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THE INFLUENCE OF FLIGHT CONTROLS ON AIRCRAFT'S EQUATIONS OF MOTION

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When designing a new aircraft with known requirements of its purpose and desired characteristics, it is strongly recomended to desingn its configuration as precisely as possible in the early design stage, and as part of that — the aircraft's control surfaces geometry and weight distribution.

The above mentioned problem especially arises concerning modern fighter plane with an active lift and sideforce controls, i.e. flaperons and canards.

To meet the requirements of stability-and-control investigations in this design stage, it is very important to possess an appropriate mathematical model of an aircraft which would permit to take the aircraft's configuration fully into consideration. For that sake equations of motion are derived from quasicoordinates and quasivelocities using Boltzmann-Hamel equations for the mechanical systems with holonomic constraints in body fixed refereence frame (F_B). We assume the aircraft as a rigid body with six degrees of freedom: longitudinal, lateral and vertical center of gravity (CG) displacement and rolling, pitching

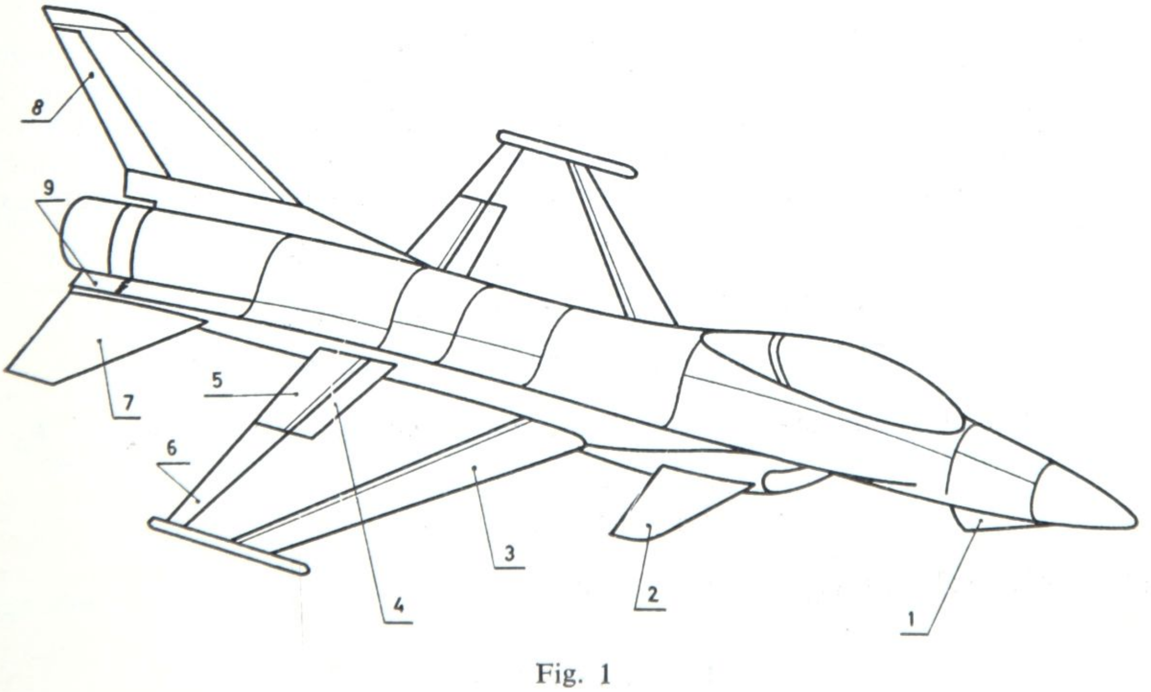

Fig. 1

and yawing. The forces acting on the aircraft are: aerodynamic, gravity, inertial and engine thrust forces. There are taken into consideration relative motions of the following control surfaces: 1) vertical canard, 2) horizontal canard, 3) leading edge, 4) spoilers, 5) inboard flaperons, 6) outboard flaperons, 7) elevator or all moving tail, 8) rudder, 9) aerodynamic brakes. The controls with an asterisk can be deflected also differentialy (one up and the other one down) depending on the maneuver encountered. In the model there is assumed that hinge axes of the controls have a dihedral and that they are swept, as can be seen on Fig. 1. It is also assumed that in filight control system the staiffens is linear and that there exists viscous damping, which is introduced by using Rayleigh's dissipation function.

There are four coordinate systems of interest in the study of a rigid body motions of aerodunamic vehicles (Fig. 2.): 1) earth fixed F_E or inertia

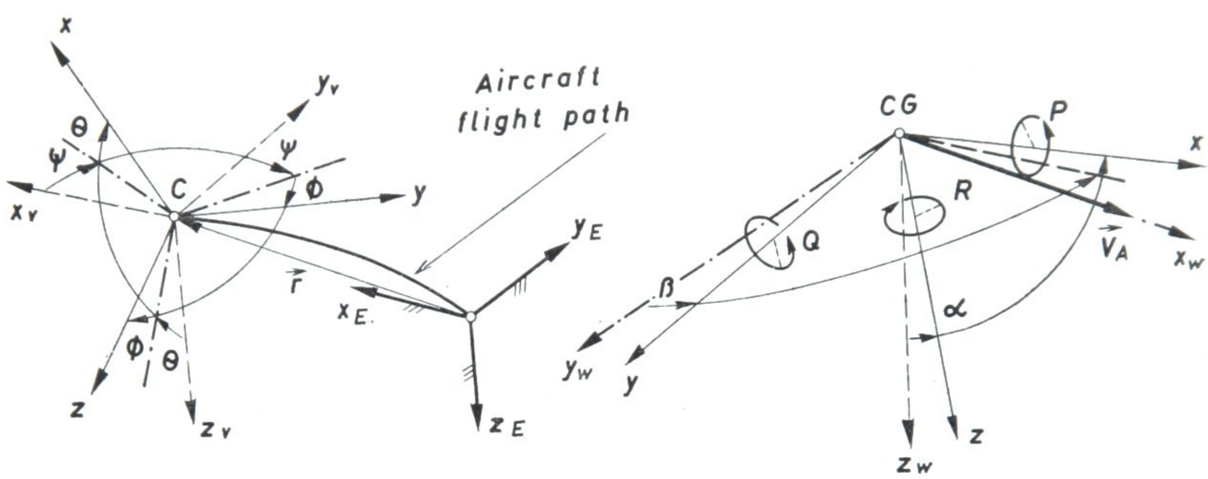

Fig. 2

F_I reference frame $Gx_E y_E z_E$ (which are equivalent when „flat-earth" assumption holds), 2) vehicle carried vertical frame $F_V(Cx_V y_V z_V)$ with axes parallel to F_E, where the origin is at the aircraft's CG, 3) air trajectory reference fame $F_W(Cx_W y_W z_W)$ with the origin at CG and x_W axis directed along aircraft's velocity vector relative to the atmosphere, 4) body fixed reference frame Cxyz.

The aircraft's position in any moment is defined (see Fig. 2.) with:

a) the position of its CG due to F_E by vector $\vec{r}(x_E, y_E, s_E)$,

b) aircraft's Euler angles: Ψ-yawing, Θ-pitching, Φ-rolling.

Vector of the aircraft's instantaneous translational velocity $\vec{V}_A$ in F_B is:

$$\vec{V}_A = U\vec{i} + V\vec{j} + W\vec{k},$$

and angular velocity $\vec{\Omega}_A$ is expreseed as:

$$\vec{\Omega}_A = P\vec{i} + Q\vec{j} + R\vec{k},$$

where kinematical relations between quasivelocities U, V, W and P, Q, R and generalized velocities $\dot{x}_E$, $\dot{y}_E$, $\dot{z}_E$ and $\dot{\Phi}$, $\dot{\Theta}$, $\dot{\Psi}$, are respectively:

(1) $$\mathrm{cal}\,[U,\ V,\ W]=L_V\,\mathrm{cal}\,[\dot{X}_E,\ \dot{Y}_E,\ \dot{Z}],$$

(2) $$\mathrm{cal}\,[P,\ Q,\ R]=L_\Omega\,\mathrm{cal}\,[\dot{\Phi},\ \dot{\Theta},\ \dot{\Psi}],$$

where matrices L_V and L_Ω are given in (1).

It is obvious that U, V, W, P, Q, R are quasivelocities, since their integrabs in time domain, i.e. quasicoordinates π_U, π_V, π_W, π_P, π_Q, π_R do not define aircraft's position explicitly.

Equations of motion are derived, as has already been mentioned, from quasicoordinates and quasivelocities using Boltzmann-Hamel equations for the systems with holonomic constraints. According to (2) they can be written as follows:

(3) $$\frac{d}{dt}\left(\frac{\partial T^*}{\partial\omega_\mu}\right)-\frac{\partial T^*}{\partial\pi_\mu}+\sum_{n=1}^{k}\sum_{\alpha=1}^{k}\gamma_{\mu\alpha}^{r}\frac{\partial T^*}{\partial\omega_r}\,\omega_\alpha=Q_\mu^*,\quad \mu=1,\ \dots\ ,\ k$$

where μ, r, $\alpha=1,\ \dots\ ,\ k$-number of degrees of freedom, π_μ-quasicoordinates, ω_μ-quasivelocities, T^*-kinetic energy, Q^*-generalized forces.

Relations between ω_σ and $\dot{q}_\sigma$ are:

$$\omega\sigma=\sum_{\alpha=1}^{k}a\sigma\alpha\,\dot{q}\alpha \qquad \dot{q}\sigma=\sum_{\mu=1}^{k}b\sigma_\mu\,\omega_\mu \quad \sigma=1,\ \dots\ ,\ k$$

where we assume that:

$$a\sigma\alpha=a\sigma\alpha(q_1,\ \dots\ ,\ q_k) \qquad b\sigma_\mu=b\sigma_\mu(q_1\ \dots\ ,\ q_k)$$

where $q_i\,(i=1,\ \dots\ ,\ k)$ are generalized coordinates. Matrices $[a\sigma_\mu]$ and $[b\sigma_\mu]$ can be written as follows:

$$[a\sigma_\mu]=\begin{bmatrix}L_V & 0 & 0\\ 0 & L_\Omega & 0\\ 0 & 0 & E\end{bmatrix} \qquad b\sigma_\mu=\begin{bmatrix}L_V^{-1} & 0 & 0\\ 0 & L_\Omega^{-1} & 0\\ 0 & 0 & E\end{bmatrix}$$

where E is entity matrix.

Boltzmann's symbols $\gamma_{\mu\alpha}^{r}$ are defined as:

$$\gamma_{\mu\alpha}^{r}=\sum_{\sigma=1}^{k}\sum_{\lambda=1}^{k}\left(\frac{\partial a_{r\sigma}}{\partial q_\lambda}=\frac{\partial a_{r\lambda}}{\partial q_\sigma}\right)b_{\sigma\mu}b_{\lambda\alpha}.$$

The quasivelocities vector of the aircraft model under consideration is:

$$\vec{\omega}=\mathrm{cal}\,[\vec{\omega}_{RB},\ \vec{\omega}_{CD}],$$

where the rigid body (RB) quasivelocities vector is:

$$\vec{\omega}_{RB}=\mathrm{cal}\,[U,\ V,\ W,\ P,\ Q,\ R]$$

and the vector of quasivelocities of relative control deflections (*CD*) is:

$$\vec{\omega}_{CD} = \mathrm{cal}\,[\dot{\delta}_{CV}, \dot{\delta}_{CR}, \dot{\delta}_{CL}, \dot{\delta}_{LE}, \dot{\delta}_{SP}, \dot{\delta}_{FI}, \dot{\delta}_{FR}, \dot{\delta}_{FL}, \dot{\delta}_{HR}, \dot{\delta}_{HL}, \dot{\delta}_{V}\ \dot{\delta}_{AB}].$$

The generalised velocity vector is:

$$\dot{\vec{q}} = \mathrm{cal}\,[\dot{\vec{q}}_{RB}, \dot{\vec{q}}_{CD}]$$

where:

$$\dot{\vec{q}}_{RB} = \mathrm{cal}\,[\dot{x}_E, \dot{y}_E, \dot{z}_E, \dot{\Phi}, \dot{\Theta}, \dot{\Psi}], \ \dot{\vec{q}}_{CD} = \vec{\omega}_{CD}.$$

Boltzmann's symbols $\gamma^{r}_{\alpha\mu}$ $(r, \alpha, \mu = 1, \dots, 6)$ which are not zero for the model under consideration are:

$$\gamma^{1}_{35} = \gamma^{1}_{62} = \gamma^{2}_{16} = \gamma^{2}_{43} = \gamma^{3}_{24} = \gamma^{3}_{51} = \gamma^{4}_{65} = \gamma^{5}_{46} = \gamma^{6}_{54} = -1,$$

$$\gamma^{1}_{26} = \gamma^{1}_{53} = \gamma^{2}_{34} = \gamma^{2}_{61} = \gamma^{3}_{15} = \gamma^{3}_{42} = \gamma^{4}_{56} = \gamma^{5}_{64} = \gamma^{6}_{45} = 1.$$

Introducing the above mentioned and computed symbols into (1) we obtain Boltzmann-Hamel equations for holonomic systems in quasicoordinates and quasivelocities for the model of an aircraft with active controls:

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial U}\right) - \frac{\partial T^*}{\partial \pi_U} - \frac{\partial T^*}{\partial V} R + \frac{\partial T^*}{\partial W} Q = Q^{\bullet}_U \tag{4}$$

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial V}\right) - \frac{\partial T^*}{\partial \pi_V} - \frac{\partial T^*}{\partial W} P + \frac{\partial T^*}{\partial U} R - Q^{\bullet}_V \tag{5}$$

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial W}\right) - \frac{\partial T^*}{\partial \pi_W} - \frac{\partial T^*}{\partial U} Q + \frac{\partial T^*}{\partial V} P = Q^{\bullet}_W \tag{6}$$

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial P}\right) - \frac{\partial T^*}{\partial \pi_P} - \frac{\partial T^*}{\partial W} W + \frac{\partial T^*}{\partial W} V - \frac{\partial T^*}{\partial Q} R + \frac{\partial T^*}{\partial R} Q = Q_P \tag{7}$$

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial Q}\right) - \frac{\partial T^*}{\partial \pi_Q} - \frac{\partial T^*}{\partial W} U + \frac{\partial T^*}{\partial U} W - \frac{\partial T^*}{\partial R} P + \frac{\partial T^*}{\partial P} R = Q^{\bullet}_Q \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial R}\right) - \frac{\partial T^*}{\partial \pi_R} - \frac{\partial T^*}{\partial U} V + \frac{\partial T^*}{\partial V} U - \frac{\partial T^*}{\partial P} Q + \frac{\partial T^*}{\partial Q} P = Q^{\bullet}_R \tag{9}$$

while for each indipendent control we introduce the equation of type:

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial \delta_\nu}\right) - \frac{\partial T^*}{\partial \delta_\nu} + \sum_{r=1}^{18} \sum_{\alpha=1}^{18} \gamma^{r}_{\nu\alpha} \frac{\partial T^*}{\partial \omega_r} \omega_\alpha = Q^* \delta_\nu, \quad (\nu = 7, \dots, 18) \tag{10}$$

The set of equations is completely defined together with the equations of kinematical relations (1) and (2).

Aerodynamic forces and moments measured in F_W (that are represented as coefficients which are functions of the incidence angle α, sideslip angle β and control deflection δ_v) are available in F_B by applying matrix transformation L_A /1/. Aircraft's velocity components i.e. quasivelocities in F_B are:

$$\text{cal}[U,\ V,\ W] = \text{cal}[\cos\alpha\cos\beta,\ \sin\beta,\ \cos\alpha\,\beta\sin\alpha]\,V_A.$$

Gravitu forces in F_B are expressed as:

$$\overrightarrow{mg} = \text{cal}[-\sin\theta,\ \cos\theta\sin\Phi\cos\theta\cos\Phi]\,mg.$$

Now, for example, we shall show how to determine the kinetic energy of the left horizontal canard showed in Fig. 3. To define canard's translational

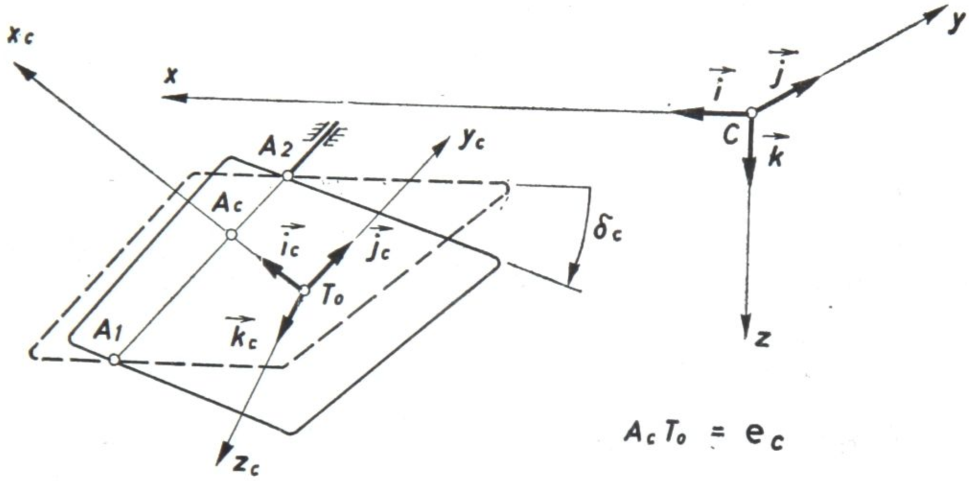

FIG.3

and angular velocity we must determine unity vectors $\vec{i}_c$, $\vec{j}_c$, $\vec{k}_c$ of the canard fixed coordinate axes with the origin conveniently situated in its CG (which of course does not have to lie on hinge axis).

The angular velocity is:

$$\vec{\Omega}_T = (\vec{\Omega}_T)_P + (\vec{\Omega}_T)_r = \vec{\Omega}_A + (\vec{\Omega}_T)_r$$

$$(\vec{\Omega}_T)_r = \dot{\delta}_C j_C$$

and translational velocity respectively:

$$\vec{V}_T = (\vec{V}_T)_P + (\vec{V}_T)_\Upsilon = \vec{V}_A + \vec{\Omega}_A \times \overrightarrow{CT} + (\vec{V}_T)_\Upsilon$$

$$(\vec{V}_T)_P = U\vec{i} + V\vec{j} + W\vec{k} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P & Q & R \\ x_{AC} - e_C i_{CX} - y_{AC} - e_C i_{Cy} & z_{AC} - e_C i_{CZ} \end{vmatrix}$$

$$(\vec{V})_r = (\vec{\Omega}_T)_r \times \overrightarrow{A_C T} = \dot{\delta}_C \vec{j}_C \times (-e_C \vec{i}_C) = \dot{\delta}_C e_C \vec{k}_C$$

Then the kinetic energy of the left horizontal canard can be written as follows:

$$T^*_{LHC} = [U + Q\,(z_{AC} - e_C\, i_{CZ}) + R\,(y_{AC} + e_C\, i_{CY}) + \dot{\delta}_C\, e_C\, k_{CX}]^2\, m_{LHC}/2 +$$

$$+ [V - P\,(z_{AC} - e_C\, i_{CZ}) + R\,(x_{AC} - e_C\, i_{CX}) = \dot{\delta}_C\, e_C\, k_{CY}]^2\, m_{LHC}/2 +$$

$$+ [W - P\,(y_{AC} + e_C\, i_{CY}) = Q\,(x_{AC} - e_C\, i_{CX}) + \dot{\delta}_C\, e_C\, k_{CZ}]^2\, m_{LHC}/2 +$$

$$+ (Pi_{CX} = Qi_{CY} + Ri_{CZ})^2\, J_{LHCX}/2 + (Pj_{CX} + Qi_{CY} + Rj_{CZ} +$$

$$+ \dot{\delta}_C)^2\, J_{LHCY}/2 + (Pk_{CX} + Qk_{CY} + Rk_{CZ})^2\, J_{LHCZ}/2 + (Pi_{CX} + Qi_{CY} +$$

$$+ Ri_{CZ}) * (Pk_{CX} + Qk_{CY} + Rk_{CZ})\, J_{LHCXZ}$$

In the same way we can determine the expressions for T^* of the other controls.

Only for she case of single vertical canard and for single rudder are the expressions for their kinetic energies somewhat simpler, because these controls have Cxz plane of symmetry.

Let us now concentrate on equations (4) + (9) on the example of right hotizontal canard. For this specific case they can be written:

$$\dot{U} m_{CR} + \dot{Q}_{Cr_3} - \dot{R}_{Cr_2} - VRm_{CR} + WQm_{CR} + PQ_{Cr_2} + RP_{Cr_3} -$$

$$- Q^2_{Cr_1} - \mathrm{R}^2_{Cr_1} - \ddot{\delta}_{CR}\, m_{CR}\, e_{CR}\, k_{CRX} - \dot{\delta}_{CR}\, m_{CR}\, e_{CR}\,(Qk_{CRZ} -$$

$$- \mathrm{R}k_{CRY}) = (Q^*_U)_{CR}$$

$$\dot{V} m_{CR} - \dot{P}_{Cr_3} + \dot{\mathrm{R}}_{Cr_1} + U\mathrm{R}m_{CR} - WPm_{CR} + PQ_{Cr_1} + Q\mathrm{R}_{Cr_3} -$$

$$- P^2_{Cr_2} - \mathrm{R}^2_{Cr_2} - \ddot{\delta}_{CR}\, m_{CR}\, e_{CR}\, k_{CRY} + \dot{\delta}_{CR}\, m_{CR}\, e_{CR}\,(Rk_{CRZ} -$$

$$- \mathrm{R}k_{CRX}) = (Q^*_V)_{CR}$$

$$\dot{W} m_{CR} + \dot{P}_{Cr_2} - \dot{Q}_{Cr_1} - UQm_{CR} + VPm_{CR} + Q\mathrm{R}_{Cr_2} + RQ_{Cr_1} -$$

$$- P^2_{Cr_3} - Q^2_{Cr_3} - \ddot{\delta}_{CR}\, m_{CR}\, e_{CR}\, k_{CRZ} - \dot{\delta}_{CR}\, m_{CR}\, e_{CR}\,(Pk_{CRY} -$$

$$- Qk_{CRX}) = (Q^*_W)_{CR}$$

$$- \dot{V}_{Cr_3} + \dot{W}_{Cr_2} + \dot{P}_{Cr_8} - \dot{Q}_{Cr_4} - \dot{\mathrm{R}}_{Cr_6} - UQ_{C1_2} - U\mathrm{R}_{Cr_3} +$$

$$+ VP_{Cr_2} + WP_{Cr_3} - PQ_{Cr_6} - QP_{Cr_{11}} + \mathrm{R}P_{Cr_4} - Q^2_{Cr_5} +$$

$$+ \mathrm{R}^2_{Cr_5} - \ddot{\delta}_{CR\; Cr_{13}} - \dot{\delta}_{CR}\, Q_{Cr_{15}} + \dot{\delta}_{CR}\, \mathrm{R}_{Cr_{14}} + \dot{\delta}_{CR}\, m_{CR}\, e_{CR} *$$

$$* (Wk_{CRY} - Vk_{CRZ}) = (Q^*_P)_{CR}$$

$$\dot{U}_{Cr_3} - \dot{W}_{Cr_1} - \dot{P}_{Cr_4} + \dot{Q}_{Cr_9} - \dot{\mathrm{R}}_{Cr_5} + UQ_{Cr_1} - VP_{Cr_1} -$$

$$-V\mathrm{R}_{Cr_3}+WQ_{Cr_3}+PQ_{Cr_5}-Q\mathrm{R}_{Cr_4}+\mathrm{R}P_{Cr_{12}}P^2_{Cr_{12}}-$$

$$-\mathrm{R}^2_{Cr_6}-\ddot{\delta}_{CR\,Cr_{14}}-\dot{\delta}_{CR}\mathrm{R}_{Cr_{13}}+\dot{\delta}_{CR}P_{Cr_{15}}-\dot{\delta}_{CR}m_{CR}e_{CR}*$$

$$*(Wk_{CRX}-Uk_{CRZ})=(Q^*_Q)_{CR}$$

$$-\dot{U}_{Cr_2}+\dot{V}_{Cr_1}-\dot{P}_{Cr_6}-\dot{Q}_{Cr_5}+\dot{\mathrm{R}}_{Cr_7}+U\mathrm{R}_{Cr_1}+V\mathrm{R}_{Cr_2}-$$

$$-WP_{Cr_1}-WQ_{Cr_2}-PQ_{Cr_{10}}+Q\mathrm{R}_{Cr_6}-\mathrm{R}P_{Cr_5}-P^2_{Cr_4}+$$

$$+Q^2_{Cr_4}-\ddot{\delta}_{CR\,Cr_{15}}-\dot{\delta}_{CR}P_{Cr_{14}}+\dot{\delta}_{CR}Q_{Cr_{13}}+\dot{\delta}_{CR}m_{CR}e_{CR}*$$

$$*(Vk_{CRX}-Uk_{CRY})=(Q^*_R)_{CR}.$$

where the meaning of the coefficients cr_1 through cr_{15} is:

$$cr_1=m_{CR}(x_{CR}-e_{CR}\,i_{CRX})\qquad cr_2=m_{CR}(y_{CR}-e_{CR}\,i_{CRY})$$

$$cr_3=m_{CR}(z_{CR}-e_{CR}\,i_{CRZ})$$

$$cr_4=cr_1(y_{CR}-e_{CR}\,i_{CRY})-J_{CRX}i_{CRX}i_{CRY}-J_{CRY}\,j_{CRX}\,j_{CRY}-$$

$$-J_{CRZ}\,k_{CRX}\,k_{CRY}-J_{CRXZ}(i_{CRX}k_{CRY}+i_{CRY}k_{CRX})$$

$$cr_5=cr_2(z_{CR}-e_{CR}\,i_{CRZ})-J_{CRX}i_{CRY}i_{CRZ}-J_{CRY}j_{CRY}j_{CRZ}-$$

$$-J_{CRZ}\,k_{CRY}k_{CRZ}-J_{CRXZ}(i_{CRY}k_{CRZ}+i_{CRZ}\,k_{CRY})$$

$$cr_6=cr_1(z_{CR}-e_{CR}\,i_{CRZ})-J_{CRX}\,i_{CRX}\,i_{CRZ}-J_{CRY}\,j_{CRX}\,j_{CRZ}-$$

$$-J_{CRZ}\,k_{CRX}\,k_{CRZ}-J_{CRXZ}(i_{CRX}\,k_{CRZ}+i_{CRZ}\,k_{CRX})$$

$$cr_7=m_{CR}(x_{CR}-e_{CR}\,i_{CRX})^2+m_{CR}(y_{CR}-e_{CR}\,i_{CRY})^2+J_{CRX}\,i^2_{CRX}+$$

$$+J_{CRY}\,j^2_{CRZ}+J_{CRZ}\,k^2_{CRZ}+2\,J_{CRXZ}\,i_{CRZ}\,k_{CRZ}$$

$$cr_8=m_{CR}(y_{CR}-e_{CR}\,i_{CRY})^2+m_{CR}(z_{CR}-e_{CR}\,i_{CRZ})^2+J_{CRX}\,i^2_{CRX}+$$

$$+J_{CRY}\,j^2_{CRX}+J_{CRZ}\,k^2_{CRX}+2\,J_{CRXZ}\,i_{CRX}\,k_{CRX}$$

$$cr_9=m_{CR}(z_{CR}-e_{CR}i_{CRZ})^2+m_{CR}(x_{CR}-e_{CR}i_{CRX})^2+J_{CRX}\,i^2_{CRY}+$$

$$+J_{CRY}\,j^2_{CRY}+J_{CRZ}\,k^2_{CRY}+2\,J_{CRXZ}\,i_{CRY}\,k_{CRY}$$

$$cr_{10}=m_{CR}(y_{CR}-e_{CR}\,i_{CRY})^2-m_{CR}(x_{CR}-e_{CR}\,i_{CRX})^2+J_{CRX}*$$

$$*(i^2_{CRX}-i^2_{CRY})+J_{CRY}(j^2_{CRX}-j^2_{CRY})+J_{CRZ}(k^2_{CRX}-k^2_{CRY})+$$

$$+2\,J_{CRXZ}(i_{CRX}k_{CRX}-i_{CRY}k_{CRY})$$

$$cr_{11} = m_{CR}(z_{CR} - e_{CR}\, i_{CRZ})^2 - m_{CR}(y_{CR} - e_{CR}\, i_{CRY})^2 + J_{CRX} *$$

$$* \, (i^2_{CRY} - i^2_{CRZ}) + J_{CRY}(j^2_{CRY} - j^2_{CRZ}) + J_{CRZ}(k^2_{CRY} - k^2_{CRZ}) +$$

$$+ 2\, J_{CRXZ}(i_{RY}\, k_{CRY} - i_{CRZ}\, k_{CRZ})$$

$$cr_{12} = m_{CR}(z_{CR} - e_{CR}\, i_{CRZ})^2 - m_{CR}(x_{CR} - e_{CR}\, i_{CRX})^2 + J_{CRX} *$$

$$* \, (i^2_{CRX} - i^2_{CRZ}) + J_{CRY}(j^2_{CRX} - i^2_{CRZ}) + J_{CRZ}(k^2_{CRX} = k^2_{CRZ}) +$$

$$+ 2\, J_{CRXZ}(i_{CRX}\, k_{CRX} - i_{CRZ}\, k_{CRZ})$$

$$cr_{13} = -cr_3\, e_{CR}\, k_{CRY} + cr_2\, e_{CR}\, k_{CRZ} + J_{CRY}\, j_{CRX}$$

$$cr_{14} = cr_3\, e_{CR}\, k_{CRX} - cr_1\, e_{CR}\, k_{CRZ} + J_{CRY}\, j_{CRY}$$

$$cr_{15} = -cr_2\, e_{CR}\, k_{CRX} + cr_1\, e_{CR}\, k_{CRY} + J_{CRY}\, j_{CRZ}.$$

It is quite obvious that we can obtain the influence of each control in Boltzmann-Hamel equations of motion of an aircraft if we repeatedly use the above shown procedure.

After some regroupings and rearrangements, there are derived complete differential equations of motion of an aircraft together with $1\,(1 = 7, \ldots, 18)$ equations of controls of tupe (10).

To perceive the advanteges of the suggested mode', it is sufficient to review the aircraft's equations of motion assuming that the controls deflection velocities are constant. Bearing in mind the above mentioned assumptions, the equations can be written as:

$$\underline{\dot{U}_{C13}} + \dot{Q}_{C3} + \dot{R}_{C2} - \underline{VR_{C13}} + \underline{WQ_{C13}} - PQ_{C2} + RP_{C3} +$$

$$+ Q^2_{C1} + R^2_{C1} - \sum_{\nu=7}^{18} \dot{\delta}_\nu\, m_\nu\, e_\nu\,(Qk_{\nu_z} - Rk_{\nu_y}) = Q^*_U$$

$$\underline{\dot{V}_{C13}} - \dot{P}_{C3} - \dot{R}_{C1} + \underline{UR_{C13}} - \underline{WP_{C13}} - PQ_{C1} + QR_{C3} +$$

$$+ P^2_{C2} + R^2_{C2} + \sum_{\nu=7}^{18} \dot{\delta}_\nu\, m_\nu\, e_\nu\,(Pk_{\nu_z} - Rk_{\nu x}) = Q^*_V$$

$$\underline{\dot{W}_{C13}} - \dot{P}_{C2} + \dot{Q}_{C1} - \underline{UQ_{C13}} + \underline{VP_{C13}} - QR_{C2} - RP_{C1} -$$

$$- P^2_{C3} - Q^2_{C3} - \sum_{\nu=7}^{18} \dot{\delta}_\nu\, m_\nu\, e_\nu\,(Pk_{\nu_y} - Qk_{\nu x}) = Q^*_W$$

$$- \dot{V}_{C3} - \dot{W}_{C2} + \underline{\dot{P}_{C8}} + \dot{Q}_{C4} + \underline{\dot{R}_{C6}} = UQ_{C2} - UR_{C3} -$$

$$- VP_{C2} + WP_{C3} + PQ_{C6} + QR_{C11} - RP_{C4} + Q^2_{C5} - R^2_{C5}$$

$$- Q \sum_{\nu=7}^{18} \dot{\delta}_{\nu\, C\nu 15} + \sum_{\nu=7}^{18} \dot{\delta}_{\nu\, C\nu 14} + \sum_{\nu=7}^{18} \dot{\delta}_\nu\, m_\nu\, e_\nu\,(Wk_{\nu_y} - Vk_{\nu_z}) = Q^*_P$$

$$\dot{U}_{C3}+\dot{W}_{C1}+\dot{P}_{C4}+\underline{\dot{Q}_{C9}}+\dot{R}_{C5}-UQ_{C1}+VP_{C1}-$$

$$-VR_{C3}+WQ_{C3}-PQ_{C5}+QR_{C4}+\underline{RP_{C12}}-\underline{P^{2}_{C6}}+\underline{R^{2}_{C6}}$$

$$-R\sum_{\nu=7}^{18}\dot{\delta}_{\nu}\,C_{\nu 13}+P\sum_{\nu=7}^{18}\dot{\delta}\,C_{\nu 15}-\sum_{\nu=7}^{18}\dot{\delta}_{\nu}\,m_{\nu}\,e_{\nu}\,(Wk_{\nu x}-Uk_{\nu z})=Q_{Q}^{*}$$

$$\dot{U}_{C2}-\dot{V}_{C1}+\underline{\dot{P}_{C6}}+\dot{Q}_{C5}+\underline{\dot{R}_{C7}}-UR_{C1}-VR_{C2}+$$

$$+WP_{C1}+WQ_{C2}+\underline{PQ_{C10}}-\underline{QR_{C6}}+RP_{C5}+P^{2}_{C4}-Q^{2}_{C4}-$$

$$-P\sum_{\nu=7}^{18}\dot{\delta}_{\nu}\,C_{\nu 14}+Q\sum_{\nu=7}^{18}\dot{\delta}_{\nu}\,C_{\nu 13}+\sum_{\nu=7}^{18}\dot{\delta}_{\nu}\,m_{\nu}\,e_{\nu}\,(Vk_{\nu x}-Uk_{\nu y})=Q_{R}^{*}$$

where in the previous models there only the underlined expressions were considered, while all the others were omitted irrespective whether the aircraft had the „xz“ plane of symmetry or not.

We shall now consider paralely inertia matrices of conventional A_c and new model A_n:

$$A_c=\begin{Bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_x & 0 & -J_{xz} \\ 0 & 0 & 0 & 0 & J_y & 0 \\ 0 & 0 & 0- & J_{xz} & 0 & J_z \end{Bmatrix}$$

$$A_n=\begin{Bmatrix} c\,13 & 0 & 0 & 0 & c\,3 & \underline{c\,2} \\ 0 & c\,13 & 0 & -c\,3 & 0 & -c\,1 \\ 0 & 0 & c\,13 & -\underline{c\,2} & c\,1 & 0 \\ 0 & -c\,3 & -\underline{c\,2} & c\,8 & \underline{c\,4} & c\,6 \\ c\,3 & 0 & -c\,1 & \underline{c\,4} & c\,9 & \underline{c\,5} \\ \underline{c\,2} & -c\,1 & 0 & c\,6 & \underline{c\,5} & c\,7 \end{Bmatrix}$$

The marked antisymmetric three-by-three sub-matrices in inertia matrih A_n represent giroscopic influences which are not taken into account in previous models. All dash-underlined coefficients in A_n differe from zero only for differential deflections of symmetric controls and / or for deflections of single controls (vertical canard or rudder).

Modern aircraft, especially those with active controls can produce strong inertial and kinematical couplings beoause of their complex configurations. The model of the aircraft under consideration in this work can handle this problem, and because of that, it permits a more complete investigation of the aircraft flight qualities in the early design stage or in an already flying aircraft.

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L'INFLUENCE DES CONTROLS DE VOL SUR LES ÉQUATIONS DIFFERENTIELLES DU MOUVEMENT D'AVION

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Résumé

L'influence des controls de vol sur les équations differentielles du mouvement d'avion est discuté par l'application des équations Boltzmann-Hamel pour les systèmes holonomiques. Il est demontré que pour le cas de vitesses relatives constantes de deflections de controls de vol on a obtenu un système d'équations differentielles du mouvement d'avion plus complet qui en outre prends en consideration aussi les influences giroscopiques.

UTICAJ KOMANDI LETA NA DIFERENCIJALNE JEDNAČINE KRETANJA AVIONA

Gorazd Paljaruci

Izvod

Razmotren je uticaj komandi leta na diferencijalne jednačine kretanja aviona primenom Bolcman-Hamelovih jednačina za holonomne sisteme. Pokazano je da je već za slučaj konstantnih relativnih brzina othlona komandi dobijen potpuniji sistem diferencijalnih jednačina kretanja aviona koji između ostalog uzima u obzir i žiroskopske uticaje.

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