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ACCELERATION WAVES IN GRANULAR MATERIALS

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1. Introduction

Recently J. W. Nunziato and K. Walsh (1) considered onedimensional acceleration waves in granular materials. Their investigations were based on a continuum theory of granular materials, developed by Cowin and Goodman (2) (3) (4). Central to their theory is the concept of distributed body which leads naturally to the introduction of an independent kinematical quantity called the volume distrubution function ν. This quantity represents the fact that the granules do not occupy the entire volume of material.

J. W. Nunziato and K. Walsh examined the behavoir of one dimensional acceleration waves in non-conducted materials with a nonuniform, initial volume distribution of granules. In particular they were interested in the influence of void compaction and material nonuniformity on the propagation of waves.

In this paper we present a theory for the behaviour of granular materials. Following a brief review of the Kinematics and balance laws for granular materials, we propuse the appropriate constitutive equations for solid-like. materials.

We then consider, from a general point of view, the behavoir of acceleration waves and discuss the special casses.

2. Thermodynamic Theory of Granular Solids

We follow the approach of Godman and Gowin (2) and assign to the solid the mathematical structure of a distributed body. The motion of such body is described by the functions:

$$x^k = x^k (X^K,\ t) \tag{2.1}$$

An important consequence of the motion of a distributed body is the fact that at any point $x(X^K)$ and time t the density ρ can be decomposed as

$$\rho = \nu \gamma \tag{2.2}$$

where $\gamma=\gamma(X^K, t)$ is the density of granules, and $\nu=\nu(X^K, t)$ $(0\leqslant\nu\leqslant 1)$ is called the volume distribution function. This distribution function represents the ratio of the volume of granules dV_g, to the volume of the material dV i.e.

$$dV_g=\nu\, dV \tag{2.3}$$

The following balance laws for granular materials are postulated to hold for all bodies irrespective of the nature of the body, its geometry and constitution:

Conservation of mass

$$\frac{d}{dt}\int\limits_{V-\sigma}\rho\, dV=0 \tag{2.4}$$

Balance of momentum

$$\frac{d}{dt}\int\limits_{V-\sigma}\rho\,\mathbf{V}\, dV=\int\limits_{\mathcal{S}-\sigma}\mathbf{t}^k\, da_k+\int\limits_{V-\sigma}\rho\,\mathbf{f}dV \tag{2.5}$$

Balance of moment of momentum

$$\frac{d}{dt}\int\limits_{V-\sigma}(\mathbf{r}\times\rho\,\mathbf{v})\, dV=\int\limits_{\mathcal{S}-\sigma}(\mathbf{r}\times\mathbf{t}_{(n)})\, da+\int\limits_{V-\sigma}(\mathbf{r}\times\rho\,\mathbf{f})\, dV \tag{2.6}$$

Conservation of energy

$$\frac{d}{dt}\int\limits_{V-\sigma}\varepsilon+\frac{1}{2}\mathbf{v}\cdot\mathbf{v}+\frac{1}{2}k\,\dot{\nu}^2\Big)dV=\int\limits_{\mathcal{S}-\sigma}(\mathbf{t}_{(n)}\cdot\mathbf{v}+h_n\dot{\nu}-q_n)\,da+ \tag{2.7}$$

$$+\int\limits_{V}\rho\,(f\cdot\mathbf{v}+l\cdot\dot{\nu}+r)\, dV$$

Balance of equilibrated force

$$\frac{d}{dt}\int\limits_{V-\sigma}\rho\, k\,\dot{\nu}\, dV=\int\limits_{\mathcal{S}-\sigma}h_{(n)}\, da+\int\limits_{V-\sigma}\rho\,(l+g)\, dV \tag{2.8}$$

Balance of equilibrated intertia

$$\frac{d}{dt}\int\limits_{V-\sigma}\rho\, k\, dV==0 \tag{2.9}$$

Entropy inequality

$$\frac{d}{dt}\int\limits_{V-\sigma}\rho\eta\, dV\geqslant-\int\limits_{\mathcal{S}-\sigma}\frac{q_n}{\theta}\, da+\int\limits_{V-\sigma}\frac{\rho\, r}{\theta}\, dV \tag{2.10}$$

where

$\mathbf{v}\,(V^k)$ velocity vector

$\mathbf{t}^k\,(t^{kl})$ stress vectors

$\mathbf{f}\,(f^k)$ body force

$\mathbf{r}\,(r^k)$ position vector

$\mathbf{t}_{(n)} = t^k n_k$ surface traction

n_k unit normal to surface

ε internal energy density

k equilibrated inertia

$\mathbf{h}\,(h^k)$ equilibrated stress vector

$\mathbf{q}\,(q^k)$ heat vector

l external equilbrated

r heat source

g intrinsic equilibrated body force

η entropy density

θ absolute temperature

$\sigma(t)$ discontinuity surface moving with a velocity U

$h_{(n)} = \mathbf{h}\cdot\mathbf{n};\ q_n = \mathbf{q}\cdot\mathbf{n}$

$\frac{d}{dt} = \dot{(\ )}$ material derivate

For a large class of problems, it is convenient to employ a formulation based on the reference configuration. Here we express the local balance laws and the jumps condition on a discontinuity surface $\Sigma\ (t)$ in the reference frame X^k:

Conservation of mass

$$\rho_0 = \rho J; \qquad [\rho_0 U_N] = 0 \qquad \text{on } \Sigma \tag{2.11}$$

Balance of momentum

$$T^{Kk}{}_{;K} + \rho_0 (f^k - \dot{V}^k) = 0 \qquad [T^{Kl}]\, N_K + \rho_0 U_N [V^l] = 0 \qquad \text{on } \Sigma \tag{2.12}$$

Balance of moment of momentum

$$T^{Kl} x^k_{;K} = T^{Kk} x^l_{;K} \tag{2.13}$$

Conservation of energy

$$\rho_0 \dot{\varepsilon} = T^{Kl} V_{l;K} - Q^K_{,K} + H^K (\dot{\nu})_{,K} + \rho_0 r - \rho_0 g \dot{\nu} \tag{2.14}$$

$$\rho_0 U_N \left[\left(\varepsilon + \frac{1}{2} V^2 + \frac{1}{2} k \dot{\nu}^2\right)\right] + \left[T^{Kl} V_l + H^K \dot{\nu} - Q^K\right] N_K = 0 \quad \text{on } \Sigma$$

Balance of equilibrated inertia

$$\dot{k} = 0; \qquad \rho_0 U_N [k] = 0 \quad \text{on } \Sigma \tag{2.15}$$

Balance of equilibrated force

$$\rho_0 k \ddot{\nu} = \rho_0 (l+g) + H^K{}_{,K}; \qquad [H^K] N_K + \rho_0 k U_N [\dot{\nu}] = 0 \quad \text{on} \quad \Sigma \tag{2.16}$$

Entropy inequality

$$\rho_0 \dot{\eta} \geqslant -\left(\frac{Q^K}{\theta}\right)_{,K} + \frac{\rho_0 r}{\theta} \tag{2.17}$$

$$\rho_0 U_N [\eta] - \left[\frac{Q^K}{\theta}\right] N_K \leqslant 0 \quad \text{on} \quad \Sigma$$

where we have introduced the material components of the stress tensor, the heat flux, the equilibrated stress vector:

$$T^{Kl} \equiv J X^K_{;k}\, t^{kl}; \quad Q^K \equiv J X^K_{;k}\, q^k; \quad H^K \equiv J X^k_{;k}\, h^k \tag{2.18}$$

We have also denoted by:

$\rho_0 \equiv$ initial mass dencity,

$$J \equiv \det\left(\frac{\partial z^k}{\partial Z^K}\right)$$

U_N = the speed of propagation of a discontinuity sufrace $\Sigma(t)$,
N = the unit normal to the surface $\Sigma(t)$,
$[\varphi] \equiv \varphi^+ - \varphi^-$ the jump of a quantity φ across (the wave) $\Sigma(t)$,
, = partial covariant derivative
; = total covariant derivative,

From (2.14) and (2.17) we obtain

$$-\rho_0 (\dot{\varepsilon} - \theta \dot{\eta}) + T^{Kl} V_{l;k} + H^K (\dot{\nu})_{,k} - \frac{1}{\theta} Q^K \theta_{,k} - \rho_0 g \dot{\nu} \geqslant 0 \tag{2.19}$$

An alternative useful form of (2.19) results if we introduce the Helmhotz free-energy function ψ by

$$\psi \equiv \varepsilon - \theta \eta \tag{2.20}$$

Eliminating between this and (2.20) we obtain:

$$-\rho_0 (\dot{\psi} + \eta \dot{\theta})) + T^{Kl} \nu_{l;K} + H^K (\dot{\nu})_{,K} - \frac{1}{\theta} Q^K \theta_{,K} - \rho_0 \dot{\nu} g \geqslant 0 \tag{2.21}$$

Inequalities (2.19) and (2.21) are the final forms of the entropy eneqality. They are known as the Clausius-Duhem inequality which is postulated to be valid for all indenpendent thermodynamical processes.

We assume that the response of a granular solids is essentially determined by the initial distribution of granules, the changes in void volume, the temperatute, the temerature gradient and the deformation gradient

$$\nu_0; \ \nu; \ \nu_{,k}; \ \theta, \ \theta_{,k}; \ x^k_{;K} \tag{2.22}$$

Thus, as our constitutive assumption we adopt the constitutive equations:

$$
\begin{aligned}
\psi &= \psi\,(_0;\ \eta;\ \eta,_X;\ \theta;\ \theta,_K;\ x^k_{:K}) \\
T^{Kl} &= T^{Kl}\,(\qquad\qquad\qquad\qquad) \\
\eta &= \eta\,(\qquad\qquad\qquad\qquad) \\
g &= g\,(\qquad\qquad\qquad\qquad) \\
H^K &= H^K\,(\qquad\qquad\qquad\qquad) \\
Q^K &= Q^K\,(\qquad\qquad\qquad\qquad)
\end{aligned}
\tag{2.23}
$$

Substituting (2.23) into (2.21) we obtain

$$
\left(T^K{}_l - \rho_0 \frac{\partial \psi}{\partial x_{l;K}}\right) v_{l;K} - \rho_0\left(\eta + \frac{\partial \psi}{\partial \theta}\right)\dot{\theta} + \left(H^K - \rho_0 \frac{\partial \psi}{\partial \nu,_K}\right)\dot{\nu},_K -
\tag{2.24}
$$

$$
-\rho_0\left(\frac{\partial \psi}{\partial \nu}\right)\dot{\nu} - \rho_0 \frac{\partial \psi}{\partial \theta,_K}\dot{\overline{\theta,_K}} - \frac{1}{\theta} Q^K Q,_K \geqslant 0
$$

This inequality is linear in the following set of variables

$$
v_{l;K};\ \dot{\nu};\ \dot{\nu},_K;\ \dot{\nu};\ \dot{\overline{\theta,_K}}
\tag{2.25}
$$

for the coefficients of these variables depend on the set (2.23).
Therefore (2.24) cannot be maintained in one sign unless the coefficient o quantities listed in (2.25), vanish, i.e.

$$
T^{Kk} = \rho_0 \frac{\partial \psi}{\partial x_{k;K}};\quad \eta = -\frac{\partial \psi}{\partial \theta};\quad g = -\frac{\partial \psi}{\partial \nu}
\tag{2.26}
$$

$$
H^K = \rho_0 \frac{\partial \psi}{\partial \nu_{,K}};\quad \frac{\partial \psi}{\partial \theta_{,K}} = 0
$$

and

$$
-\frac{Q^K \theta_{,K}}{\theta} \geqslant 0
\tag{2.27}
$$

From the principle of objectivity it follows that (2.26) can depend on $x^k_{;K}$ only trough the right Caunchy-Green tensor:

$$
C_{KL} = g_{kl}\, x^k_{;K}\, x^l_{;L}
\tag{2.28}
$$

We can thus write the constitutive relations (2.26) in the equivalent form:

$$
\begin{aligned}
T^{KL} &= T^{KL}(\nu_0;\ \nu;\ \nu,_K;\ \theta;\ C_{KL})\\
H^K &= H^K(\qquad\qquad\qquad)\\
g &= g\ (\qquad\qquad\qquad)\\
\eta &= \eta\ (\qquad\qquad\qquad)\\
\psi &= \psi\ (\qquad\qquad\qquad)\\
Q^K &= Q^K(\nu_0;\ \nu;\ \nu,_K;\ \theta;\ \theta,_K\ C_{KL})
\end{aligned}
\tag{2.29}
$$

where we introduced the Piola-Kirchhoff stress tensor:

$$T^{KL} = T^{Kl} X^L_{;l} \tag{2.30}$$

In isotropic thermoelastic granular solids the constitutive functions (2.29) are isotropic scalar, vector or tensor functions of their variables with respect to orthogonal transformations of the reference configuration. Assuming that these functions are polynomials, we can make use of the representation theory of such functions to find the following more explicit forms for T^{KL}, H^K, Q^K (see I. Müller (6)).

$$
\begin{aligned}
T_{KL} &= P_\alpha \overset{\alpha}{C}_{KL} + \underset{\alpha}{R}\, \nu,_{(K}\overset{\alpha}{C}{}^{M}{}_{)M}\nu_M + \underset{\sigma}{S}\, \nu,^M C_{M(K}\overset{\sigma}{C}{}_{L)}{}^N \nu,_N +\\
&\quad + S_0\, \nu;\, C^2_{M(K} C^{2\ N}_{L)}\ \nu,_N \qquad\qquad \alpha = 0, 1, 2\\
H_K &= \bar{H}_\alpha \overset{\alpha}{C}{}_{K.}^{L}\, \nu,_L;\quad Q = \bar{Q}_\alpha \overset{\alpha}{C}{}_{K}^{.L}\, \nu,_L + \underset{\alpha}{\bar{\bar{Q}}}\, \overset{\alpha}{C}{}_{K.}^{L}\, \theta,_L \qquad \sigma = 1, 2
\end{aligned}
\tag{2.31}
$$

The free energy ψ, specific entropy η, the intrinsic equilibrated body force g as well as all scalar coefficients in $(2.31_{1,2})$ can depend on the variables

$$
\begin{aligned}
&\nu_0;\quad \nu;\quad \theta\\
&I_p = \overset{p}{C}{}^K_K;\quad J = \overset{\alpha}{C}{}^{KL}\nu,_K \nu,_L\\
&(p = 1, 2, 3;\qquad \alpha = 0, 1, 2)
\end{aligned}
\tag{2.32}
$$

while the coefficients $\bar{Q}_\alpha$ and $\bar{\bar{Q}}$ depend on (1.32) and

$$K_\alpha = \theta,_A \overset{\alpha}{C}{}^{AB}\theta,_B;\quad M_\alpha = \theta,_\alpha \overset{\alpha}{C}{}^{AB}\nu,_B \tag{2.33}$$

3. General properties of Acceleration Waves

A propagating surface Σ in an acceleration wave is the fields $x^k = x^k(X^K; t)$; $\theta = \theta(x^k; t)$; $\nu = \nu(x^k; t)$ have the following properties:

$$x^k;\quad \dot{x}^k;\quad x^k_{;K};\quad \theta;\quad \dot{\theta};\quad \theta,_K \tag{3.1}$$

are continuous functions of X^K and t jointly for all X^L and t;

(3.2) $$\ddot{x}^k;\quad \dot{x}^k_{;K};\quad x^k_{;KL};\quad \nu,_K;\quad \dot{\nu};\quad \ddot{\theta};\quad \theta,_K;\quad \theta,_{KL}$$

and all higher order derivatives of these quantities suffer, at most, jump discontinuities across Σ, but are continuous in X^L and t jointly everywhere else.

The compatibility conditions of the surface of discontinuity are

(3.3) $$[x^k_{;KL}] = S^k N_K N_L,\ [\dot{x}^k_{;K}] = U_N S^k N_K,\ [\ddot{x}^k] = U_N^2 S^k;$$

$$[\nu,_{KL}] = \omega N_K N_L,\ [\dot{\nu},_K] = -U_N \omega N_K,\ [\ddot{\nu}] = U_N^2 \omega;$$

$$[\theta,_{KL}] = \tau N_K N_L,\ [\dot{\theta}_K] = -\tau U_N N_K,\ [\ddot{\theta}] = U_N^2 \tau$$

where we have denoted by: S^k — vector amplitude of the discontinuity, ω — amplitude of void compaction, τ — amplitude of temperature.

In the absence of heat conduction, we assume that the response of a granular solid is essentially determined by the initial distribution of granules, the changes in void volume, and the deformation gradient, i.e.

(3.4) $$\nu_0;\quad \nu;\quad \nu,_X;\quad x^k_{;K}$$

Hence, we assume that the response of the body is determined by:

(3.5)
$$\psi = \psi\,(\nu_0;\ \nu;\ \nu,_K;\ C_{KL})$$
$$\eta = \eta\,(\qquad\qquad)$$

(3.5)
$$T^{KL} = T^{KL}\,(\qquad\qquad)$$
$$H^K = H^K\,(\qquad\qquad)$$
$$g = g\,(\qquad\qquad)$$

According to (2.20) we can write (2.14), in the following form:

(3.6) $$\rho_0\,\theta\dot{\eta} = \rho\,r$$

In view of our assumption 1 and 2 we have:

(3.7)
$$[\varepsilon] = 0;\quad [\eta] = 0$$
$$[T^{Kl}]\,N_K = 0;\quad [k] = 0;\quad [H^K]\,N_K = 0$$

From (3.7) aud (2.29) we can conclude that:

(3.8) $$[\dot{\nu}] = 0;\quad [\nu,_K] = 0$$

Now, from (3.6) we see that:

(3.9) $$[\dot{\eta}] = 0;\quad [\eta,_K] = 0$$

i.e., every acceleration wave propagating in a granular solid that does not conduct heat is homentropic.

Now, taking the jump of field epuations (2.12) and (2.26) we obtain (3.10)

$$[T^{Kk}_{;K}] = \rho_0 [\ddot{x}^k] \quad \rho_0 k]\ddot{\nu}] = [H^{K\cdot\cdot}_{;X}] \tag{3.10}$$

where we have assumed that body forces are continuous.

A straightforward calculation, using $(3.5)_{3,4}$, leads us to the following relation:

$$T^{Kk}_{;K} = \frac{\partial T^{Kk}}{\partial \nu_0} \nu_{0,K} + \frac{\partial T^{Kk}}{\partial \nu} \nu,_K + \frac{\partial T^{Kk}}{\partial \nu,_L} \nu,_{LK} + \frac{\partial T^{Kk}}{\partial x^p_{;P}} x^p_{;PK}$$

$$H^K_{,K} = \frac{\partial H^K}{\partial \nu_0} \nu_{0,K} + \frac{\partial H^K}{\partial \nu} \nu,_K + \frac{\partial H^K}{\partial \nu,_L} \nu,_{LK} + \frac{\partial H^K}{\partial x^p_{;P}} x^p_{;PK}$$

Assuming that

$$\frac{\partial T^{Kk}}{\partial \nu_0}; \quad \frac{\partial T^{Kk}}{\partial \nu}; \quad \frac{\partial T^{Kk}}{\partial \nu,_L}; \quad \frac{\partial T^{Kk}}{\partial x^p_{;P}}$$

$$\frac{\partial H^K}{\partial x^p_{;P}}; \quad \frac{\partial H^K}{\partial \nu_0}; \quad \frac{\partial H^K}{\partial \nu}; \quad \frac{\partial H^K}{\partial \nu,_L}; \quad \frac{\partial H^K}{\partial x^p_{;P}}$$

are continuous functions on the surface of singularity, we obtain:

$$\begin{aligned} [T^{Kk}{}_{;K}] &= \omega A^{KkL} N_K N_L + A^{Kk\cdot P}_{\ \ p} S^p N_P N_K \\ [H^K_{,K}] &= \omega B^{KL} N_K N_L + B^{K\cdot P}_{\ \ p} S^p N_P N_K \end{aligned} \tag{3.11}$$

where, by definition:

$$\begin{aligned} A^{KkL} &= \frac{\partial T^{Kk}}{\partial \nu,_L}; \quad A^{Kk\cdot P}_{\ \ p} = \frac{\partial T^{Kk}}{\partial x^p_{;P}} \\ B^{KL} &= \frac{\partial H^K}{\partial \nu,_L}; \quad B^{K\cdot P}_{\ \ p} = \frac{\partial H^K}{\partial x^p_{;P}}. \end{aligned} \tag{3.12}$$

If we substitute (3.11) and (3.3) into (3.10) we obtain:

$$\begin{aligned} &\omega A^k + (A^k_{\cdot p} - \rho_0 \delta^k{}_p U_N^2) S^p = 0 \\ &(B - \rho_0 U_N^2)\, \omega + B_p S^p = 0, \end{aligned} \tag{3.13}$$

where we have used the following notations:

$$\begin{aligned} &A^k = A^{KkL} N_K N_L; \quad A^k_{\cdot p} = A^{Kk\cdot P}_{\ \ p} N_K N_P \\ &B = B^{KL} N_K N_L; \quad B_p = B^{K\cdot P}_{\ \ p} N_K N_P \end{aligned} \tag{3.14}$$

The acceleration and void compaction S^p and ω must satisfy the four homogeneous linear equations (3.13). If these equations are to have a non-trivial solution the determinant of the coeficient must vanish and this leads to an equation for the determination of the speed of propagation U_N. In general, the equation for U_N will be of order 12. The remainder of this paper is devoted to the study of a number of special cases.

3.1. Material without voids

In that case $\nu=\nu_0=1$, $\rho_0=\gamma_0$, and the free energy does not depend on ν and $\nu,_K$. From (2.26) there tollows $g=0$; $H_K=0$ and from (2.16) $l=0$.

Because of continuity of function ν as well as its derivatives upon the surface $\Sigma(t)$, equations $(3.3)_{4,5,6}$ now read: $\omega=0$

and the set of equation (3.13) takes the form: $\left(A^{k}_{\cdot p}-\rho_0\,\delta^k{}_p\,U_N^2\right)S^p=0$.

These are very well known expressions derived by Truesdell (5), and they refer to the propagation of waves in elastic materials.

3.2. Materials with incompressible granules

In this case we have $\gamma=\gamma_0$. Upon substituting this into (2.2) and (2.11) we obtain:

$$\dot{\nu}+\nu\,v^{k}_{,k}=0 \tag{3.2.1}$$

Now taking the jump of this field equation, we get $[v^{k}_{,k}]=0$ or equivalently: $S^k n_k=0$.

Hence, we can conclude that in this material only transverse wave may exist.

3.3. Wave propagation of void compaction disturbances

In that case we assume that the response of material does not depend on $x^k_{;K}$. Then $A^{k}_{\cdot p}=0$; $B_p=0$ hold.

The speed of propagation of this waves is determined by:

$$U_N^2=\frac{B}{\rho_0} \tag{3.3.1}$$

under condition that: $A^k=A^{KkL}N_K N_L=0$

C o n c l u s i o n. As can be seen, even in the degree of generality maintained up to now in this analysis, it is possible to obtain some information from equations (3.13).

We now turn our attention to the study of wave propagation in isotropic materials. In this case (3.13) can be written in a diffrent form making use of (2.31) so obtained system of equations is simple and easy to investigate. Finally, we refer the reader ton the resalts of J. W. Nunziato and S. C. Cowin [7] which we have seen after the paper has been completed.

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ВОЛНЫ УСКОРЕНИЯ В ЗЕРНИСТЫХ МАТЕРИЯЛАХ

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Резюме

В работе Нунциато и Волша [1] рассматривется распространение одномерных волн ускорения. Их работа происходит из теории сплошной зернестой среды которая разработана Ковином и Гудманом [2], [3], [4].

В нашей работе рассматривается природа волн ускорения с общей точки зрения. Получены резултаты общего характера. Показано что в этих материялах существуют три волны ускорения которые, в общем случае, не поперечные и не продольные. Полученные результаты представльаыт основу для дальнейшего исследования.

TALASI UBRZANJA U GRANULARNIM MATERIJALIMA

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Izvod

Jednodimenzionalne talase ubrzanje u granularnim materijalima su istraživali J. W. Nunziato i K. Walsh (1.) Njihov rad se zasniva na teoriji granularnih materijala koju su izveli Cowin i Goodman u radovima (2, 3, 4.) J. W. Nunziato i K. Walsh su ispitivali svojstva jednodimenzionalnih talasa ubrzanja u materijalima sa neravnomernom početnom rastresitošću čestica granularnog skeleta.

U ovom radu razmatramo svojstva talasa ubrzanja sa najopštije tačke gledišta u slučaju toplotno provodljivih granularnih materijala. S obzirom na složenost i obimnost ovog problema posebno se razmatra slučaj odsustva toplotnih efekata. Pokazuje se da u takvim materijalima postoje tri akustična talasa koji, u opštem slučaju, ne moraju biti ni transverzalni ni longitidunalni.

Dalje se analiziraju specijalni slučajevi i pokazuju vrednosti brzine prostiranja talasa. Specijalan interes je pokazan za uticaj rasporeda i međusobne povezanosti šupljina u granularnim materijalima na rasprostiranje talasa.

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