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ELASTIC STABILITY OF AN EXCENTRIC CIRCULAR ANNULUS

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1. Introduction

Let us take a wall in the form of an excentric circular annulus which is on the outer boundary L_2 loaded by a constant compression $(-P_Z)$ and on the inner boundary L_1 by a constant compression $(-P_N)$, Fig. 1. Let the wall have a simply supported inner and outer boundary. The loading of the wall perpendicularly to its plane shall be equal to zero. The wall is isothropic and homogeneous and its thickness h must be such that the plane stress state always remains in the elastic domain [1].

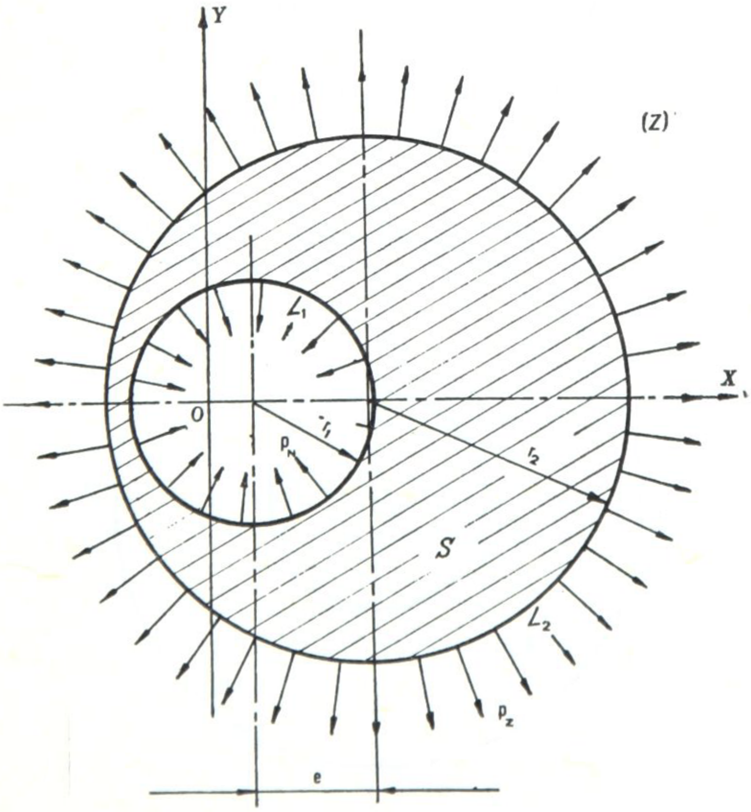

Fig. 1

In a similar way as at simply connected walls, a state of instability appears also in a finite simply or doubly connected wall where the holes lower the degree of stability of the wall. For the above chosen example of the wall it will be determined those vector values of the extermal load at which a state of instability arises in the wall and where at an arbitrary small disturbance the process of buckling appears.

In order to determine the state of instability in the wall, the fundamental differential equation of the elastic stability of a wall was used (3) containing the elements of the tensor of plane stress state σ_{ij}.

2. Determination of the plane stress tensor

Let us take a three-dimensional orthogonal coordinate system with a complex plane (z) and a real axis Z. The wall middle plane is set into the complex plane (z) with the domain S and is conformally mapped into a new complex plane (ζ), [4] on a centrical circular annulus with the domain Σ, Fig. 2. The centres of the circles shall be at the beginning of the plane (ζ), $\zeta_0 = 0$, thus, that the real axis $\operatorname{Im} z = 0$ is mapped on the real axis $\operatorname{Im} \zeta = 0$.

This requirement can be fulfilled by a bilinear mapping function [5].

(1) $$z = x + iy = \omega(\zeta) = \omega(\rho\, e^{i\vartheta}) = \omega(\xi + iy) = \frac{a_1 \zeta + b_1}{c_1 \zeta + d_1}$$

Choosing the constant: $a_1 = b_1 = C$, $c_1 = 1$, $d_1 = -1$ where C is a real function of the wall geometry, the mapping function is

(2) $$z = \omega(\zeta) = C\frac{\zeta + 1}{\zeta - 1}$$

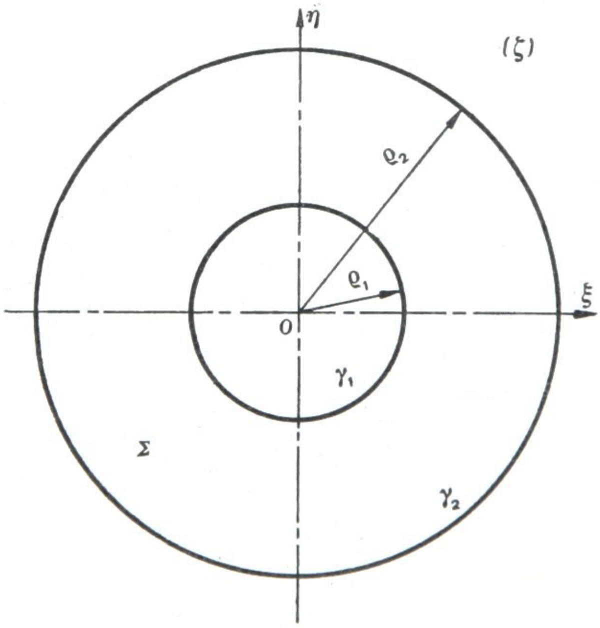

Fig. 2

due to $a_1 d_1 - b_1 c_1 < 0$ the semi-plane Im $z > 0$ is mapped by the function (2) into the semi-plane Im $\rho < 0$.

The real constant C and the radii of the centric circular annulus $\rho_2 \leqslant (\zeta) \leqslant \rho_1$, are:

$$C = \sqrt{\left[\frac{e^2 - r_1^2 - r_2^2}{2e}\right]^2 - \left[\frac{r_1 r_2}{e}\right]^2} \tag{3}$$

$$\rho_1 = \frac{C}{r_1} + \sqrt{\left[\frac{C}{r_1}\right]^2 + 1}$$

$$\rho_2 = \frac{C}{r_2} + \sqrt{\left[\frac{C}{r_2}\right]^2 + 1} \tag{4}$$

Due to the simple pole in the point $\zeta = 1$, the domain Σ is chosen, defined by the inequation $1 \leqslant \rho_2 \leqslant /\zeta/ \leqslant \rho_1$.

The principle vector resultant force on the inner boundary is equal to zero, $F_1 = 0$, therefore the functions $\Phi(\zeta)$ and $\Psi(\zeta)$ in the domain Σ are just equal to holomorphic functions [4].

$$\Phi(\zeta) = \Phi_*(\zeta) = \sum_{n=-\infty}^{n=\infty} a_n \zeta^n$$

(5)

$$\Psi(\zeta) = \Psi_*(\zeta) = \sum_{n=-\infty}^{n=\infty} b_n \zeta^n$$

The function $\Phi_*(\zeta)$ is introduced into the differential equation of the plane stress state in the centric circular annulus, [6]

$$\frac{d}{d\zeta}\left\{\Phi_*(\zeta)\left[\bar{\omega}\left(\frac{\rho_2^2}{\zeta}\right) - \bar{\omega}\left(\frac{\rho_1^2}{\zeta}\right)\right]\right\} + \bar{\Phi}_*\left(\frac{\rho_1^2}{\zeta}\right)\frac{\rho_1^2}{\zeta^2}\bar{\omega}'\left(\frac{\rho_1^2}{\zeta}\right) -$$

(6)

$$-\bar{\Phi}^*\left(\frac{\rho_2^2}{\rho}\right)\frac{\rho_2^2}{\rho^2}\bar{\omega}'\left(\frac{\rho_2^2}{\zeta}\right) = f(\xi)_1 \frac{\rho_1^2}{\zeta^2}\bar{\omega}'\left(\frac{\rho_1^2}{\zeta}\right) - f(\zeta)_2 \frac{\rho_2^2}{\zeta^2}\bar{\omega}'\left(\frac{\rho_2^2}{\zeta}\right)$$

If the functions of external load on the inner boundary $f(\zeta)_1 = P_N$ and on the outer boundary $f(\zeta)_2 = P_Z$ are taken into account, then the solution of the equation (6) is

$$\Phi_*(\zeta) = -\frac{h_1 \lambda}{\zeta^2} + \frac{2 h_1 \lambda}{\zeta} + (1-\lambda) h_1 + h_0 - 2 h_1 \zeta + h_1 \zeta^2 \tag{7}$$

where the constants are

$$\lambda = \rho_1^2 \rho_2^2; \quad K_3 = \rho_1/\rho_2; \quad p = p_N/pz$$

$$h_0 = \frac{pk_3^2(\rho_2^2-1)^2+(k_3^2\rho_2^2-1)^2}{2\,[(k_3^2+1)(1+k_3^2 e_2^4)-4k_3^2\rho_2^2]}\cdot p_z = k_0 \cdot p_z \tag{8}$$

$$h_1 = \frac{k_3^2(p-1)}{(k_3^2-1)\,[(k_3^2+1)(1+k_3^2\rho_2^4)-4k_3^2\rho_2^2]}\cdot p_z = k_1 \cdot p_z$$

and the second holomorphic function (6)

$$\Psi_*(\zeta) = \frac{\rho_1^2}{\zeta}\,\frac{\overline{\omega}'\left(\frac{\rho_1^2}{\zeta}\right)}{\omega'(\zeta)}\left[\Phi_*(\zeta)+\overline{\Phi}_*\left(\frac{\rho_1^2}{\zeta}\right)-f(\zeta)_1\right]-\frac{\overline{\omega}\left(\frac{\rho_1^2}{\zeta}\right)}{\omega'(\zeta)}\,\Phi_*'(\zeta) \tag{9}$$

The complex system of the stress tensor components of the plane state

$$\sigma + \sigma_\rho = 2\,[\Phi(\zeta)+\overline{\Phi(\zeta)}]$$

$$\sigma_\vartheta - \sigma_\rho + 2\,i\,\tau_{\rho\vartheta} - \frac{2\,\rho^2}{\rho^2\cdot\overline{\omega'(\zeta)}}\,[\overline{\omega(\zeta)}\,\Phi'(\zeta)+\omega'(\zeta)\,\Psi(\zeta)]$$

is now, taking into accouns the functions (7) and (9), (7):

$$\sigma_\vartheta + \sigma_\rho = 4\,h_0 + 2\,h_1\left[(\zeta-1)^2\left(1-\frac{\lambda}{\zeta^2}\right)+\left(\frac{\rho^2}{\zeta}-1\right)^2\left(1=\frac{\lambda}{\rho^4}\zeta^2\right)\right. \tag{10}$$

$$\sigma_\vartheta - \sigma_\rho = 2\,i\,\tau_{\rho\vartheta} = 2\frac{(\rho^2-\zeta)^2}{\rho^2}\left\{\frac{(2\,h_0-p\cdot p_z)\,\rho_1^2}{(\rho_1^2-\zeta)^2}+h_1(\zeta-1)\left(1-\frac{\lambda}{\zeta^3}\right)\cdot\right.$$

$$\left.\left(\frac{\rho_1^2+\zeta}{\rho_1^2-\zeta}-\frac{\rho^2-\zeta}{\rho^2-\zeta}\right)+h_1\left[\frac{(\zeta-1)^2\rho_1^2}{(\rho_1^2-\zeta)^2}\left(1-\frac{\lambda}{\zeta^2}\right)+\frac{\rho_1^2-\rho_2^2\zeta^2}{\zeta^2}\right]\right\} \tag{11}$$

3. Determination of the critical external load

The critical external load on boundaries is determined by solving the fundamental equation of elastic stability (3).

$$w_{\zeta\zeta\bar\zeta\bar\zeta}\,a(\zeta,\bar\zeta) - w_{\zeta\zeta\bar\zeta}\,b(\zeta,\bar\zeta) - w_{\zeta\bar\zeta\bar\zeta}\,\overline{b(\zeta,\bar\zeta)} + w_{\zeta\bar\zeta}\,c_3(\zeta,\bar\zeta) +$$

$$+k\left\{-w_{\zeta\bar\zeta}\,c_4(\zeta,\bar\zeta)+w_{\zeta\zeta}\,\overline{d(\zeta,\bar\zeta}+w_{\bar\zeta\bar\zeta}\,d(\zeta,\bar\zeta)-w_\zeta\,e(\zeta,\bar\zeta)-\right.$$

$$\left.-w_{\bar\zeta}\,e(\zeta,\bar\zeta)+\frac{1}{h}\,q_2(\zeta,\bar\zeta)\right\}=0 \tag{12}$$

In this case the new complex plane (ζ) was determined by the bilinear function (1) in which $a_1=1$, $b_1=0$, $c_1=-a$, $d_1=1$ were chosen as constants and where a is a real constant.

Thus the mapping function becomes equal to:

$$z=x+iy=re^{id}=\omega(q)=\omega[\xi+iy]=\omega(\rho e^{i})=\frac{\zeta}{1-a\zeta} \tag{13}$$

because $a_1 d_1 - b_1 c_1 > 0$, the semi-plane Im $\zeta>0$ is mapped into the semi-plane Im $z>0$ and the real axis Im $\zeta=0$ on the real axis Im $z=0$.

The domain S of the excentric circular annulus which is in plane (z) limited by the curves L_1 and L_2, Fig. 1, is now mapped onto domain Σ in plane (ζ) by the function (13). The domain Σ is defined by the inequation $\rho_1 \leqslant |\zeta| \leqslant \rho_2$, Fig. 3, where the constant is

$$a=\frac{e}{\sqrt{[(r_2-r_1)^2-e^2]\,[(r_2+r_1)^2-e^2]}} \tag{14}$$

and the radii

$$\rho_1=\frac{2r_1}{\sqrt{1+4a^2r_1^2}+1} \qquad \rho_2=\frac{2r_2}{\sqrt{1+4a^2r_2^2}+1}$$

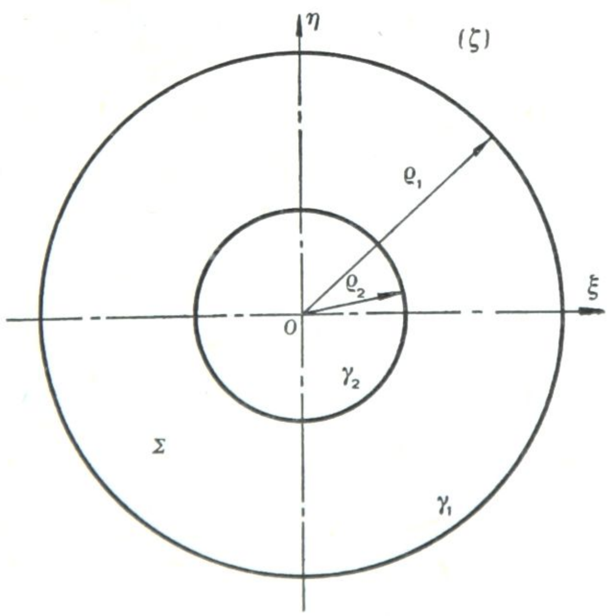

Fig. 3

In domain Σ the function (13) must not have any poles, therefore the following inequation must be valid

$$a\rho<1$$

Taking into account the stress tensor components (10). constants (3, 4, 8, 14) and the mapping function (13), the coefficients at derivatives in the differential equation (12) take the following form:

$$a(\xi, \bar{\zeta}) = \frac{1}{(1-a\zeta)^2(1-a\bar{\zeta})^2}, \qquad b(\zeta, \bar{\zeta}) = \frac{2a}{(1-a\zeta)^2(1-a\bar{\zeta})^3}$$

$$c_3(\xi\, \bar{\xi}) = \frac{4a^2}{(1-a\zeta)^3(1-a\bar{\zeta})^3}, \qquad A(\zeta) = \frac{C+(1-Ca)\zeta}{-C+(1+Ca)\zeta}$$

$$C_4(\zeta, \bar{\zeta}) = \frac{2}{(1-a\zeta)^4(1-a\bar{\zeta})^4}\left\{4k_0 + 2k_1[A(\zeta)-1]^2\left[1-\frac{\lambda}{A^2(\zeta)}\right]+\right.$$

$$\left.+2k_1[\overline{A(\zeta)}-1]^2\cdot\left[1-\frac{\lambda}{\overline{A^2(\zeta)}}\right]\right\}$$

$$B(\zeta, \bar{\zeta}) = 2[\overline{A(\zeta)}-1]\left[\frac{1-a\zeta}{1-a\bar{\zeta}}\right]^2\cdot\left\{\frac{(2k_0p)\rho_1^2}{[\rho_1^2-A(\zeta)]^2}+\right.$$

$$+k_1[A(\zeta)-1]\left[1-\frac{\lambda}{A^3(\zeta)}\right]\left[\frac{\rho_1^2+A(\zeta)}{\rho_1^2-A(\zeta)}-\right.$$

$$\left.-\frac{\overline{A(\zeta)}+1}{A(\zeta)+1}\right]+k_1\left[\frac{[A(\zeta)-1]^2\rho_1^2}{[\rho_1^2-A(\zeta)]^2}\left[1-\frac{\lambda}{A^2(\zeta)}\right]+\frac{\rho_1^2}{A^2(\zeta)}-\rho_2^2\right]\right\}$$

$$d(\zeta, \bar{\zeta}) = \frac{B(\zeta, \bar{\zeta})}{(1-a\zeta)^6(1-a\bar{\zeta})^2}$$

$$e(\zeta, \bar{\zeta}) = \frac{2a\cdot B(\zeta, \bar{\zeta})}{(1-a\bar{\zeta})^3(1-a\zeta)^6}$$

$$(15)\quad q_2(\zeta, \bar{\zeta}) = \frac{1}{pz}q(\zeta, \bar{\zeta}) = 0; \qquad k = \frac{pzh}{16D}$$

In the case of a simply supported boundary (3), the equations of the mixed boundary value problem on the boundary γ_k with points $\sigma \in \gamma_k$, $k = 1, 2$, considering the mapping function (13), take the form:

$$\frac{\nu-1}{2\rho_k^2(\nu-1)}\left\{\left[w_{\sigma\sigma}-\omega_\sigma\frac{\omega''(\sigma)}{\omega'(\sigma)}\right]\sigma^2+\left[w_{\bar{\sigma}\bar{\sigma}}\, w_{\bar{\sigma}}\frac{\overline{\omega''(\sigma)}}{\overline{\omega'(\sigma)}}\right]\bar{\sigma}^2\right\}-w_{\sigma\bar{\sigma}} = 0$$

$$(16)\quad w(\sigma, \bar{\sigma}) = u(s)_k = 0$$

With regard to the above coefficients at derivatives, the partial differential equation (12) takes the form which is in analytical way probably difficult to solve, therefore the critical external load is determined numerically by the use of the finite difference method in polar coordinates, (7).

In the complex plane (ζ) the equation (12) can be written for zero point in the differential form, Fig. 4.

(17) $$\sum_{i=1}^{i=4} D_i(\rho,\vartheta)\cdot\sum_{j=0}^{j=12} w_j f_{ij}(\rho,\vartheta)+k\sum_{i=5}^{i=9} D_i(\rho,\vartheta)\sum_{j=0}^{j=8} w_j g_{ij}(\rho,\vartheta)=0$$

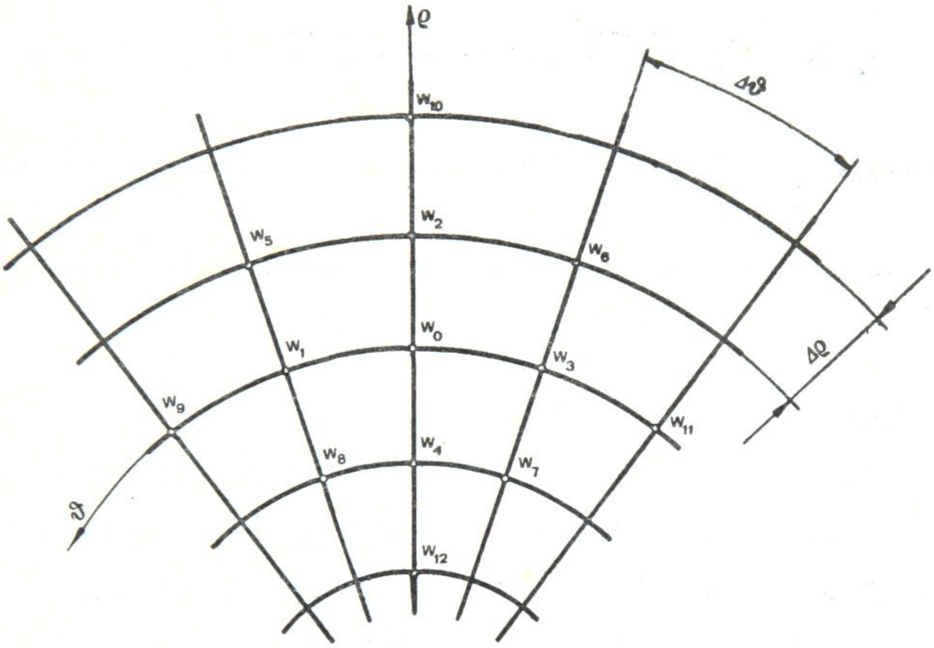

Fig. 4

where wj means the middle plane displacement of the wall in the direction of the real axis Z in point j. The functions $f_{ij}(\rho,\vartheta)$ and $g_{ij}(\rho,\vartheta)$ contain the expressions which appear at the transformation of the differential equation (12) into the equation written by finite differences.

Due to the equalities: $a_2(\rho,\vartheta)=a(\zeta,\overline{\zeta})$, $b_1(\rho,\vartheta)=b(\zeta,\overline{\zeta})$, $c_{32}(\rho,\vartheta)=c_3(\zeta,\overline{\zeta})$, $c_{42}(\rho,\vartheta)=c_4(\zeta\,\overline{\zeta})$, $d_1(\rho,\vartheta)=d(\zeta,\overline{\zeta})$ and $e_1(\rho,\vartheta)=e(\zeta,\overline{\zeta})$, the functions $Di(\rho,\vartheta)$, $i=1, 2, \ldots, 9$ can be written in the form:

$$D_1(\rho,\vartheta)=\frac{1}{16\rho^3\Delta\vartheta^3\sqrt{a_2(\rho,\vartheta)}}$$

$$D_2(\rho,\vartheta)=-\frac{1}{8\rho^2\Delta\vartheta^2}\{[b_1(\rho,\vartheta)+\overline{b_1(\rho,\vartheta)}]\cos\vartheta+i[\overline{b_1(\rho,\vartheta)}-b_1(\rho,\vartheta)\sin\vartheta\}$$

$$D_3(\rho,\vartheta)=-\frac{1}{8\rho^2\Delta\vartheta^2}\{i[\overline{b_1(\rho,\vartheta)}-b_1(\rho,\vartheta)]\cos\vartheta-[b_1(\rho,\vartheta)+\overline{b_1(\rho,\vartheta)}\sin\vartheta\}$$

$$D_4(\rho,\vartheta)=\frac{a^2}{\rho\Delta\vartheta}$$

$$D_5(\rho, \vartheta) = -\frac{1}{4\rho\Delta\vartheta} \cdot \frac{c_{42}(\rho, \vartheta)}{a_2(\rho, \vartheta)\sqrt{a_2(\rho, \vartheta)}}$$

$$D_6(\rho, \vartheta) = \frac{1}{4\rho\Delta\vartheta}\{[d_1(\rho, \vartheta) + \overline{d_1(\rho, \vartheta)}\cos 2\vartheta - i[\overline{d_1(\rho, \vartheta)} - d_1(\rho, \vartheta)]\sin 2\vartheta\}$$

$$D_7(\rho, \vartheta) = \frac{1}{4\rho\Delta\vartheta}\{[d_1(\rho, \vartheta) + \overline{d_1(\rho, \vartheta)}]\sin 2\vartheta + i[\overline{d_1(\rho, \vartheta)} - d_1(\rho, \vartheta)[\cos 2\vartheta\}$$

$$D_8(\rho, \vartheta) = \frac{1}{2}[e_1(\rho, \vartheta) + \overline{e_1(\rho, \vartheta)}]\sin\vartheta + \frac{i}{2}[\overline{e_1(\rho, \vartheta)} - e_1(\rho, \vartheta)]\cos\vartheta$$

(18) $$D_9(\rho, \vartheta) = \frac{1}{2}[e_1(\rho, \vartheta) + \overline{e_1(\rho, \vartheta)}\cos\vartheta - \frac{i}{2}[\overline{e_1(\rho, \vartheta)} - e_1(\rho, \vartheta)]\sin\vartheta$$

After the arrangement the difference equation (17) in zero point can de written

(19) $$(S_{0t} + k Z_{0t}) w_t = 0, \qquad t = 0, 1, 2, \ldots, 12$$

where the functions S_{0t} and Z_{0t} for zero point are equal

$$S_{00} = D_1(\rho, \vartheta)[4 i_0^2 \Delta\vartheta^2 (3 i_0^2 + 1) + 16\Delta\vartheta^2 (i_0^2 - 1) + 12 +$$
$$+ D_2(\rho, \vartheta)[-4 i_0^2 \Delta\vartheta^3 + 8\Delta\vartheta] + D_4(\rho, \vartheta)(-4 i_0^2 \Delta\vartheta^2 - 4)$$

$$S_{01} = D_1(\rho, \vartheta)\, 8[\Delta\vartheta^2 (1 - i_0^2) - 1] + D_2(\rho, \vartheta)(-4\Delta\vartheta) +$$
$$+ D_3(\rho, \vartheta)(-2 i_0^2 \Delta\vartheta^2 - 2) + D_4(\rho, \vartheta) \cdot 2$$

$$S_{02} = D_1(\rho, \vartheta)[4\Delta\vartheta^2 + \Delta\vartheta^4)(1 - 2 i_0) i_0 - 4\Delta\vartheta^4 (2 i_0 + 1) i_0^3] +$$
$$+ D_2(\rho, \vartheta)[-2 i_0^3 \Delta\vartheta^3 - i_0 \Delta\vartheta^3 + 2 i_0^2 \Delta\vartheta^2 - 2 i_0 \Delta\vartheta] +$$
$$+ D_4(\rho, \vartheta)[2 i_0^2 \Delta\vartheta^2 + i_0 \Delta\vartheta^2]$$

$$S_{03} = D_1(\rho, \vartheta)\, 8[\Delta\vartheta^2 (i_0^2) - 1] + D_2(\rho, \vartheta)(-4\Delta\vartheta) +$$
$$+ D_3(\rho, \vartheta)[2 i_0^2 \Delta\vartheta^2 + 2] + D_4(\rho, \vartheta) \cdot 2$$

$$S_{04} = D_1(\rho, \vartheta)[4 i_0^3 \Delta\vartheta^4 (1 - 2 i_0) - (\Delta\vartheta^4 + 4\Delta\vartheta^2)(1 + 2 i_0) i_0] +$$
$$+ D_2(\rho, \vartheta)[2 i_0^3 \Delta\vartheta^3 + i_0 \Delta\vartheta^3 + 2 i_0^2 \Delta\vartheta^3 + 2 i_0 \Delta\vartheta] +$$
$$+ D_4(\rho, \vartheta([2 i_0^2 \Delta\vartheta^2 - i_0 \Delta\vartheta^2]$$

$$S_{05} = D_1(\rho, \vartheta) \cdot 2 i_0 \Delta\vartheta^2 \cdot (2 i_0 - 1) + D_2(\rho, \vartheta) i_0 \Delta\vartheta +$$
$$+ D_3(\rho, \vartheta)[i_0^2 \Delta\vartheta^2 + 0{,}5\, i_0 \Delta\vartheta^2]$$

$$S_{06} + D_1(\rho, \vartheta)[2 i_0 \Delta\vartheta^2 (2 i_0 - 1)] + D_2(\rho, \vartheta) i_0 \Delta\vartheta +$$
$$+ D_3(\rho, \vartheta)[-i_0^2 \Delta\vartheta^2 - 0{,}5\, i_0 \Delta\vartheta^2]$$

$$S_{07} = D_1(\rho, \vartheta)[2 i_0 \Delta\vartheta^2 (2 i_0 + 1)] + D_2(\rho, \vartheta)(-i_0 \Delta\vartheta) +$$
$$+ D_3(\rho, \vartheta)[-i_0^2 \Delta\vartheta^2 + 0{,}5\, i_0 \Delta\vartheta^2]$$

$$S_{08}=D_1(\rho,\vartheta)\,[2\,i_0\Delta\vartheta^2(2\,i_0+1)]+D_2(\rho,\vartheta)\,(-i_0\,\Delta\vartheta)+$$
$$+D_3(\rho,\vartheta)\,[i_0^2\Delta\vartheta^2-0{,}5\,i_0\,\Delta\vartheta^2]$$

$$S_{09}=D_1(\rho,\vartheta)\cdot 2+D_3(\rho,\vartheta)$$

$$S_{10}=D_1(\rho,\vartheta)\,[2\,i_0^3\Delta\vartheta^4(1+i_0)]+D_2(\rho,\vartheta)\,(i_0^3\Delta\vartheta^3)$$

$$S_{011}=D_1(\rho,\vartheta)\cdot 2+D_3(\rho,\vartheta)\,(-1)$$

$$S_{012}+D_1(\rho,\vartheta)\,[2\,i_0^3\Delta\vartheta^4(i_0-1)]+D_2)\,\rho,\vartheta)\,(-i_0^3\Delta\vartheta^3)$$

$$Z_{00}=D_5(\rho,\vartheta)\,[-4\,i_0^2\Delta\vartheta^2-4]+D_6(\rho,\vartheta)\,[-4\,i_0^2\Delta\vartheta^2+4]$$

$$Z_{01}+D_5(\rho,\vartheta)\cdot 2+D_6(\rho,\vartheta)\,(-2)+D_7(\rho,\vartheta)\,2\,\Delta\vartheta+D_8(\rho,\vartheta)$$

$$Z_{02}=D_5(\rho,\vartheta)\,[2\,i_0^2\Delta\vartheta^2+i_0\,\Delta\vartheta^2]+D_6(\rho,\vartheta)\,[2\,i_0^2\Delta\vartheta^2-i_0\,\Delta\vartheta^2]+$$
$$+D_9(\rho,\vartheta)\,(-1)$$

$$Z_{03}=D_5(\rho,\vartheta)\cdot 2+D_6(\rho,\vartheta)\,(-2)+D_7(\rho,\vartheta)\,(-2\,\Delta\vartheta)+D_8(\rho,\theta)\,(-1)$$

$$Z_{04}=D_5(\rho,\vartheta)\,[2\,i_0^2\Delta\vartheta^2-i_0\,\Delta\vartheta^2]+D_6(\rho,\vartheta)\,[2\,i_0^2\Delta\vartheta^2+i_0\,\Delta\vartheta^2]+$$
$$+D_9(\rho,\vartheta)\,(i_0\,\Delta\vartheta)$$

$$Z_{05}=D_7(d,\vartheta)\,(-i_0\,\Delta\vartheta)$$

$$Z_{06}=D_7(\rho,\vartheta)\,(i_0\,\Delta\vartheta)$$

$$Z_{07}=D_7(\rho,\vartheta)\,(-i_0\,\Delta\vartheta)$$

$$Z_{08}=D_7(\rho,\vartheta)\,(i_0\,\Delta\vartheta)$$

$$Z_{09}=Z_{010}=Z_{011}=Z_{012}+0 \tag{20}$$

Since the real axis $\operatorname{Im} z=0$ is mapped by the function (13) onto the real axis $\operatorname{Im}\zeta=0$, and stress tensor σ_{ij} on the real axis is symmetrical, the domain Σ in plane (ζ) can be divided into a symmetrical polar net with n-knots, referring to the symmetry axis of the excentric circular annulus $\operatorname{Im} z=0$. Thus the net can be made denser in the wall domain $\operatorname{Im} z>0$, and at the same time we are in this way confined only to the cases when the wall is buckled symmetrically to the real axis $\operatorname{Im} z=0$.

For each knot point of the polar net the difference equation (19) is written, and thus a homegeneous system is obtained containing n-linear equations and n-unknowns, representing the displacement function w_t $t=1, 2, \ldots, n$

$$(S_{jt}+k\cdot z_{jt})\,w_t=0 \tag{21}$$

The condition for an untrivial solution of the homogeneous system of equations (21) is

$$|S_{jt}+k\,z_{jt}|=0 \tag{22}$$

On the inner and outer wall boundary the displacement functions w_t which are located outside the domain Σ, can de expressed by wall displacements within the domain Σ using the expressions (16) of a simply supported boundary written in the difference form.

The size of the elements S_{jt} and Z_{jt}, the lowest root of the algebraic equation (22), $K_{\min}$ and the buckling coefficient $\alpha = \sqrt{K_{\min}}$ are all defined by an electronic computer *IBM* 1130. For this purpose a computer program in FORTRAN programming language was worked out containing seventeen sub-programs and a main program. Two of the sub-programs were used for inversion and multiplication of matrices, five for the calculation of the stress-tensor elements σ_{ii}, nine for the determination of elements S_{jt} and Z_{jt} and one subprogram for the determination of the buckling coefficient α. The discussed example of wall was calculated numerically for homogeneous ($P_N = P_Z < 0$) and unhomogeneous ($P_N = 0$, $P_Z < 0$) plane stress state at different eccentricites (e) and redii ratios ($\mu = r_1/r_2$). As Poisson's ratio for steel $\nu = 0,3$ was chosen. The values of buckling coefficients for homogeneous stress state are assembled in Table I, Fig. 5, and for unhomogeneous stress state in Table II, Fig. 6.

Table I

e^{μ}	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
0,00	4,09	4,27	4,70	5,35	6,33	7,85	10,41	15,56	31,11	∞
0,75	3,75	4,03	4,41	4,94	5,68	6,76	8,41	11,34	17,6	
1,5	3,51	3,77	4,07	4,49	5,04	5,81	6,96	8,83		
2,25	3,28	3,49	3,73	4,05	4,47	5,04	5,88			
3,0	3,02	3,20	3,37	3,67	3,96	4,40				
3,75	2,75	2,89	3,05	3,25	3,50	3,84				
4,5	2,45	2,58	2,73	2,89	3,08					
5,25	2,13	2,27	2,40	2,53						
6,0	1,79	1,96	2,1							
6,75	1,4	1,7	1,88							

Table II

e^{μ}	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
0,00	4,16	4,56	5,14	6,01	7,08	8,74	11,8	18,6	39,2	∞
0,75	3,81	4,19	4,70	5,35	6,22	7,57	9,83	14,0	23,0	
1,5	3,58	3,87	4,26	4,77	5,47	6,53	8,19	10,9		
2,25	3,3	3,56	3,85	4,26	4,83	5,68	6,95			
3,0	3,04	3,24	3,47	3,81	3,29	4,97				
3,75	2,75	2,92	3,12	3,40	3,80	4,35				
4,5	2,45	2,6	2,78	3,02	3,35					
5,25	2,13	2,28	2,46	2,67						
6,0	1,78	1,96	2,15							
6,75	1,4	1,69	1,92							

If a comparison is drawn between analytically defined buckling coefficient α for a centric circular annulus by Škerlj (8) and the numerical result at $e=0$, the deviations do not exceed the value $-2{,}6\%$. It is found out that

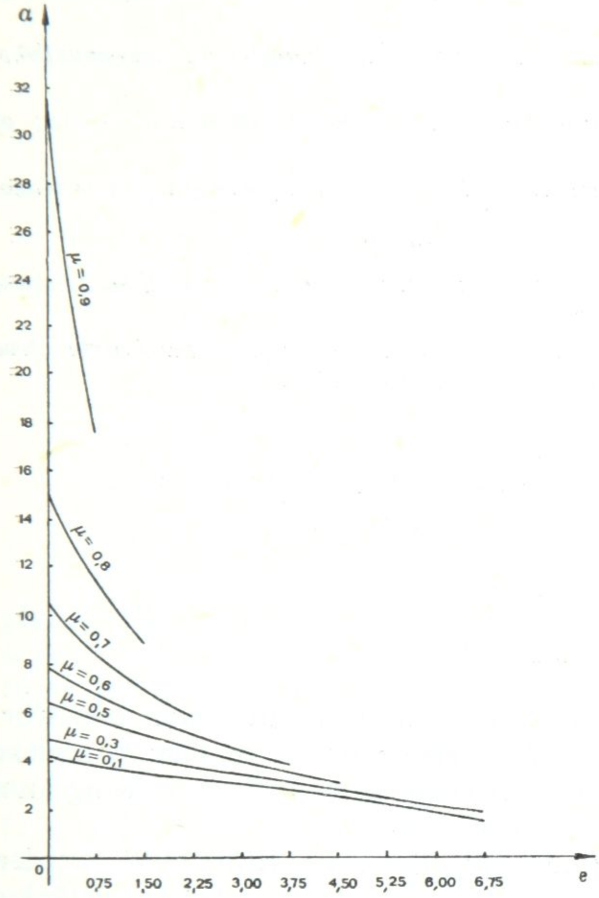

Fig. 5

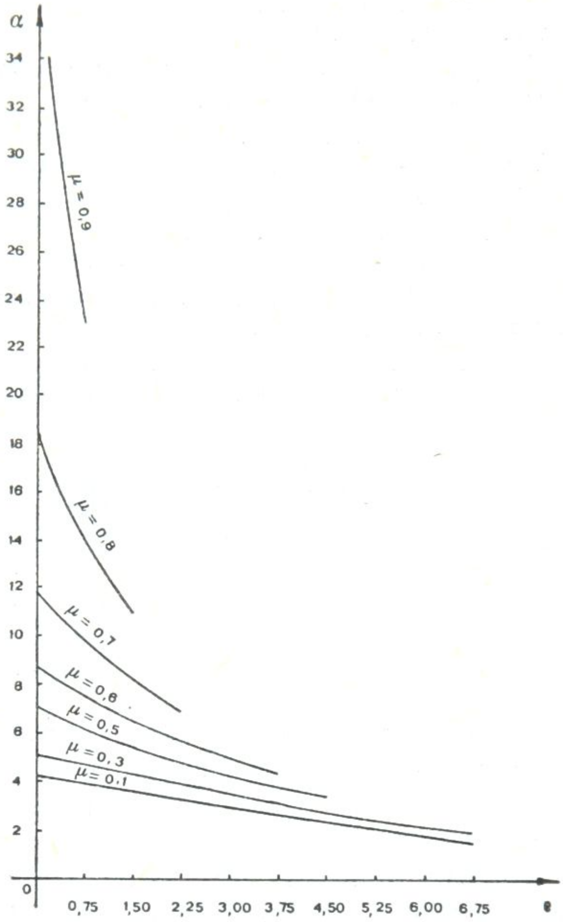

Fig. 6

at decreasing radii ratio μ and at increasing eccentricity e in the same polar net, the error grows up. In Fig. 7, the function $\alpha=\alpha(e)$ defined by a net of

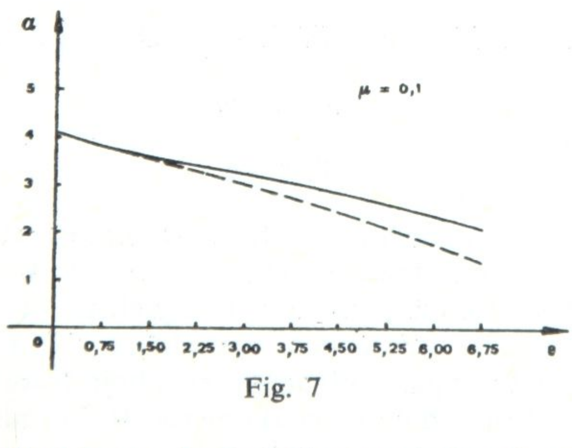

Fig. 7

twenty knots is presented by a dashed line, and the function $\alpha=\alpha(e)$ defined by a net of forty knots by a full line, at a homogeneous stress state and radii ratio $\mu=0{,}1$.

REFERENCES

[1] B. Pertot, *Problemi elasto-plastičnih deformacij pri ravninskih elementih s posebnim ozirom na stabilnostne razmere*, doktorska disertacija, FS, Ljubljana, 1972

[2] N. Yamaki, *Buckling of a Thin Anullar Plate Under Uniform Compression*, J. Appl. Mech. 25, 1958

[3] F. Kosel, *Reševanje stabilnosti sten z uporabo funkcij kompleksne spremanljivke*, Strojniški vestnik, Ljubljana, 1975/1—2

[4] N. I. Mushelišvili, *Some Basic Problems of the Mathematical Theory of Elasticity*, P. Noordhoff, Groningen, 1958

[5] A. V. Bicadze, *Osnovi teorii analitičeskih funkcij kompleksnoga peremennogo*, F, Ml. Nauka, Moskva, 1972

[6] A. I. Lurie, *Teorija uprugosti*, Nauka, Moskva, 1970

[7] F. Kosel, *Stabilnostni problem dvakrat sovisne stene z uporabo funkcij kompleksne spremenljivke*, Doktorska disertacija, FS, Ljubljana, 1974

[8] M. Škerlj, *Splošna rešitev osnosimetričnega stabilnostnega problema krožne stene s centrično krožno odprtino*, Doktorska disertacija, FS, Ljubljana, 1976.

ELASTISCHE STABILITÄT EINES EXZENTRISCHEN KREISSFÖRMIGEN RINGRAUMS

F. Kosel

Zusammenfassung

In diesem Beitrag wird elastische Stabilität eines exzentrischen kreissförmigen Ringraums behandelt. Die zentrale Wandebene wurde in eine komplexe Zahlenebene (z) gelegt und auf einen exzentrischen kreissförmigen Ringraum einer komplexen Zahlenebene (ζ) konform abgebildet.

Mit Hilfe der Differenzialmethode und des Rechners IBM 1130 werden die Beulbeiwerte numerisch definiert so für eine von beiden Seiten gelänkig gelagerte Wand, die durch eine gleichmässige Druckspannung ($P_N = P_Z < 0$) auf beiden Ränder oder durch eine gleichmässige Druckspannung ($P_N = 0$), ($P_Z < 0$) nur auf den äusseren Rand belasted wird. Die Ergebnisse und Abweichungen sind aus den Tabellen und Diagramen ersichtlich.

ELASTIČNA STABILNOST EKSCENTRIČNEGA KROZNEGA KOLOBARJA

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Rezime

V tem članku je obravnavana elastična stabilnost ekscentričnega krožnega kolobarja. Osrednja ravnina stene je bila položena v kompleksno ravnino (z) in preslikana na centrični krožni kolobar kompleksne ravnine (ζ).

Numerično, z uporabo diferenčne metode in računalnika IBM 1130, so določene velikosti izbočitvenega koeficienta za obojestransko prosto oprto steno, ki je obremenjena na obeh robovih z enakomerno tlačno napetostjo ($P_N = P_Z < 0$) oziroma samo z enakomerno tlačno napetostjo na zunanjem robu ($P_N = 0$, $P_Y < 0$).

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