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# AN APPLICATION OF COMPLEX ANALYSIS OF THREE-DIMENSIONAL FLOW OF A FLUID

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## 1. **Introduction**

Let fluid be incompressible and viscous and its flow slow. In the case that outer forces are not taken into account, the Navier-Stokes's equation can be written as

$$(1—1) \qquad \Delta \vec{U} - \frac{1}{\mu} \operatorname{grad} p = 0$$

where

$\vec{U}$ = velocity field of the fluid flow

$p$ = hydrodynamic pressure of the fluid

$\mu$ = constant

and

due to incompressibility

$$(1—2) \qquad \operatorname{div} \vec{U} = 0$$

In the methodological sense, the solution of the system of equations (1—1) and (1—2) in the case of a plane flow: $\vec{U} = \vec{U}(x, y)$ and $p = p(x, y)$ is of great interest. This problem is *particly* treated in the works [1], [2], [3], [4—7]. An analogy to the conditions at plane elasticity and at bending of a thin or moderately thick plate is found. All the known Muskhelishvili's methods [10], [11] can be applied here too. The problem is treated in all its complexity *the most* in papers [8] and [9].

In our presentation a possible extension of the use of these methods will be given. The concession that the velocitty field is linearly dependent upon the third coordinate $Z$ will be made.

## 2. Fundamental expressions of $\vec{U}$ and $p$

In order to get a simpler expression let a scalar field first be introduced

(2—1) $$V = -\frac{1}{\mu} p$$

then the system of equations (1 – 1) and (1 – 2) is expressed as

(2—2) $$\Delta \vec{U} - \operatorname{grad} V = 0$$

(2—2′) $$\operatorname{div} \vec{U} = 0$$

This system will be solved at the afore mentioned supposition

(2—3) $$\frac{\partial^2 \vec{U}}{\partial Z^2} = 0$$

whereout after a short calculation follows

(2—4) $$\vec{U} = \vec{\psi}_1 (x, y) \cdot Z - \vec{\psi}_2 (x, y)$$

From (2—2) we still get

(2—5) $$V = aZ^2 + \varphi_1 (x, y) Z + \varphi_2 (x, y)$$

where $a$ is an arbitrary constant, $\varphi_1$ and $\varphi_2$ scalar functions, and $\vec{\psi}_1$ and $\vec{\psi}_2$ vector functions all independent upon $Z$.
(2—2′) still yields

(2—6) $$\vec{U} = \operatorname{rot} \vec{W}$$

From the comparison of (2—4) and (2—6) after a short calculation again follows

(2—7) $$\vec{W} = \vec{A} (x, y) \cdot Z + \vec{B} (x, y)$$

If we write

(2—8) $$\vec{A} (x, y) = (A_1 (x, y), A_2 (x, y), A_3 (x, y))$$

(2—9) $$\vec{B} (x, y) = (B_1 (x, y), B_2 (x, y), B_3 (x, y))$$

then the following expression of velocity field is obtained from (2—6)

$$\vec{U} = \operatorname{rot} \vec{W} = \left( \frac{\partial A_3}{\partial y}, -\frac{\partial A_3}{\partial x}, \frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} \right) \cdot Z +$$

(2—10) $$+ \left( \frac{\partial B_3}{\partial y} - A_2, -\frac{\partial B_3}{\partial x} + A_1, \frac{\partial B_2}{\partial x} - \frac{\partial B_1}{\partial y} \right)$$

If this is compared still to the expression (2—4) and written as

(2—11) $$\vec{\psi}_1(x, y) = (D_1(x, y), D_2(x, y), D_3(x, y))$$

(2—12) $$\vec{\psi}_2(x, y) = (C_1(x, y), C_2(x, y), C_3(x, y))$$

then in addition follows

(2—13) $$D_1 = \frac{\partial A_3}{\partial y}, \quad D_2 = -\frac{\partial A_3}{\partial x}, \quad D_3 = \frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}$$

(2—14) $$C_1 = \frac{\partial B_3}{\partial y} - A_2, \quad C_2 = -\frac{\partial B_3}{\partial x} + A_1, \quad C_3 = \frac{\partial B_2}{\partial x} - \frac{\partial B_1}{\partial y}$$

The vectors $\vec{\psi}_1$ and $\vec{\psi}_2$ have to satisfy the equation (2—2), whereout the following equations are obtained

(2—15) $$\frac{\partial(\Delta A_3)}{\partial y} + \frac{\partial \varphi_1}{\partial x} = 0 \Rightarrow \Delta D_1 + \frac{\partial \varphi_1}{\partial x} = 0$$

(2—16) $$-\frac{\partial(\Delta A_3)}{\partial x} + \frac{\partial \varphi_1}{\partial y} = 0 \Rightarrow \Delta D_2 + \frac{\partial \varphi_1}{\partial y} = 0$$

(2—17) $$\Delta D_3 + 2a = 0$$

(2—18) $$\Delta C_1 + \frac{\partial \varphi_2}{\partial x} = 0, \quad \Delta C_2 + \frac{\partial \varphi_2}{\partial y} = 0$$

(2—19) $$\Delta C_3 + \varphi_1(x, y) = 0$$

And whereout immediately

(2—20) $$\Delta\Delta A_3 = 0$$

and

(2—21) $$\Delta\varphi_1 = 0 \Rightarrow \varphi_1 = \int(-\Delta D_1\, dx - \Delta D_2\, dy)$$

Now it is suitable to introduce a complex way of expressing. Therefore with the designation $z = x + iy$ it can be written

(2—22) $$A_3 = \bar{z}\varphi(z) + z\overline{\varphi(z)} + \chi(z) + \overline{\chi(z)}$$

(2—23) $$i(D_1 + iD_2) = \frac{\partial A_3}{\partial x} + i\frac{\partial A_3}{\partial y} = 2\frac{\partial A_3}{\partial \bar{z}} = 2[\varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)}]$$

(2—24) $$\psi(z) = \chi'(z)$$

(2—25) $$\left(\frac{\partial D_1}{\partial y}-\frac{\partial D_2}{\partial x}\right)+i\varphi_1(x,\ y)=8\varphi'(z)+ic$$

(2—25)' $$\varphi_1(x,\ y)=8\, Im\,[\varphi'(z)]+c$$

where $c$ is still a real constant.

Further there are still

(2—26) $$D_3=-\frac{a}{2}\, z\bar{z}+\varepsilon(z)+\overline{\varepsilon(z)}$$

(2—27) $$C_3=i\,[\bar{z}\,\varphi(z)\, z\,\overline{\varphi(z)}]+\eta(z)+\overline{\eta(z)}-\frac{c}{4}\, z\bar{z}$$

where $\varepsilon(z)$ and $\eta(z)$ are two further analytical functions.

From the equation

$$\frac{\partial A_2}{\partial x}-\frac{\partial A_1}{\partial y}=-\frac{a}{2}\, z\bar{z}+\varepsilon(z)+\overline{\varepsilon(z)}$$

and

(2—28) $$\xi'(z)=\varepsilon\,()$$

follows

(2—29) $$A_1=\frac{\partial U}{\partial x}+\frac{a}{6}\, y^3+\frac{i}{2}\,[\xi(z)-\overline{\xi(z)}]$$

(2—29') $$A_2=\frac{\partial U}{\partial y}-\frac{a}{6}\, x^3+\frac{1}{2}\,[\xi(z)+\overline{\xi(z)}]$$

Using the desigation $F=B_3-U$ we still get

(2—30) $$C_1=\frac{\partial F}{\partial y}+\frac{a}{6}\, x^3-\frac{1}{2}\,[\xi(z)+\overline{\xi(z)}]$$

(2—30') $$C_2=-\frac{\partial F}{\partial x}+\frac{a}{6}\, y^3+\frac{i}{2}\,[\xi(z)-\overline{\xi(z)}]$$

Now the equation (2—18) yields

(2—31) $$\frac{\partial(\Delta F)}{\partial y}+\frac{\partial\varphi_2}{\partial x}+\frac{a}{2}\, x=0$$

(2—31') $$-\frac{\partial(\Delta F)}{\partial x}+\frac{\partial\varphi_2}{\partial y}+\frac{a}{2}\, y=0$$

whereout

(2—32) $$\Delta\Delta F = 0$$

(2—33) $$F = \bar{z}\,\varphi_1(z) + z\,\overline{\varphi_1(z)} + \chi_1(z) + \overline{\chi_1(z)}$$

$$i(C_1 + iC_2) = \frac{\partial F}{\partial x} + i\frac{\partial F}{\partial y} + \frac{ai}{6}(x^3 - iy^3) - i\,\xi(z) =$$

(2—34) $$= 2\,[\varphi_1(z) + z\,\overline{\varphi_1'(z)} + \overline{\psi_1(z)}] + \frac{ai}{6}(x^3 + iy^3) - i\,\xi(z)$$

(2—34′) $$\psi_1(z) = \chi_1'(z)$$

Finally it is still

(2—35) $$\varphi_2(x, y) = 8\,Im\,[\varphi_1'(z)] - \frac{a}{2}\,z\bar{z} + b$$

where $b$ is some further constant.

From the above equations follows that six analytical functions $\varphi(z)$, $\psi(z)$, $\varepsilon(z)$, $\eta(z)$, $\varphi_1(z)$ and $\psi_1(z)$ are needed for the determination of velocity field $\vec{U}$. It is possible still to rearrange considerably all the above expressions. First let us choose an arbitrary point in the domain $D$ where fluid is being investigated. Let it be named $T(x_0, y_0, 0)$. Due to $\frac{\partial V}{\partial Z} = 2aZ + \varphi_1(x, y)$ there shall be

(2—36) $$\frac{\partial^2 V}{\partial Z^2} = 2a_0, \quad \frac{\partial V}{\partial Z} = \varphi_1(x_0, y_0) = c_0, \quad V = \varphi_2(x_0, y_0) = b_0$$

where $a_0$, $b_0$ and $c_0$ are arbitrarily prescribed real constants. Now $a_0 = a$ and

(2 – 25″) $$\varphi_1(x, y) = 8\,Im\,[\varphi'(z)] - 8\,Im\,[\varphi'(z_0)] + c_0$$

(2—27′) $$C_3 = i\,[\bar{z}\,\varphi(z) - z\,\overline{\varphi(z)}] + 2\,Im\,[\varphi'(z_0)]\,z\bar{z} - \frac{c_0}{4}\,z\bar{z} + \eta(z) + + \overline{\eta(z)}$$

(2—35′) $$\varphi_2(x, y) = 8\,Im\,[\varphi_1'(z)] - 8\,Im\,[\varphi_1'(z_0)] - \frac{a_0}{2}(z\bar{z} - z_0\bar{z}_0) + b_0$$

Thus the velocity field $\vec{U}$ and field $V$ are in accordance with (2—36) determined by the equations (2—23), (2—26), (2—34), (2—27′), (2—25″) and (2—35′), while for the determination of vectors $\vec{A}$ and $\vec{B}$ still remain, some arbitrary functions what is however not inportant for further deduction.

## 3. Fundamental boundary-value problem

Let $C$ be the boundary curve of a simply connected domain $D$ lying in plane $Z=0$. The fundamental boundary value problem will be defined in the following way:

Determine at the given point $(x_0, y_0) \in D$ and at arbitrary given constants $a_0$, $b_0$ and $c_0$ the velocity field $\vec{U}$ which will satisfy the equations (2—2) and (2—3) so that $\vec{\psi}_1(x, y)$ and $\vec{\psi}_2(x, y)$ on $C$ take the prescribed values.

The boudary values of vectors $\vec{\psi}_1(x, y)$ and $\vec{\psi}_2(x, y)$ have not to be arbitrary. Namely from (2—2′) the following requirements immediately appear:

(3—1) $$\frac{\partial D_1}{\partial x} + \frac{\partial D_2}{\partial y} = 0$$

(3—2) $$\frac{\partial C_1}{\partial x} + \frac{\partial C_2}{\partial y} + D_3 = 0$$

whereout immediately follows:

(3—3) $$\int_C D_n ds = \int_C (D_2 dx + D_1 dy) = Re\Big[\int_C i(D_1 + iD_2)d\bar{z}\Big] = 0$$

and

$$\int_C C_n ds = \int_C (-C_2 dx + C_1 dy) = Re\Big[\int_C i(C_1 + iC_2)d\bar{z}\Big] =$$

(3—4) $$= -\iint_D D_3 dx\, dy$$

where by $D_n$ and $C_n$ the normal vector components of $(D_1, D_2, 0)$ and $(C_1, C_2, 0)$ on $C$ are meant. Both equations have a simpler physical interpretation.

If the boundary-value of the expression $i(D_1 + iD_2)$ is taken with $2f_{D12}(z)$, then from (2—23) and (3—3) follows:

(3—5) $$\varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)} = f_{D12}(z), \ z \in C$$

(3—5′) $$Re\Big[\int_C f_{D12}(z)\, d\bar{z}\Big] = 0$$

Further a special analytical function $\rho_0$ is introduced of which the real part suits the following prescription

(3—6) $$\Delta Re(\rho_0) = 0 \wedge Re(\rho_0) z\bar{z}, \ z \in C$$

Then the equation (2—26) can be written in the following equivalent way

$$D_3 = \frac{a_0}{2}[Re(\rho_0) - z\bar{z}] + \varepsilon(z) + \overline{\varepsilon(z)} \tag{3—7}$$

If the boundary value $D_3$ is designated by $2f_{D3}(z)$ then from (3—7) it can be seen that this is the case of a boundary-value problem

$$Re[\varepsilon(z)] = f_{D3}(z), \quad z \in C \tag{3—8}$$

Thus in $\varepsilon(z)$ the constant $a_0$ does not appear!

Similarly the equation (2—27) can be written suitably more as

$$C_3 = \frac{c_0}{4}[Re(\rho_0) - z\bar{z}] - 2\,Im[\varphi'(z_0)] \cdot [Re(\rho_0) - z\bar{z}] +$$

$$+ i[\bar{z}\varphi(z) - z\overline{\varphi(z)}] + \eta(z) + \overline{\eta(z)} \tag{3-9}$$

If we denote the boundary value of $C_3$ by $2f_{C3}(z)$, then from (3—9) follows:

$$Re[\eta(z)] = f_{C3}(z) - \frac{i}{2}[\bar{z}\varphi(z) - z\overline{\varphi(z)}], \quad z \in C \tag{3—10}$$

In $\eta(z)$ $c_0$ and $z_0$ do not apper here too.

Finally let the boundary value of the expression $i(C_1 + C_2)$ be denoted by $2f_{C12}(z)$. Then from (2—24) follows

$$\varphi_1(z) + z\overline{\varphi_1'(z)} + \overline{\psi_1(z)} = f_{C12}(z) - \frac{a_0 i}{12}(x^3 + iy^3) + \frac{i}{2}\xi(z) =$$

$$= f_0(z), \quad z \in C \tag{3—11}$$

In a very simple way from equation (3—4) follows that the latter is equivalen to the requirement

$$Re\left[\int_C f_0(z)\,d\bar{z}\right] = 0 \tag{3—12}$$

From the upper statements the following method of solving of the boundary-value problem is obtained if it is solvable at all:

First (3—5) and (3—5′), then (3—8) and (3—10) and at last (3—11) and (3—11′) have to be solved.

### 4. Uniqueness theorem and existence theorem

First the following fact has to be proved:

Let $\vec{U}$ be a velocity field given by analytical functions $\varphi(z)$, $\psi(z)$, $\varepsilon(z)$, $\eta(z)$, $\varphi_1(z)$, $\psi_1(z)$ at constants $z_0$, $a_0$, $b_0$ and $c_0$. If these functions are additively changed in turn for:

(4—1) $$\tilde{\varphi}(z) = iK_0 z + K_1, \quad \overline{K_0} = K_0$$

(4—2) $$\tilde{\varphi}'(z) = -\overline{K_1}$$

(4—3) $$\tilde{\varepsilon}(z) = iK_2, \quad \overline{K_2} = K_2$$

(4—4) $$\tilde{\xi}(z) = iK_2 z + K_3$$

(4—5) $$\tilde{\eta} z = i\overline{K_1} z + iK_4, \quad \overline{K_4} = K_4$$

(4—6) $$\tilde{\varphi}_1(z) = -\frac{1}{4} K_2 z + iK_5 z + K_6, \quad \overline{K_5} = K_5$$

(4—7) $$\tilde{\psi}_1(z) = -\frac{i}{2}\overline{K_3} - \overline{K_6}$$

$$z \in D$$

where $K_0 - K_6$ are constants,

then the new functions define the same velocity field $\vec{U}$ at the same values of $z_0$, $a_0$, $b_0$ and $c_0$. From here the proof of uniqueness theorem for a simply connected domain $D$, [10] can immediately be obtained:

From requirements

1. $a_0 = b_0 = c_0 = 0$
2. $D_1 = D_2 = D_3 = C_1 = C_2 = C_3 = 0$ on $C$

equations (4—1) to (4—7) follow.

If the domain is infinite, it additionally follows

$$K_0 = K_1 = K_2 = K_5 = 0$$

To prove the existence of the boundary-value problem solution an extension of Šerman's way [10], [11] can be used

(4—8) $$\varphi(z) = \frac{1}{2\pi i}\int_C \frac{\omega_0(t)\, dt}{t - z}$$

(4—9) $$\psi(z) = \frac{1}{2\pi i}\int_C \frac{\overline{\omega_0(t)} - \bar{t}\,\omega_0'(t)}{t - z}\, dt$$

(4—10) $$\varepsilon(z) = \frac{1}{2\pi i}\int_C \frac{\omega_1(t)\, dt}{t - z}, \quad \overline{\omega_1(t)} = \omega_1(t)$$

(4—11) $$\eta(z)=\frac{1}{2\pi i}\int_C \frac{\omega_2(t)\,dt}{t-z}, \quad \overline{\omega_2(t)}=\omega_2(t)$$

(4—12) $$\varphi_1(z)=\frac{1}{2\pi i}\int_C \frac{\omega_3(t)\,dt}{t-z}$$

(4—13) $$\psi_1(z)=\frac{1}{2\pi i}\int_C \frac{\overline{\omega_3(t)}-\bar{t}\,\omega_3'(t)}{t-z}\,dt$$

$$z\in D$$

If these expressions are introduced into (3—5), (3—8), (3—10) and (3—11), only Fredholm's integral equations of the second order are obtained. There equations at the conditions (3—5'), (3—11') are always solvable [10], [11].

## 5. The boundary value problem in the multiple connected domain

In such a domain the form of all analytical functions has to be defined at first. From the equations (2—23), (2—25), (2—26), (2—27), (2—34) and (2—35) immediately follow:

(5—1) $$\varphi(z)=\sum_{k=1}^{k=n}\beta_k\ln(z-z_k)+\varphi_0(z)$$

(5—2) $$\psi(z)=\sum_{k=1}^{k=n}\bar{\beta}_k\ln(z-z_k)+\psi_0(z)$$

(5—3) $$\varepsilon(z)=\sum_{k=1}^{k=n}\gamma_k\ln(z-z_k)+\varepsilon_0(z), \quad \bar{\gamma}_k=\gamma_k$$

(5—4) $$\xi(z)=\sum_{k=1}^{k=n}(\gamma_k z+m_k)\ln(z-z_k)+\xi_0(z)$$

(5—5) $$\eta(z)=\sum_{k=1}^{k=n}(-i\bar{\beta}_k z+\vartheta_k)\ln(z-z_k)+\eta_0(z), \quad \bar{\vartheta}_k=\vartheta_k$$

(5—6) $$\varphi_1(z)=\sum_{k=1}^{k=n}\left(\frac{i}{4}\gamma_k z+\frac{i}{4}m_k+\nu_k\right)\ln(z-z_k)+\varphi_{10}(z)$$

(5—7) $$\psi_1(z)=\sum_{k=1}^{k=n}\left(\frac{i}{4}\bar{m}_k+\bar{\nu}_k\right)\ln(z-z_k)+\psi_{10}(z)$$

where the points $z_k$ have an analogous meaning as in [10], and the functions with index $o$ are all holomorphic in domain $D$.

Now the validity of the first two theorems from the previous chapter follows for a multiple connected domain too [10].

At proving the existence theorem it is necessary to proceed in an analogous way as in the case of the multiple connected domain in plane elasticity, [10].

The expressions from (4—8) to (4—13) have to be changed correspondingly. The expressions for the constants $\beta_k$, $\gamma_k$, $\vartheta_k$ and $\nu_k$ are quite analogue to [10]. Only for $m_k$ it has to be taken into account that the expression for this constant follows form the already known function $\varepsilon(z)$. Using these corrections the existence of the boundary-value problem solution for multiple connected finite and infinite domain $D$ can be proved in analogous way as in [10].

## 6. The interpretation of constants $a_0$, $b_0$ and $c_0$

The constant $b_0$ appears only in (2—35) in the expression $\varphi_2(x, y)$ and is additive. The meaning of the other two is obtained from the expression of the rate of flow $\Phi$ through an area parallel to $D$ at a general $Z$:

(6—1)
$$\Phi = Z \cdot \iint_D D_3\, dx\, dy + \iint_D C_3\, dx\, dy$$

From (3—7) follows

$$\iint_D D_3\, dx\, dy = \frac{a_0}{2} \iint_D [Re(\rho_0) - z\bar{z}]\, dx\, dy +$$

(6—2)
$$+ 2 \iint_D Re(\varepsilon(z))\, dx\, dy = a_0\, \alpha + \beta$$

where $\alpha$ is dependent only upon domain $D$ and $\beta$ only upon domain $D$ and boundary conditions for $D_3$.

Similarly for the second integral

(6—3)
$$\iint_D C_3\, dx\, dy = \frac{1}{2} c_0\, \alpha + \gamma$$

where $\gamma$ depends only upon boundary conditions for $D_1$, $D_2$, $C_3$, domain $D$ and point $z_0$.

From the upper equations results that the prescription for $a_0$ and $c_0$ can be substituted by the prescription for the rate of flow $\Phi$.

For a flow in a cylinder with a rigid boundary the fulfilment of the following conditions has to be required

(6—4) $$a_0 \alpha + \beta = 0$$

(6—5) $$\frac{1}{2} c_0 \alpha + \gamma = \Phi$$

where the rate of flow $\Phi$ can be arbitrarily given.

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# EINE ANWENDUNG DER KOMPLEXEN ANALISE BEI RAUMSTRÖMUNG VON FLÜSSIGKEIT

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## Zusammenfassung

In diesem Aufsatz wird die Raumströmung einer inkompressiblen und zähen Flüssigkeit behandelt. Die Strömungsgeschwindigkeit ist klein und linear von der dritten Koordinaten $Z$ abhängig angenommen. Es wurde gezeigt, dass

bei solchen Bendingungen alle bekannten Methoden von Mushelišvili erfolgreich angewendet werden können.

## NEKA UPORABA KOMPLEKSNE ANALIZE PRI PROSTORKEM TOKU FLUIDA

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Povzetek

V tem sestavku je obravnavan prostorski tok fluida, ki je nestisljiv in viskozen, hitrost toka pa majhna in lilearno od tretje koordinate $Z$ odvisna. Dokazano, je da je možno uspešno uporabljati pri popisovanju takih razmer vse znane Mushelišvilijeve metode.

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