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DONSTITUTIVE EQUATIONS FOR PLASTIC BODIES

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1. **Introduction**

As is well known nonliear theory of elastic-plastic continuum of the "rate type" includes explicite restricitions derived from thermodynamics. On the other hand Thomas (1), has suggested that plastic flow occurs in such a way as to minimize a certain integral taken over the region of flow and over time. The main purpose of this paper is to develop a general form of constitutive equations for elastic-plastic flow of material with couple stresses by using a type of constitutive functional and, as we shall show, thes constitutive aquations result from this "variational" formulation.

2. **Mathematical preliminaries**

In the present paper we use mainly direct vestor and tensor notations. The components of vector and tensor ere referred to the natural bases of local coordinates. We fix a rectangular Cartesian coordinates system in the physical space E so that E is represented in a definite way by the Euclidean space R which constits of ordered triples of real numbers. We assumed tht summation convention will be implied for repeated indices. For the purpnse of the present paper we introduce some simple

D e f i n i t i o n 1. Let $|f>$ be set of elements as

(0) $$|f> = \{f_A , f, f_{AB} \cdot \cdot\}^T$$

where $f, f_A, f_{AB} \cdots$ be scalar, vector, tensor, respective and latin indices $A, B, \cdot\cdot$ run over range $1, 2, \ldots n$, then we it call block or covariant block tensor. On the same way the set of elements $<f| = \{f, f^A, f^{AB}, \cdots\}$, we call contravariant block tensor.

D e f i n i t i o n 2. Iner product we call a product of the form

(1) $$<f|\varphi> = \{f\varphi + f_A \varphi^A + f^{AB} + \cdots\}.$$

3. Variotional formulation

Let we call $< \xi | = \{0,\ e_{ABC},\ e_{AB}\}$ covariant strain block and suppose that it can decomposition on the form

$$< \xi |' = < \xi | - < \xi ||'' \tag{2}$$

where $< \xi |'$ is the plactic strain block and $< \xi |''$ is strain block which is zero when the stresses are zero. On the sama manner we can write time rat of stress block

$$| \dot{F} > = \{\dot{T},\ \dot{S}^{AB},\ \dot{S}^{ABC}\}. \tag{3}$$

Next we admit the existence of a scalar function so called a yield function and it is such that equation

$$\Phi (T,\ \xi''_{\lambda},\ F^{\lambda}) = \varkappa \tag{4}$$

for $\varkappa =$ cons, represents a hypersurface in F^{λ} — spaces. We also postulate constitytive assumptions for Φ and $\varkappa$ as follows

$$\xi_{\lambda}' = \xi_{\lambda}' (T,\ \varkappa,\ \xi''_{\lambda},\ F^{\lambda}) \tag{5}$$

and

$$\varkappa = \varkappa (T,\ \xi''_{\lambda},\ F^{\lambda}) \tag{6}$$

subject to conditions $\xi_{\lambda} = 0$ when $\varkappa = 0$.

In our paper we restrict to the case in which $\dot{\xi}''_{\lambda}$ is of the form

$$\dot{\xi}''_{\lambda} = \alpha_{\lambda\rho} \dot{F}^{\rho} + \alpha_{\lambda} \dot{T} \tag{7}$$

Affter all we can say that condition, for material to be in plastic state is

$$\frac{DI}{Dt} = \dot{I} = \int_{L}^{t} \int_{V} \left[\left(\dot{T} \frac{\partial}{\partial T} + \dot{\xi}''_{\lambda} \frac{\partial}{\partial \xi''_{\lambda}} + F^{\lambda} \frac{\lambda \partial}{\partial F^{\lambda}} \right) \Phi - \dot{\varkappa} \right] dv\, dt = 0 \tag{8}$$

where

$$\dot{\varkappa} = \frac{D}{Dt} \varkappa (T,\ \xi''_{\lambda},\ F^{\lambda}). \tag{9}$$

In the fact if we take time rate of I we have

$$X (\Phi - \varkappa) = 0, \tag{10}$$

where

$$X = \dot{T} \frac{\partial}{\partial T} + \dot{\xi}''_{\lambda} \frac{\partial}{\partial \xi''_{\lambda}} + \dot{F}^{\lambda} \frac{\partial}{\partial F^{\lambda}}. \tag{11}$$

It is known from (1) that $\dot{\varkappa}$ have form $\dot{\varkappa} = \lambda \mathrm{h}^{\lambda} \mu_{\lambda} X \Phi$, then (10) become

$$\lambda \mu_{\lambda} X \Phi \left\{ \mathrm{h}^{\lambda} - \frac{\partial \Phi}{\partial \xi_{\lambda}''} \right\} = X \Phi. \tag{12}$$

If we choose $\lambda \Phi = \wedge$ then (12) can be written

$$\mu_{\lambda} \left(h^{\lambda} - \frac{\partial \Phi}{\partial \xi_{\lambda}''} \right) = +1 \tag{13}$$

or

$$\dot{\xi}_{\lambda}'' = \lambda \mu_{\lambda} \tag{14}$$

where $\lambda > 0$.

From (14) we cand find that

$$\wedge = \frac{\dot{\xi}_{\lambda}'' \mu^{\lambda}}{\mu_{\nu} \mu^{\nu}}, \tag{15}$$

on in developed form

$$\wedge = \frac{\mu_{AB} \dot{e}_{AB}'' + \mu_{ABC} \dot{e}_{ABC}''}{\mu_{LM} \mu_{L}^{LM} + \mu_{LMN} \mu_{LMN}}. \tag{16}$$

If we come back to expresion (15) we can written

$$\dot{e}_{AB}'' = \wedge \mu_{AB}; \quad \dot{e}_{ABC} = \wedge \mu_{ABC}. \tag{17}$$

Conclusion. In the present brief analysis we have derived constitutive equationes for plastic deformation using variational formulationes.

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СВЗЯМИ МЕЖДУ НАПРЯЖЕНЙИЫМИ И ДЕФОРМАЦИЯМИ ДЛЯ ПЛАСТИЧНИХ СРЕД

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Резюме

В этой работе расматривается вопрос определяющих уравнения у пластичных сред учитивающие моменнтые напряжения. Показиывается что

путем испвльзования сот. (8), и несложных вычислительных процедур и на основе ,,вериационной" формулировки задач (12), изложен алгоритм и разработан метод описания определяющих управнения (17), у пластичних сред са моментнымй напряжениями.

KONSTITUTIVNE JEDNAČINE ZA PLASTIČNA TELA

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Izvod

U ovom radu trhtiran je problem formiranja konstitutivnih jednačina za slučaj plastičnih materijala u koji je dopuštena pojava naponskih spregova. Nakon uvođenja pojma bloka tenzora (0), i formiranja izraza (12) običnim postupkom dolazi se do relacija (17), koje predstavljaju tražene veze.

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