes for w also the boundedness of the expressions $\frac{\partial w}{\partial x}+i\frac{\partial w}{\partial y}$ and $\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}+i\frac{\partial w}{\partial y}\right)$ is required.

From the solution of the boundary value problem (1—2) it follows that it is possible to express the functions $\Phi_0(z)$ and $\Psi_0(z)$ in the following form

$$\Phi_0(z)=\frac{1}{2\pi i}\int_L \frac{\omega_1(\xi)}{\xi-z}\,d\xi \tag{4—9}$$

$$\Psi_0(z)=\frac{1}{2\pi i}\int_L \frac{\omega_2(\xi)}{\xi-z}\,d\xi - z\,\Phi_0'(z) \tag{4—10}$$

In order to moke the above functions stress functions, we must ensure that the following is fulfilled:

$$\omega_1(t)=o\left(\frac{1}{t^{1+\varepsilon}}\right) \tag{4—11}$$

$$\omega_2(t)=o\left(\frac{1}{t^{1+\varepsilon}}\right) \tag{4—12}$$

$$\int_{Lk}\omega_1(\xi)\,d\xi=0,\quad 0\leqslant k\leqslant n \tag{4—13}$$

$$\int_{Lk}\omega_2(\xi)\,d\xi=0,\quad 0\leqslant k\leqslant n \tag{4—14}$$

$$\int_{Lk}\xi\,Re\,[\omega_2(\xi)+2\,\omega_1(\xi)]\,d\xi=0,\quad 1\leqslant k\leqslant n \tag{4—15}$$

Now, if a bounded load function $f_0(x)$ is to be obtained, then the function $\Phi_0''(z)$ should be bounded in the neighbourhood of each point c. This will not be true unless the fulfillment of the following equations is required:

$$\omega_1(c)=0 \tag{4—16}$$

$$\omega_1'(c)=0 \tag{4—17}$$

$$\omega_1''(c)=0 \tag{4—18}$$

Further, also $\Psi_0(z)$ has to be a bounded function, thus

$$\omega_2(c)=0 \tag{4—19}$$

In addition,, it is also demanted that thee functions $\omega_1''(x)$ and $\omega_2(x)$ everywhere on L fulfill Hölder's condition H_ε.

If it is written

$$(4\text{—}20) \qquad 2\tilde{\omega}_1(t) - -\varkappa\omega_1(t) + \overline{\omega_1(t)} + \overline{\omega_2(t)} + \alpha_0 \overline{\omega_1''(t)}$$

$$(4\text{—}20') \qquad -\tilde{\omega}_2(t) = \varkappa\omega_1(t) + \overline{\omega_1(t)} + \overline{\omega_2(t)} + \alpha_0 \overline{\omega_1''(t)}$$

then, using Plemelj's formulae, we get:

$$(4\text{—}21) \qquad f_0^{\pm}(\xi) = \pm\tilde{\omega}_1(\xi) + \int_L \frac{\tilde{\omega}_2(t)\,dt}{t-\xi}$$

Wherefrom it follows that in solving the boundary value problem (1—3) the obtained expressions w, $\frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y}$ and $\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y}\right)$ are bounded everywhere on L if the load function $f_0(x)$ takes the form (4—21) in all the above required conditions for the functions $\omega_1(x)$ and $\omega_2(x)$.

From the eqnation (4—7) it results that the load function $f_0(x)$ in the neighbourhood of points c can be unbounded. As an example it will be taken that

$$(4\text{—}22) \qquad f_0^{\pm}(x) = \frac{A_c}{(x-c)^2} + \frac{B_c}{(x-c)} + C_c \ln|x-c| + D_c + o(1)$$

Now the following funotions can be defined

$$(4\text{—}23) \qquad \Phi_*^{(k)}(z) = \frac{1}{2\pi i}\int_{L_k} \frac{\omega(t)\,dt}{t-z}$$

$$(4\text{—}24) \qquad \Psi_*^{(k)}(z) = -z\,\Phi_*^{(k)\prime}(z)$$

where $\omega(t)$ is polynom of the sixth degree the coefficients of which can be calculated with the requirements:

$$(4\text{—}25) \qquad c = b_k : -\frac{1}{2\pi i}\,\omega(b_k) = \overline{A}_c$$

$$(4\text{—}25') \qquad -\frac{1}{2\pi i}\,\omega'(b_k) = \overline{B}_c$$

$$(4\text{—}25'') \qquad \frac{1}{2\pi i}\left[\alpha_0\,\omega''(b_k) + \varkappa\overline{\omega(b_k)} + \omega(b_k)\right] = \overline{C}_c$$

$$(4\text{—}26) \qquad c = a_k : \frac{1}{2\pi i}\,\omega(a_k) = \overline{A}_c$$

(4—26′) $$\frac{1}{2\pi i}\,\omega'(a_k)=\overline{B}_c$$

(4—26″) $$-\frac{1}{2\pi i}\left[\alpha_0\,\omega''(a_k)+\overline{\varkappa\omega(a_k)}+\omega(a_k)\right]=\overline{C}_c$$

and

(4—27) $$\int_{Lk}\omega(t)\,dt=0$$

where the following condition has to be fulfilled at $c=a_k$ and $c=b_k$;

(4—28) $$D_c^+-D_c^-=0$$

what means

(4—29) $$C_c=2\varkappa\overline{A}_c$$

At $k=0$ let $L_0=[2b_0,\ b_0]$ and

(4—30) $$\omega(2b_0)=\omega'(2b_0)=\omega''(2b_0)=0$$

Using the designations

(4—31) $$\Phi_*(z)=\sum_{k=0}^{k=n}\Phi_*^{(k)}(z)$$

(4—31′) $$\Psi_*(z)=\sum_{k=0}^{k=n}\Psi_*^{(k)}(z)$$

now $\Phi_0(z)$ and $\Psi_0(z)$ can be expressed in the form

(4—32) $$\Phi_0(z)=\Phi_*+\Phi_{0*}(z)$$

(4—32′) $$\Psi_0(z)=\Psi_*(z)+\Psi_{0*}(z)$$

By introducing

(4—33) $$f_{0*}^{\pm}(x)=f_0^{\pm}(x)-F^{\pm}\,\Phi_*(z),\ \ \Psi_*(z)]$$

the boundary value problem (2—36) in fulfilling the condition (4—29) is reduced to the boundary value problem

(4—34) $$F^{\pm}[\Phi_{0*}(z),\ \Psi_{0*}(z)]=f_{0*}^{\pm}(x)$$

which can be solved in a quite analogous way as the boundary value problem (2—36) at the bounded load function $f_0(x)$, since the function $f_{0*}(x)$ in the neighbourhood of all points c is bounded.

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ABOUT SOME PROPERTY OF THE SOLUTION OF THE BOUNDARY VALUE PROBLEMS OF THE BENDING OF A PLATE BY THE IMPROVED THEORY

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Summary

In this paper the beoding of a infinite plate cut by a finite number of cuts along one line is discussed. One of the cuts is infinite. Special attention is drawn to the fact that at given loads on cuts the bending in the end points of cuts is in general unbounded. In order to deal with these particularities, special forms of stress functions and conditions are found that have to be fulfilled by the load function on cuts to make the bending and its derivatives bounded everywhere on cuts.

O NEKI LASTNOSTI REŠITVE ROBNIH PROBLEMOV UPOGIBA PLOŠČE PO IZBOLJŠANI TEORIJI

Bogdan Krušić

V tem sestavku je obravnavan upogib neskončne plošče, prerezane s končno mnogo prerezi na eni premici. Eden od prerezov je neskončen. Opozorjeno je, da je ob danih obreminitvah na zarezah upogib v splošnem neomejen v krajiščih prerezov. Za obravnavo te posebnosti so najdene posebne oblike napetostnih funkcij in najdeni pogoji, ki jih mora obremenitvena funkcija na zarezah izpolniti, da so upogib in njegovi odvodi povsod na zarezah omejeni

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