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LINEARIZED EQUATIONS OF THE COUPLED THERMOMECHANICAL GENERAL MEMBRANE PROBLEM IN THE FINITE ELEMENT METHOD

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1. Introduction

It is well-known that an engineering idealisation of aerospace and similar structures leads to a finite-element model which is, usually, an assemblage of membranes and bars. In the most general case, such structures are dynamically and thermally loaded. Consequently, structural analysis software should be based on a sound enough thermomechanical foundation. Although, in past, the finite element structural dynamics and heat conduction problems were frequently solved, the coupled situation was studied only in some special cases[1].

The aim of this paper is to develop the linearized equations of the coupled thermomechanical problem. These equations are necessary, if we use the implicit approach in the solution of the coupled problem. There is no evidence in the available sources that such equations were published earlier.

2. Field equations

We start from the membrane equations of motion

$$(x^i_\alpha N^{\alpha\beta})|_\beta + \bar{\rho} F^i = \bar{\rho} \ddot{x}^i \tag{2.1}$$

having the boundary conditions

$$N^i - x^i_\alpha N^{\alpha\beta} \nu_\beta = 0 \tag{2.2}$$

and from the energy balance equations

$$\bar{\rho}\theta\dot{\eta} - q^\beta|_\beta - \bar{\rho} r - \partial = 0 \tag{2.3}$$

with boundary conditions

$$\bar{q} = \bar{q}^\beta \nu_\beta \tag{2.4}$$

In these equations

$$x^i = x^i(\xi^\alpha, t) \quad i = 1, 2, 3$$

are the Cartesian coordinates of a point at a membrane,

(2.5) $$x^i_\alpha = \partial x^i / \partial \xi^\alpha$$

are the coordinates of a base vector,

(2.6) $$\xi^\alpha, \ \alpha = 1, 2$$

are the intrinsic coordinates of a point at a membrane,

$N^{\alpha\beta}$ — the tensor of the membrane forces,

$\bar{\rho}$ — the surface density of a membrane,

F^i — the surface forces,

x^i — the acceleration at a point x^i,

N^i — the boundary forces,

ν_β — the outward normal vector to the boundary,

θ — the absolute tempe ature,

η — the specific entropy,

q — the heat flux,

r — the heat source,

(2.7) $$\partial \equiv \frac{1}{2} N^{\alpha\beta} \dot{a}_{\alpha\beta} - \bar{\rho}(\dot{\psi} + \eta \dot{\theta})$$

the internal dissipation,

(2.8) $$a_{\alpha\beta} = \delta_{ij} x^i_\alpha x^j_\beta$$

the fundamental metric tensor of a membrane, and

ψ — the free energy.

The upright line denotes the covariant differentiation with respect to ξ^α and over dot the time differentiation.

3. Constitutive equations

3.1. *Thermomechanically simple materials.* It can be shown[1,3] that thermomechanically simple materials are characterized by two constitutive functionals. Particularly, for membranes these functionals have a form[7]

(3.1) $$\psi = \underset{s=0}{\overset{\infty}{\psi}} [a_{\alpha\beta}(t-s), \ \theta(t-s); \ a_{\alpha\beta}(t), \theta(t)],$$

(3.2) $$q^\alpha = \underset{s=0}{\overset{\infty}{q^\alpha}} [a_{\alpha\beta}(t-s), \ \theta(t-s); \ a_{\alpha\beta}(t), \ \theta(t), \ \theta_\alpha(t)]$$

and describe a free energy and a heat flux, respectively. In (3.2) $\theta_\alpha = \partial\theta/\partial\xi^\alpha$.

The membrane forces, entropy and internal dissipation are now determined by the functionals

$$N^{\alpha\beta}=2\,\bar{\rho}\,D_{a_{\alpha\beta}}\,\psi \tag{3.3}$$

$$\eta=-D_\theta\,\psi; \tag{3.4}$$

$$\partial=-\bar{\rho}\delta\psi \tag{3.5}$$

In these expressions $D_{a_{\alpha\beta}}$ and D_θ are the partial derivatives with respect to $a_{\alpha\beta}(t)$ and $\theta(t)$, while $\delta\psi$ is a Fréchet differential, linear in $\dot{a}_{\alpha\beta}(t-s)$ and $\dot{\theta}(t-s)$. In subsequent work we also need a time derivatives of the functionals ψ and q.

$$\dot{\psi}=\delta\psi+\dot{a}_{\alpha\beta}\,D_{a_{\alpha\beta}}\psi+\dot{\theta}\,D_\theta\,\psi \tag{3.6}$$

$$\dot{q}^\alpha=\delta\,q^\alpha+a_{\alpha\beta}\,D_{a_{\alpha\beta}}q^\alpha+\dot{\theta}\,D_\theta\,q^\alpha+\dot{\theta}_\alpha D_{\theta_\alpha}\,q^\alpha. \tag{3.7}$$

3.2 *Isotropic linearly thermoelastic material.* For an isotropic, linearly thermoelastic membrane[7], the functionals (1) and (2) become the functions

$$\begin{aligned}\bar{\rho}_0\,\psi=&\frac{\mu D}{2}\,\frac{1+\nu}{1-\nu}-\left(2\,\mu\,D\frac{1+\nu}{1-\nu}\,\alpha+CD\right)\theta-\\&-\frac{\mu D}{2}\,\frac{1+\nu}{1-\nu}\,(1-2\,\alpha\Theta)\,A^{\alpha\beta}\,a_{\alpha\beta}+\left(2\,\mu\,D\frac{1+\nu}{1-\nu}\,\alpha+CD\right)\theta+\\&+\frac{1}{4}\,\frac{\mu D}{1-\nu}\,[\nu\,A^{\alpha\beta}\,A^{\chi\psi}+(1-\nu)\,A^{\alpha\chi}\,A^{\beta\psi}]\,a_{\alpha\beta}\,a_{\chi\psi}-\\&-\mu D\frac{1+\nu}{1-\nu}\,\alpha\,A^{\alpha\beta}\,a_{\alpha\beta}\,\theta-CD\,\theta\ln\frac{\theta}{\Theta}\end{aligned} \tag{3.8}$$

$$\bar{q}^\alpha=D\,\varkappa\,A^{\alpha\beta}\,\theta_\beta \tag{3.9}$$

where D, $A^{\alpha\beta}$ and $\bar{\rho}_0$ are respectively the membrane thickness, contravariant metric tensor and surface den ity in the reference configuration, μ the shear modulus, ν the Poisson's cœfficient, α the cœfficient of thermal expansion, C the specific heat, $\varkappa$ the heat conduction coefficient, and Θ the reference temperature.

4. Finite elements

4.1. *General.* Let approximate a membrane by an assembly of elements, interconnected at their boundaries. To each element we associate a separate set of convected coordinates ξ^α. The Cartesian coordinates of a point at a middle surface of the membrane element are

$$x^i=P^K(\xi^\alpha)\,x^i_K(t);\ K=1,\ldots,n \tag{4.1}$$

where P^K are the suitably selected interpolation functions, x^i_K are the nodal coordinates, and n is a total number of nodes. The nodes are the characteristic points necessary for interpolation. The velocities and accele ations of an arbitary membrane point are now

(4.2) $$\dot{x}^i = P^K \dot{x}^i_K; \quad \ddot{x}^i = P^K \ddot{x}^i_K$$

where $\dot{x}^i_K$ and $\ddot{x}^i_K$ are the nodal velocities and accelerations. The base vectors (2.5) become now

(4.3) $$x^i_\alpha = P^K_\alpha x^i_K; \quad P^K_\alpha = \partial P^K / \partial \xi^\alpha$$

According to (2.8) the fundamental metric tensor will be

(4.4) $$a_{\alpha\beta} = \delta_{ij} P^K_\alpha P^L_\beta x^i_K x^j_L$$

and its time derivative

(4.5) $$\dot{a}_{\alpha\beta} = 2\, \delta_{ij} B^{KL}_{\alpha\beta} x^i_K \dot{x}^j_L$$

where

(4.6) $$B^{KL}_{\alpha\beta} = \frac{1}{2} (P^K_\alpha P^L_\beta + P^L_\alpha P^K_\beta)$$

There is no obstacle to approximate the absolute temperature by the same set of interpolation functions, so that

(4.7) $$\theta = P^K \theta_K$$

(4.8) $$\dot{\theta} = P^K \dot{\theta}_K$$

(4.9) $$\theta_\alpha = P^K_\alpha \theta_K$$

where θ_K and $\dot{\theta}_K$ are the nodal temperatures and their time rates, respectively.

4.2. *Equations of motion.* In the Galerkin method[2], we approximate the weak solution of the equations (2.1) by the set of linearly independent functions $P^K(\xi^\alpha)$.

(4.10) $$\sum_e \int_S [(x^i_\alpha N^{\alpha\beta})|_\beta + \rho (F^i - \ddot{x}^i)] P^L dS = 0$$

Making use of a divergence theorem, the boundary conditions (2.2) and the relationships (1) and (2) we get the equations of motion in a discrete form

(4.11) $$M^{KL} \ddot{x}^i_K + S^{KL} x^i_K = R^{Li}$$

where

(4.12) $$M^{KL} = \sum_e \int_S \bar{\rho} P^K P^L dS$$

are the entries of the mass matrix of a complete assembly. Further

(4.13) $$S^{KL} = \sum_e \int_S N^{\alpha\beta} P^K_\alpha P^L_\beta dS$$

are the members of a force matrix, and

$$R^{Li} = \sum_e \int_S \bar{\rho} F^i P^L dS + \sum_e \int_S N^i P^L ds \tag{4.14}$$

are the nodal forces. In these expressions the integration is carried out over the domain of each element, and summation includes all the elements of a system.

4.3. *Heat conduction equations.* We approximate the weak solution of the equation (2.3) using the same system of functions P^K as before.

$$\sum_e \int_S (\bar{\rho}\,\theta\,\dot{\eta} - \bar{q}^{\beta}|_{\beta} - \bar{\rho}\,r - \partial)\,P^L dS = 0 \tag{4.15}$$

Applying the divergence theorem, the boundary conditions (2.4) and the expressions (6), (7) and (8) we obtain the heat conduction equations in a discrete form

$$O^{KL}\theta_K + G^L - Q^L - \partial^L = 0 \tag{4.16}$$

where

$$O^{KL} = \sum_e \int_S \bar{\rho}\,\dot{\eta}\,P^K P^L dS \tag{4.17}$$

$$G^L = \sum_e \int_S \bar{q}^{\beta} P^L_{\beta} dS \tag{4.18}$$

$$Q^L = \sum \int_S \bar{\rho}\,r\,P^L dS + \sum_e \int_C \bar{q}\,P^L dS \tag{4.19}$$

$$\partial^L = \sum_e \int_S P^L \partial\, dS \tag{4.20}$$

The heat conduction equations, basically equivalent to (16), were originally developed by Oden[1]. These equations are very general, and their form is independent on the material properties. Unfortunately, from the point of view of the effective computation, these equations are not very practical, because the temperature rate is contained implicitly in (16).

We consider now the special case of heat conduction equations, for a thermomechanically simple material. Making use of (3.1), (3.4) and (3.6), we can write that

$$\dot{\eta} = -\delta\,D_{\theta}\psi - \dot{a}_{\chi\psi}\,D_{a_{\chi\psi}}\,D_{\theta}\psi - \dot{\theta}\,D^2_{\theta}\psi \tag{4.21}$$

Having further in mind (4.5) and the symmetry of the expression $D_{a_{\alpha\beta}}\psi$, in accordance with (3.3) we can write the equations (15) in a form

$$D^{KiL}\dot{x}_{Ki} + U^{KL}\dot{\theta}_K + J^{KL}\theta_K + G^L - Q^L - \partial^L = 0 \tag{4.22}$$

where

(4.23) $$D^{KiL} = -2\sum_e \int_S \bar{\rho}\theta x^i_\chi P^L P^K_\psi D_{a\chi\psi} D_\theta \psi \, dS$$

(4.24) $$U^{KL} = -\sum_e \int_S \bar{\rho}\theta P^K P^L D^2_\theta \psi \, dS$$

(4.25) $$J^{KL} = -\sum_e \int_S \bar{\rho} P^K P^L D_\theta \delta\psi \, dS$$

(4.26) $$\partial^L = -\sum_e \int_S \bar{\rho} P^L \delta\psi \, dS$$

The equations (22) were originally developed by the present author[6]. Their advantage is the explicit appearance of the temperature rate $\dot{\theta}_K$, a fact of a paramount importance in the explicit numerical solution procedures.

5. Linearization of the equations of motion and heat conduction

If one wants to solve (4.11) and (4.22), using implicit methods, it is necessary first to linearize these equations We rewrite (4.11) and (4.22) in a form

(5.1) $$\Phi^{Jj} = M^{IJ} \ddot{x}^j_I + S^{IJ} x^j_I - R^{Jj} = 0$$

(5.2) $$\Phi^J = D^{IiJ} \dot{x}_{Ii} + U^{IJ} \dot{\theta}_I + J^{IJ} \theta_I + G^J - Q^J - \partial^J = 0$$

describing the state of a system at a moment t.

The state at a moment $t+h$ can be determined making use of a Taylor expansion

$$\Phi^{Jj}_h = \Phi^{Jj} + M^{IJ} (\ddot{x}^j_{Ih} - \ddot{x}^j_I) + h\dot{S}^{IJ} x^I_I + S^{IJ} (x^j_{Ih} - x^j_I) -$$

(5.3) $$- R^{Jj}_h + R^{JI} + \text{ higher order terms in } h = 0$$

$$\Phi^J_h = \Phi^J + h\, \dot{D}^{IiJ} \dot{x}_{Ii} + D^{IiJ} (\dot{x}^h_{Ii} - \dot{x}_{Ii}) +$$

$$+ h\, \dot{U}^{IJ} \dot{\theta}_I + U^{IJ} (\dot{\theta}^h_I - \dot{\theta}_I) +$$

$$+ h\, \dot{J}^{IJ} \theta_I + J^{IJ} (\theta^h_I - \theta_I) +$$

$$+ h\, \dot{G}^J - (Q^J_h - Q^J) - h\dot{\partial}^J +$$

(5.4) $$+ \text{higher order terms in } h = 0$$

After the effective determination of the time derivatives, and some lengthy but simple algebra, neglecting higher order terms in h, we finally obtain the linearized equations of motion and heat conduction

$$M^{IiJj}\ddot{x}^h_{Ii}+(K^{IiJj}+S^{IiJj})\,x^h_{Ii}+L^{IJj}\,\theta^h_I=$$

$$=R^{Jj}_h+K^{IiJj}\,x_{Ii}+L^{IJj}\,\theta_I-h\,\delta R^{Jj} \tag{5.5}$$

$$D^{IiJ}\dot{x}_{Iih}+F^{IiJ}\,x_{Iih}+U^{IJ}\,\dot{\theta}_{Ih}+[H^{IJ}+J^{IJ}]\,\theta_{Ih}=$$

$$=Q^J_h+F^{IiJ}\,x_{Ii}+H^{IJ}\,\theta_I-G^J+\partial^J+h\,\Delta^J \tag{5.6}$$

In these expressions

$$M^{IiJj}=\sum_e\int_S\bar{\rho}\,\delta^{ij}\,P^I P^J\,dS \tag{5.7}$$

$$S^{IiJj}=2\sum_e\int_S\bar{\rho}\,\delta^{ij}\,P^I_\alpha P^J_\beta\,D_{a_{\alpha\beta}}\,\psi\,dS \tag{5.8}$$

$$K^{IiJj}=4\sum_e\int_S\bar{\rho}\,x^j_\alpha\,x^i_\chi\,P^J_\beta P^I_\psi\,D_{a_{\alpha\beta}}\,D_{a_{\chi\psi}}\,\psi\,dS \tag{5.9}$$

$$\delta R^{Jj}=\sum_e\int_S\bar{\rho}\,x^j_\alpha\,P^J_\beta\,D_{a_{\alpha\beta}}\,\delta\psi\,dS \tag{5.10}$$

$$L^{IJj}=\sum_e\int_S\bar{\rho}\,x^j_\alpha\,P^J_\beta P^I\,D_{a_{\alpha\beta}}\,D_\theta\,\psi\,dS \tag{5.11}$$

$$D^{IiJ}=-2\sum_e\int_S\bar{\rho}\,\theta\,x^i_\chi\,P^J P^I_\psi\,D_{a_{\chi\psi}}\,D_\theta\,\psi\,dS \tag{5.12}$$

$$F^{IiJ}=-2\sum_e\int_S\bar{\rho}\,\dot{\theta}\,x^i_\chi\,P^J P^I_\psi\,D_{a_{\chi\psi}}\,D_\theta\,\psi\,dS-$$

$$-2\sum_e\int_S\bar{\rho}\,\theta\,\dot{x}^i_\chi\,P^J P^I_\psi\,D_{a_{\chi\psi}}\,D_\theta\,\psi\,dS-$$

$$-2\sum_e\int_S\dot{\bar{\rho}}\,\theta\,x^i_\chi\,P^J P^I_\psi\,D_{a_{\chi\psi}}\,D_\theta\,\dot{\psi}\,dS-$$

$$-2\sum_e\int_S\dot{\bar{\rho}}\,\theta\,x^i_\chi\,P^J P^I_\psi\,D_{a_{\chi\psi}}\,D_\theta\,\delta\psi\,dS+$$

$$+2\sum_e\int_S\bar{\rho}\,x^i_\chi\,P^J P^I_\psi\,D_{a_{\chi\psi}}\,\delta\psi\,dS+$$

$$+2\sum_e\int_S x^i_\chi\,P^J_\beta P^I_\psi\,D_{a_{\chi\psi}}\,\bar{q}^\beta\,dS \tag{5.13}$$

$$U^{IJ} = -\sum_e \int_S \bar{\rho}\,\theta\, P^I P^J D_\theta^2\, \psi\, dS \tag{5.14}$$

$$H^{IJ} = -\sum_e \int_S \bar{\rho}\,\dot{\theta}\, P^I P^J D_\theta^2\, \psi\, dS$$

$$-\sum_e \int_S \bar{\rho}\,\theta\, P^I P^J D_\theta^2\, \dot{\psi}\, dS$$

$$-\sum_e \int_S \bar{\rho}\,\theta\, P^I P^J D_\theta^2\, \delta\psi\, dS$$

$$+\sum_e \int_S \bar{\rho}\, P^I P^J D_\theta\, \delta\psi\, dS$$

$$+\sum_e \int_S P^I P^J_\beta D_\theta\, \bar{q}^\beta\, dS$$

$$+\sum_e \int_S P^I_\alpha P^J_\beta D_{\theta_\alpha}\, \bar{q}^\beta\, dS \tag{5.15}$$

$$J^{IJ} = -\sum_e \int_S \bar{\rho}\, P^I P^J D_\theta\, \delta\psi\, dS \tag{5.16}$$

$$Q^J = \sum_e \int_S \bar{\rho}\, P^J r\, dS + \sum_e \int_C P^J \bar{q}\, ds \tag{5.17}$$

$$\partial^J = -\sum_e \int_S \bar{\rho}\, P^J \delta\psi\, dS \tag{5.18}$$

$$G^J = \sum_e \int_S P^J_\beta \bar{q}^\beta\, dS \tag{5.19}$$

$$\Delta^J = -\sum_e \int_S \bar{\rho}\,\theta\, P^J D_\theta\, \delta^2\, \psi\, dS$$

$$+\sum_e \int_S \bar{\rho}\, P^J \delta^2\, \psi\, dS$$

$$+\sum_e \int_S P^J_\beta\, \delta\, \bar{q}^\beta\, dS \tag{5.20}$$

6. Example

6.1. *Isotropic linearly thermoelastic material.* Using the constitutive equations (3.8) and (3.9) we reduce the expressions (5.7) — (5.11), (4.14) and (5.12) — (5.19) to

(5.7′) $$M^{IiJj} = \sum_e \int_{S_0} \rho_0 \, \delta^{ij} D P^I P^J dS_0$$

(5.8′) $$S^{IiJj} = \sum_e \int_{S_0} \delta^{ij} P^I_{\alpha} P^J_{\beta} \frac{\mu D}{1-\nu} \{[\nu A^{\alpha\beta} A^{\chi\psi} + (1-\nu) A^{\alpha\chi} A^{\beta\psi}] a_{\chi\psi} - (1+\nu)[1+2\alpha(\theta-\Theta)] A^{\alpha\beta}\} dS_0$$

(5.9′) $$K^{IiJj} = 2 \sum_e \int_{S_0} x^j_{\alpha} x^i_{\chi} P^J_{\beta} P^I_{\varphi} \frac{\mu D}{1-\nu} [\nu A^{\alpha\beta} A^{\chi\psi} + (1-\nu) A^{\alpha\chi} A^{\beta\psi}] dS_0$$

(5.10′) $$\delta R^{Jj} = 0$$

(5.21) $$R^{Jj}_h = \sum_e \int_{S_0} P^J \rho_0 D F^j_h \, dS_0 + \sum_e \int_{C_0} P^J N^j_h \, ds_0$$

(5.22) $$R^{Jj} = \sum_e \int_{S_0} P^J \rho_0 D F^j dS_0 + \sum_e \int_{C_0} P^J N^J ds_0$$

(5.11′) $$L^{IJj} = -2 \sum_e \int_{S_0} x^j_{\alpha} P^J_{\beta} P^I \alpha \mu D \frac{1+\nu}{1-\nu} A^{\alpha\beta} dS_0$$

(5.12′) $$D^{IiJ} = 2 \sum_e \int_{S_0} \theta \, x^i_{\chi} P^J P^I_{\psi} \alpha\mu D \frac{1+\nu}{1-\nu} A^{\chi\psi} dS_0$$

(5.13′) $$F^{IiJ} = 2 \sum_e \int_{S_0} \dot{\theta} \, x^i_{\chi} P^J P^I_{\psi} \alpha\mu D \frac{1+\nu}{1-\nu} A^{\chi\psi} dS_0 + 2 \sum_e \int_{S_0} \theta \, \dot{x}^i_{\chi} P^J P^I_{\psi} \alpha\mu D \frac{1+\nu}{1-\nu} A^{\chi\psi} dS_0$$

(5.14′) $$U^{IJ} = \sum_e \int_{S_0} P^I P^J DC \, dS_0$$

(5.15′) $$H^{IJ} = \sum_e \int_{S_0} P^I_{\alpha} P^J_{\beta} D \varkappa A^{\alpha\beta} dS_0$$

(5.17′) $$Q^J = \sum_e \int_{S_0} P^J \rho_0 D r \, dS + \sum P^J \bar{q} \, dS_0$$

(5.19′) $$G^J = \sum \int_{S_0} \theta_\alpha P^J_\beta D \varkappa A^{\alpha\beta} dS_0$$

(5.16′) $$J^{IJ} = 0$$

(5.18′) $$\partial^J = 0$$

(5.20′) $$\Delta^J = 0$$

In these expressions, K^{IiJj} are the entries in complete stiffness matrix. For the membrane finite elements, K^{IiJj} were published earlier[6]. The same is true for the mass and force matrix entries. The members of thermal matrices U^{IJ} and H^{IJ} are the membrane specializations of the otherwise well known quantites[5]. However, the connecting terms L^{IJj}, D^{IiJ} and F^{IiJ} were not published before. All integrations in this section are performed in the reference configuration.

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ЛИНЕАРИЗИРОВАННИЕ УРАВНЕНИЯ СОПРЯЖЕННОЙ ТЕРМОМЕХАНИЧЕСКОЙ ПРОБЛЕМЫ ОБЩЕЙ БЕЗМОМЕНТНОЙ ОБОЛОЧКИ В МЕТОДЕ КОНЕЧНИХ ЭЛЕМЕНТОВ

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Резюме

В настоящей работе рассматривается проблема конечних элементов безмоментной оболочки. Начиная с уравнениями поля и определяющими уравнениями безмоментних оболочек, и используя процедуру Галеркина, выведени нелинейние уравнения движения и теплопроводимости в конечно-элементной форме. Потом, используя развитие Тейлора по времени, эти уравнения линеаризированни. Конечно, в качестве примера, анализированни уравнения описивающие поведение линейной термоупругой безмоментной оболочки.

ЛИНЕАРИЗОВАНЕ ЈЕДНАЧИНЕ СПРЕГНУТОГ ТЕРМОМЕХАНИЧКОГ ПРОБЛЕМА ОПШТЕ МЕМБРАНЕ У МЕТОДИ КОНАЧНИХ ЕЛЕМЕНАТА

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Извод

У овом раду је размотрен проблем коначних елемената мембрана, интересантан и са гледишта практичних примена у области танкозидних конструкција. Циљ рада био је да се изведу линеаризоване једначине спрегнутог проблема, неопходне при имплицитном решавању.

Полазећи од једначина поља са граничним условима (2.1 — 4), конститутивних једначина за термомеханички прост материјал (3.4 — 7), коначно-елементних апроксимација (4.1 — 9) и Галеркинове процедуре апроксимирања слабог решења, добијене су дискретне нелинеарне једначине кретања и провођења топлоте (4.11) и (4.16) респективно. За термомеханички прост материјал показано је да једначине провођења топлоте могу да се реше експлицитно по првом изводу температуре по времену (4.22).

Развитком у ред Taylora по времену и линеаризацијом, једначине кретања и провођења топлоте су сведене на облик (5.5) и (5.6) респективно, при чему су експлицитно дати изрази за израчунавање коефицијената добијеног система линеарних д.ј. другог и првог реда (5.7 — 5.20). У својству примера размотрен је случај изотропног линеарно термоеластичног материјала, и показано је да се коефицијенти једначина за случај неспрегнутог проблема своде на познате изразе (8′), (9′) (14′), (15′), док повезујући коефицијенти (11′), (12′), (13′) и за овај релативно једноставан случај нису били раније публиковани.

Овај рад је део истраживачког пројекта финансираног преко П. М. Ф.-а од стране Заједнице науке С. Р. Србије.

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