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UNSTEADY FLOW OF AN ELASTICO-VISCOUS FLUID PAST AN INFINITE FLAT PLATE WITH SUCTION

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1. Introduction

Unsteady oscillatory flows with constant suction, past an infinite flat porous plate have been studied by Stuart (1955), Watson (1958), Kelly (1965). The problem was solved by assuming the unsteady flow to be superimposed on the mean flow past an infinite plate with constant suction which was studied by Griffith and Meredith (1936). However, in the absence of the free-stream oscillations, the unsteady flow past an infinite porous plate with a step-change in the constant suction is also of importance from an application point of view. In bio-physics, the purification of blood is always done by passing the blood over a porous plate. Such a process is generally known as the dialysis. In case of Newtonian fluids, such a study was recently presented by Purohit and Goyal (1975). Laplace-transform technique was used and the problem was solved. However, while taking the inverse Laplace transform, the solution was presented in the form of an infinite series containing incomplete Gamma functions for the general value of the suction and the exact solution was derived through inverse Laplace-transform for a particular value of the suction velocity as 2. However, it has been shown by Soundalgekar and Uplekar (to be published) that inverse Laplace transform can be found for all values of the suction parameter λ in a closed form. But from bio-physics point of view, the study of a Newtonian fluid has limited applications. Most of the fluids in bio-physics are of rheological nature and hence Non-Newtonian in character. Many authors like Stokes (1966), Walters (1962) etc. have tried to give constitutive equations to describe the Non-Newtonian fluids and it has been observed that the constitutive equations given by Walters (1962), on the assumption of vanishing memory, not only govern the flow of some polymers like polyacrylamied P 250, polyisobutylene B 1000, but also help us to understand the flow of biophysical liquids like Saliva, blood etc. Hence in the unsteady flow of such elastico-viscous fluids past an infinite porous plate with constant suction, it will be most useful to study the effects of a step-change in the suction-velocity. This has been discussed here. In Sec. 2, the mathematical analysis has been presented followed by a discussion.

2. Mathematical Analysis

We consider a two-dimensional flow of an elastico-viscous fluid (Walters liquid B') past an infinite porous plate with x'-axis taken along the plate in the direction of the flow and the y'axis is taken normal to the plate. It is assumed that for $t' \leq 0$, the flow is under constant normal suction velocity say $v_1 (<0)$ and from $t' \geq 0$, the normal suction velocity is changed to $v_2 (<0)$ and is maintained the same. The free-stream velocity is assumed to be constant say U_∞. Under these conditions, the physical variables are functions of t' and y' only and it has been shown in Soundalgekar and Puri (1969) that the flow of an elastico-viscous fluid is governed by the following equations:

(1) $$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - K_0^* \left(\frac{\partial^3 u'}{\partial t \, \partial y^2} + v' \frac{\partial^3 u'}{\partial y'^3} \right)$$

(2) $$\frac{\partial v'}{\partial y'} = 0$$

The boundary conditions are

(3) $$\begin{aligned} &u'(o, t) = 0, \quad u'(\infty, t) = U_\infty \\ &v' = v_1 \text{ (constant)}, \quad t \leq 0 \\ &\quad = v_2 \text{ (constant)}, \quad t > 0 \end{aligned}$$

On introducing the following non-dimensional quantities

$$y = y' \, |v_1| / \nu, \quad t = t' \, |v_1|^2 / \nu, \quad u = u' / U_\infty, \quad k = K_0^* \, |v_1|^2 / \nu^2$$

in equation (1) and (3), we get

(4) $$\frac{\partial u}{\partial t} + \frac{v'}{|v_1|} \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - k \left(\frac{\partial^3 u}{\partial t \, \partial y^2} + \frac{v'}{|v_1|} \frac{\partial^3 u}{\partial y^3} \right)$$

(5) $$\begin{aligned} &u(o, t) = 0, \quad u(\infty, t) = 1 \\ &\frac{v'}{|v_1|} = -1 \quad t \leq 0 \\ &\quad = -\lambda \quad t > 0, \quad \lambda = \frac{v_2}{|v_1|} \end{aligned}$$

The initial condition is taken as the steady flow past an infinite porous plate of an elastico-viscous fluid and is given by

(6) $$u(y, 0) = 1 - e^{-y} + k y e^{-y}$$

We have now to solve equation (4) subject to the boundary conditions (5) and (6). To simplify the mathematical procedure, we expand u in the powers of k, the elastic parameter, which is $\ll 1$ for the elastico-viscous fluids. So we assume

$$u = u_0 + k u_1 \tag{7}$$

Substituting (7) in (4), equating the coefficients of different powers of k, neglecting those of k^2, we get

$$\frac{\partial u_0}{\partial t} + \frac{v'}{|v_1|}\frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} \tag{8}$$

and

$$\frac{\partial u_1}{\partial t} + \frac{v'}{|v_1|}\frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial u^2} - \left(\frac{\partial^3 u_0}{\partial t\,\partial y^2} + \frac{v'}{|v_1|}\frac{\partial^3 u_0}{\partial y^3}\right) \tag{9}$$

These equations are solved by the usual Laplace-Transform technique and the solutions for u_0 and u_1 are written as follows:

$$u_0 = 1 - \exp\{(1-\lambda)t - 2\eta\sqrt{t}\} + \frac{1}{2}\exp\{(1-\lambda)t - 2\eta\lambda\sqrt{t}\}.$$

$$\left[e^{\eta(\lambda-2)\sqrt{t}}\,\mathrm{erfc}\left(\eta + \frac{\lambda-2}{2}\sqrt{t}\right) + e^{-\eta(\lambda-2)\sqrt{t}}\,\mathrm{erfc}\left(\eta - \frac{\lambda-2}{2}\sqrt{t}\right)\right]$$

$$-\frac{1}{2}\exp\{-\eta\lambda\sqrt{t}\}\left[e^{\lambda\eta\sqrt{t}}\,\mathrm{erfc}\left(\eta + \frac{\lambda\sqrt{t}}{2}\right) + e^{-\eta\lambda\sqrt{t}}\,\mathrm{erfc}\left(\eta - \frac{\lambda\sqrt{t}}{2}\right)\right] \tag{10}$$

$$u_1 = A_1 e^{-(\lambda\eta\sqrt{t}} + t e^{\{(1-\lambda)t - 2\eta\sqrt{t}\}} - \eta\sqrt{t}\,e^{-\lambda\eta\sqrt{t}}\left[A_2\left(\frac{\lambda^4}{2} + \frac{(1-\lambda)\lambda^2}{2} + \right.\right.$$

$$\left. + (1-\lambda)^2 + \frac{3\lambda^2(1-\lambda)}{2}\right) + A_3(2\lambda(1-\lambda) + \lambda^3) +$$

$$\left. + A_4(1-\lambda) - \frac{\lambda^6}{2}A_5 - \lambda^3 A_6\right] \tag{11}$$

where

$$A_1 = \frac{t e^{(1-\lambda)t}}{2}\left[\left(1 - \frac{2\eta}{(\lambda-2)\sqrt{t}}\right)e^{-\eta(\lambda-2)\sqrt{t}}\,\mathrm{erfc}\left(\eta - \frac{(\lambda-2)}{2}\sqrt{t}\right) + \right.$$

$$\left. + \left(1 + \frac{2\eta}{(\lambda-2)\sqrt{t}}\right. e^{\eta(\lambda-2)\sqrt{t}}\cdot\mathrm{erfc}\left(\eta + \frac{\lambda-2}{2}\sqrt{t}\right)\right]$$

$$A_2 = \frac{e^{(1-\lambda)t}}{\lambda-2}\left[e^{-\eta(\lambda-2)}\mathrm{erfc}\left(\eta - \frac{\lambda-2}{2}\sqrt{t}\right) - e^{\eta(\lambda-2)\sqrt{t}}\cdot\mathrm{erfc}\left(\eta + \frac{\lambda-2}{2}\sqrt{t}\right)\right]$$

$$A_3 = \frac{\lambda - 2}{2} A_2$$

$$A_4 = \frac{1}{\sqrt{\pi t}} e^{-\left(\eta^2 + \frac{\lambda^2 t}{4}\right)}, \quad \eta = y/2\sqrt{t}$$

$$A_5 = \frac{1}{\lambda}\left[e^{-\eta\lambda\sqrt{t}} \operatorname{erfc}\left(\eta - \frac{\lambda\sqrt{t}}{2}\right) - e^{\eta\lambda\sqrt{t}} \cdot \operatorname{erfc}\left(\eta + \frac{\lambda\sqrt{t}}{2}\right)\right]$$

$$A_6 = \frac{1}{2}\left[e^{\eta\lambda\sqrt{t}} \cdot \operatorname{erfc}\left(\eta + \frac{\lambda\sqrt{t}}{2}\right) + e^{-\eta\lambda\sqrt{t}} \operatorname{erfc}\left(\eta - \frac{\lambda\sqrt{t}}{2}\right)\right]$$

Knowing u_0 and u_1, we have calculated the numerical values of the velocity profiles from (7) and they are shown on figure 1. We observe from this figure that the velocity increases with increasing λ, both in case of

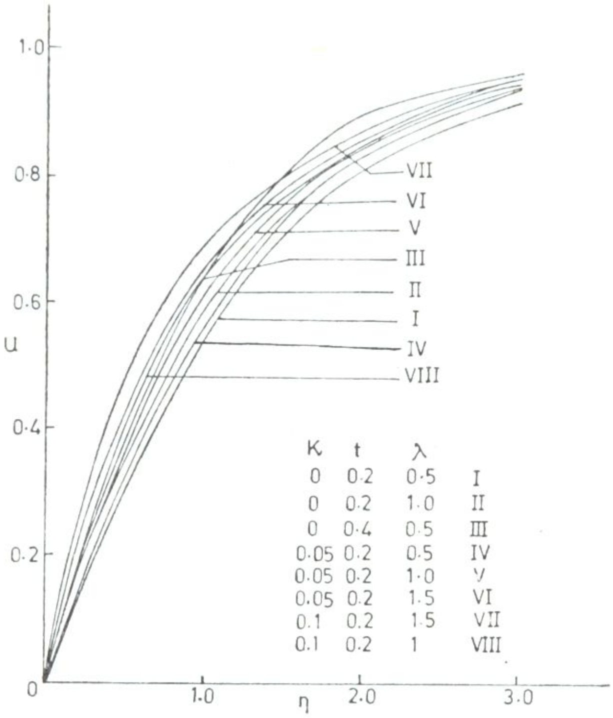

FIG. 1. VELOCITY PROFILES

Newtonian and Non-Newtonian fluids. The value of the velocity at any point is more in case of an elastico-viscous fluids than that in case of Newtonian fluids. An increase in K, leads to an increase in the velocity.

Knowing the velocity field, we can now calculate the shearing stress. It is given in non-dimensional form as

$$P_{xy} = \frac{1}{2\sqrt{t}} \left| \frac{\partial u}{\partial \eta} - K \frac{\partial^2 u}{\partial \eta \, \partial t} \right|_{\eta=0} \tag{12}$$

and in view of (7), (12) reduces to

$$P_{xy} = \frac{1}{2\sqrt{t}} \left| \frac{\partial u_0}{\partial \eta} + K\left(\frac{\partial u_1}{\partial \eta} - \frac{\partial^2 u_0}{\partial \eta \, \partial t} - \frac{\partial^2 u_0}{\partial \eta^2}\right) \right|_{\eta=0} \tag{13}$$

Substituting (10) and (11) in (13), we can calculate the expression for P_{xy}. To save space, it is not mentioned here. The numerical values of P_{xy} are entered in Table I.

Table I

Values of Shearing Stress

K	$t \backslash \lambda$	0.5	1	1.5	2.5
0	0.2	1.1974	1.0000	0.8576	0.7418
0	0.4	1.3238	1.0000	0.7969	0.7202
0.05	0.2	1.1394	0.9328	0.7386	0.3701
0.05	0.4	1.2445	0.9250	0.6765	0.3653
0.1	0.2	1.0814	0.8656	0.6196	−0.0016
0.1	0.4	1.1652	0.8501	0.5562	0.0103

We observe from this table that in case of an elastico-viscous fluids, the value of the shearing stress is less than that for Newtonian fluids. It increases with time but decreases with increasing λ.

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НЕСТАЦИОНАРНЫЙ ПОТОК ЭЛАСТИЧНО-ВЯЗКОЙ ЖИДКОСТИ МИМО БЕСКОНЕЧНОЙ ПЛОСКОЙ ПЛАСТИНЫ СО ВСАСЫВАНИЕМ

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Резюме

Разработано точное решение для нестационарного потока жидкости, являющейся эластично-вязкой, мимо бесконечной пористой пластины с учётом ступенчатых изменений скорости всасывания. Обнаружено, что увеличение коэффициента всасывания λ приводит к уменьшению срезывающего напряжения.

NESTACIONARNO STRUJANJE ELASTIČNO-VISKOZNE TEČNOSTI DUŽ BESKONAČNE RAVNE PLOČE SA USISIVANJEM

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Rezime

Prikazano je tačno rešenje nestacionarno strujanje tečnosti koja je elastično-viskozno duž beskonačne porozne ploče, uzevši u obzir stepenaste promene brzine usisivanja. Utvrđeno je da povećanje koeficijenata usisivanja λ dovodi do smanjenja smanjivanja tangencijalnog napona.

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