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UNIVERSAL EQUATIONS OF LAMINAR BOUNDARY LAYER FOR THE CASE OE "FROZEN" FLOW OF AN IDEALLY DISSOCIATED GAS AND THEIR PARAMETRIC SOLUTIONS

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1. System of basic equations

In this paper the basic equations of laminar two-dimensional boundary layer for the case of "frozen" flow of an ideally dissociated gas are reduced to the universal form according to Lojcjanskij's definition [1]. In addition, by introducing new purposeful transformations the generalization of the obtained system of universal equations of the problem concerned is achieved.

The above cited case of flow, on one hand, and the case of flow at the equilibrium dissociation, on the other hand, represent boundary cases for the problem of flow of the ideally dissociated gas in the boundary layer. The real flow of the ideally dissociated gas is, however, realized under the conditions between these two cases.

For that reason, the general case of flow of the ideally dissociated gas represented with the following system of equations according to [2] will be firstly considered:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right),$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0,$$

$$\rho u \frac{\partial \alpha}{\partial x} + \rho v \frac{\partial \alpha}{\partial y} + \frac{\partial}{\partial y}\left(\rho D \frac{\partial \alpha}{\partial y}\right) + \dot{W}_A, \tag{1}$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = -u \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \mu \left(\frac{\partial u}{\partial y}\right)^2 -$$

$$- (h_A - h_M) \dot{W}_A + \rho D (c_{pA} - c_{pM}) \frac{\partial \alpha}{\partial y} \frac{\partial T}{\partial y},$$

$$p = \rho T (1 + \alpha) \frac{k}{m_A} = (1 + \alpha) R_M;$$

with corresponding boundary conditions:

$$
\begin{aligned}
&u=v=0, \quad &&\alpha=\alpha_w, \quad &&T=T_w \quad &&\text{at} \quad y=0,\\
(2) \qquad &u\to u_e(x), \quad &&\alpha\to\alpha_e(x), \quad &&T\to T_e(x) \quad &&\text{at} \quad y\to\infty,\\
&u=u_0(y), \quad &&\alpha=\alpha_0(y), \quad &&T=T_0(y) \quad &&\text{at} \quad x=x_0.
\end{aligned}
$$

Where the well known symbols from the theory of boundary layer are used:

x — for the longitudinal coordinate,
y — for the transversal coordinate,
$u(x, y)$ — for the longitudinal velocity component in the boundary layer,
$v(x, y)$ — for the transversal velocity component in the boundary layer,
$\rho(x, y)$ — for the density of the ideally dissociated gas,
$\mu(x, y)$ — for the dynamic viscosity coefficient,
$\alpha(x, y)$ — for the coefficient of the atomic component mass of the ideally dissociated gas,
$D(x, y)$ — for the diffusion coefficient,
$\dot{W}_A$ — for the velocity of the atomic mass formation at chemical reactions,
$T(x, y)$ — for the absolute temperature,
$\lambda(x, y)$ — for the coefficient of thermal conductivity,
$c_p(T)$ — for the specific heat at constant pressure,
$h(x, y)$ — for the enthalpy,
$p(x, y)$ — for the pressure,
R — for the gas constant,
m_A — for the atomic mass,
k — for Boltzmann' constant.

The following indices designate:

w — the conditions at the surface of the body flowed over by the gas,
e — the conditions at the external limit of the boundary layer,
0 — the distribution of physical values in a given section of the boundary layer determined with the absissa $x=x_0$,
A — the atomic component,
M — the molecular component of the ideally disscciated gas.

2. Transformation of variables

N. V. Krivcova was the first to point out the possibility of reducing the system (1), (2) to the universal form in her work [3]. Namely, she has successfully achieved the universalization of equations of the boundary layer of the ideally dissociated gas but for the special case of equilibrium dissociation. Afterwards she has determined parametric solutions of these equations by means of numerical integration.

With regard to the analogy of the problems considered from the physical viewpoint, the transformations of variables are applied in this paper like those in [3]. For that reason, instead of physical coordinates x, y, new transformations are introduced as Dorodnicyn — Lees variables [3].

$$s(x)=\frac{1}{\rho_n \mu_n}\int_0^x \rho_w \mu_w \, dx, \qquad z(x, y)=\frac{1}{\rho_n}\int_0^y \rho \, dy, \tag{3}$$

where ρ_n and μ_n represent the known values of density and dynamic viscosity coefficient.

After introducing the stream function $\psi(s, t)$ by means of the relations

$$u=\frac{\partial \psi}{\partial z}, \qquad \bar{v}=\frac{\rho_n \mu_n}{\rho_w \mu_w}\left(u \frac{\partial z}{\partial x}+\frac{\rho}{\rho_n} v\right)=-\frac{\partial \psi}{\partial s}, \tag{4}$$

in the governing equations (1) with the corresponding boundary conditions (2), the following system is obtained:

$$\frac{\partial \psi}{\partial z}\frac{\partial^2 \psi}{\partial s \partial z}-\frac{\partial \psi}{\partial s}\frac{\partial^2 \psi}{\partial z^2}=\frac{\rho_e}{\rho} u_e \frac{du_e}{ds}+\nu_n \frac{\partial}{\partial z}\left(N \frac{\partial^2 \psi}{\partial z^2}\right),$$

$$\frac{\partial \psi}{\partial z}\frac{\partial \alpha}{\partial s}-\frac{\partial \psi}{\partial s}\frac{\partial \alpha}{\partial z}=\nu_n \frac{\partial}{\partial z}\left(\frac{N}{S_m}\frac{\partial \alpha}{\partial z}\right)+\frac{\rho_n \mu_n}{\rho_w \mu_w}\frac{\dot{W}_A}{\rho},$$

$$c_p\left(\frac{\partial \psi}{\partial z}\frac{\partial T}{\partial s}-\frac{\partial \psi}{\partial s}\frac{\partial T}{\partial z}\right)=-\frac{\rho_e}{\rho} u_e \frac{du_e}{ds}\frac{\partial \psi}{\partial z}+\nu_n \frac{\partial}{\partial z}\left(\frac{N}{P_r} c_p \frac{\partial T}{\partial z}\right)+$$

$$+\nu_n N\left(\frac{\partial^2 \psi}{\partial z^2}\right)^2+\nu_n \frac{N}{S_m}(c_{pA}-c_{pM})\frac{\partial \alpha}{\partial z}\frac{\partial T}{\partial z}-\frac{\rho_n \mu_n}{\rho_w \mu_w}(h_A-h_M)\frac{\dot{W}_A}{\rho}; \tag{5}$$

$$\psi=\frac{\partial \psi}{\partial z}=0, \qquad \alpha=\alpha_w, \qquad T=T_w \quad \text{at} \quad z=0,$$

$$\frac{\partial \psi}{\partial z}\to u_e(s), \qquad \alpha\to\alpha_e(s), \qquad T\to T_e(s) \quad \text{at} \quad z\to\infty,$$

$$\frac{\partial \psi}{\partial z}=u_0(z), \qquad \alpha=\alpha_0(z), \qquad T=T_0(z) \quad \text{at} \quad s=s_0.$$

where the functions N, Schmidt's number S_m and Prandtl's number P_r are determined with the expressions

$$N=\frac{\rho \mu}{\rho_w \mu_w}, \qquad N=1 \quad \text{at} \quad z=0, \qquad N=\frac{\rho_e \mu_e}{\rho_w \mu_w}=N(s) \quad \text{at} \quad z\to\infty,$$

$$S_m=\frac{\mu}{\rho D}, \qquad P_r=\frac{\mu c_p}{\lambda}. \tag{6}$$

According to the ideas presented in [3], another transformation of variables has been applied to the system of equations (5) as follows:

(7) $$s=s, \qquad \eta(s,\,z)=\frac{B_L}{\Delta^{**}}z, \qquad \psi(s,\,z)=\frac{u_e\Delta^{**}}{B_L}\Phi(s,\,\eta), \qquad \bar{T}=\frac{T}{T_1}$$

where

- B_L — „standard" Lojcjanskij constant,
- Φ — the dimensionless stream function,
- T_1 — the temperature at the front stagnation point of the body flowed over by the gas (the total temperature),
- $\bar{T}$ — the dimensionless temperature,
- Δ^{**} — momentum thickness defined with the expression

(8) $$\Delta^{**}(s)=\int_0^\infty \frac{u}{u_e}\left(1-\frac{u}{u_e}\right)dz.$$

It should be noted that for this case of the compressible fluid flow the respective impulse equation can be written in one of the following forms according to [3]:

(9) $$\frac{dZ^{**}}{ds}=\frac{F}{u_e(s)}, \qquad \frac{df}{ds}=\frac{u_e'}{u_e}F+\frac{u_e''}{u_e'}f, \qquad \frac{\Delta^{**\prime}}{\Delta^{**}}=\frac{u_e'F}{2u_ef},$$

which formally coincide with the respective forms of the impulse equation for the incompressible fluid. On the basis of one parametric methods of the boundary layer theory, the following parameter has been taken into account

(10) $$f=\frac{\Delta^{**2}}{\nu_n}\frac{du_e}{ds},$$

as well as the well-known values:

(11) $$Z^{**}=\frac{\Delta^{**2}}{\nu_n}, \qquad \zeta=\left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})}\right]_w, \qquad H=\frac{\Delta^*}{\Delta^{**}},$$

$$F=2\,[\zeta-(2+H)f], \qquad \Delta^*=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{u}{u_e}\right)dz.$$

By means of the new-introduced variables (7) and by using the relations (9), (10) and (11), the system of equations (5) is reduced to this form:

$$\frac{\partial}{\partial\eta}\left(N\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{F+2f}{2B_L^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B_L^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]=$$

$$=\frac{u_ef}{u_e'B_L^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial s\,\partial\eta}-\frac{\partial\Phi}{\partial s}\frac{\partial^2\Phi}{\partial\eta^2}\right),$$

$$\frac{\partial}{\partial\eta}\left(\frac{N}{S_m}\frac{\partial\alpha}{\partial\eta}\right)+\frac{F+2f}{2B_L^2}\Phi\frac{\partial\alpha}{\partial\eta}+\frac{1}{\nu_n}\left(\frac{\Delta^{**}}{B_L}\right)^2\frac{\rho_n\mu_n}{\rho_w\mu_w}\frac{\dot{W}_A}{\rho}=$$

$$=\frac{u_e f}{u_e' B_L^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial s}-\frac{\partial\Phi}{\partial s}\frac{\partial\alpha}{\partial\eta}\right),$$

$$\frac{\partial}{\partial\eta}\left(\frac{N}{P_r}c_p\frac{\partial\overline{T}}{\partial\eta}\right)+\frac{F+2f}{2B_L^2}c_p\Phi\frac{\partial\overline{T}}{\partial\eta}-\frac{\rho_e}{\rho}\frac{u_e^2}{T_1}\frac{f}{B_L^2}\frac{\partial\Phi}{\partial\eta}+N\frac{u_e^2}{T_1}\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+$$

$$(12)\qquad +\frac{N}{S_m}(c_{pA}-c_{pM})\frac{\partial\alpha}{\partial\eta}\frac{\partial\overline{T}}{\partial\eta}-\frac{1}{\nu_n}\left(\frac{\Delta^{**}}{B_L}\right)^2\frac{\rho_n\mu_n}{\rho_w\mu_w}\frac{h_A^0}{T_1}\frac{\dot{W}_A}{\rho}=$$

$$=\frac{u_e f}{u_e' B_L^2}c_p\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{T}}{\partial s}-\frac{\partial\Phi}{\partial s}\frac{\partial\overline{T}}{\partial\eta}\right);$$

$$\Phi=\frac{\partial\Phi}{\partial\eta}=0,\qquad \alpha=\alpha_w,\qquad \overline{T}=\overline{T}_w\qquad \text{at}\quad \eta=0,$$

$$\frac{\partial\Phi}{\partial\eta}\to 1,\qquad \alpha\to\alpha_e,\qquad \overline{T}\to\overline{T}_e\qquad \text{at}\quad \eta\to\infty,$$

$$\Phi=\Phi_0(\eta),\qquad \alpha=\alpha_0(\eta),\qquad \overline{T}=\overline{T}_0(\eta)\qquad \text{at}\quad s=s_0.$$

In addition to Prandtl's number P_r, Schmidt's number S_m (which are considered as constant values), function N, specific heats c_p, c_{pA} and c_{pM}, this evidently complex system of equations (12) contains the terms which, by means of the velocity of the atomic mass formation $\dot{W}_A$, characterize the thermochemical reactions of general case of the ideally dissociated gas flow. However, the momentum equation in (12) (like other equations of this system except the above cited terms enables to introduce the set of Lojcjanskij parameters [2] in the system (12) in order to get it universalized. It is, therefore, quite logical at the first step toward the simplification of the previous system to eliminate these members, i.e. to suppose that

$$(13)\qquad \dot{W}_A=0.$$

In this way we have to consider the "frozen flow" [2] because the condition (13) just defines this boundary case of the ideally dissociated gas flow.

3. Distribution forms of physical values; introduction of usual assumptions

a) It should be noted that up to now the energy equation of the system (12) has not be reduced to the dimensionless form while it has been already done for momentum and diffusion equations. As the dimensionality is the consequence of the factor c_p which exists in this equation, that is, of the difference $c_{pA}-c_{pM}$, it is reduced to the dimensionless form after dividing it by c_{p1} (c_{p1} — specific heat at the front stagnation point), According to [2], the ratios of these heats are expressed as follows:

$$\frac{c_p}{c_{p1}}=\frac{C^*}{C_1^*}, \qquad \frac{c_{pA}-c_{pM}}{c_{p1}}=\frac{D^*}{C_1^*}, \tag{14}$$

where the functions C^*, C_1^* and D^* are determined by the expressions

$$C^*=\frac{10}{7}\alpha+(1-\alpha)\left[1+\frac{2}{7}e^{-(\overline{T}_V/\overline{T})^2}\right],$$

$$C_1^*=\frac{10}{7}\alpha_1+(1-\alpha_1)\left[1+\frac{2}{7}e^{-\overline{T}_V^2}\right]; \tag{15}$$

$$D^*=\frac{3}{7}\alpha-\frac{2}{7}e^{-(\overline{T}_V/\overline{T})^2}\Bigg], \qquad \overline{T}_V=T_V/T_1,$$

in which $T_V=800\,\mathrm{K}$ is the characteristic air temperature.

b) On the other hand, after dividing the energy equation by c_{p1} the parameter $u_e^2/c_{p1}T_1$, i.e. the dimensionless parameter $u_e^2/2c_{p1}T_1$ appears which, being only the function of the longitudinal coordinate s, is directly related with the external temperature T_e of the boundary layer. Assuming that the specific heat c_{pe} at the outer edge of the boundary layer is constant and equal to c_{p1} [2], the equation of the total temperature T_1 can be written as follows:

$$\frac{T_e}{T_1}+\frac{u_e^2}{2c_{p1}T_1}=1.$$

from where

$$\overline{T}_e=1-f_0, \tag{16}$$

where

$$f_0=\frac{u_e^2}{2c_{p1}T_1}, \tag{17}$$

the so-called local compressibility parameter [3]. On the basis of the previous relations the conclusion can be drawn that the range of the possible variation of this parameter is determined by the interval

$$0\leqslant f_0\leqslant 1. \tag{18}$$

c) The nature of the problem considered requires [2] the conditions for the existence of the "frozen" flow to be satisfied along the outer edge of the boundary layer, too. This leads to $\alpha_e=\text{const}=\alpha_1$, that is, with the condition of temperature constancy T_w at the body the density ratio at the outer edge and in the boundary layer for the case of the ideal catalytic wall ($\alpha_w = 0$) is given with the expression

$$\frac{\rho_e}{\rho} = \frac{1+\alpha}{1+\alpha_1} \frac{\overline{T}}{1-f_0}. \tag{19}$$

The nondimensional function N in the system (12) which has been introduced by the relation (6) gets the following form according to [4]:

$$N = (1+\alpha)^{-1,5} \frac{\overline{T}_w}{\overline{T}} \Pi(\overline{T}), \tag{20}$$

where the function $\Pi(\overline{T})$ is determined as:

$$\Pi(\overline{T}) = \left(\frac{\overline{T}}{1-f_0} \frac{T_1}{300}\right)^{1,5} \frac{413}{\dfrac{\overline{T} T_1}{1-f_0} + 113} + \tag{21}$$
$$+ 3,7 \left(\frac{\overline{T}}{1-f_0} \frac{T_1}{10\,000}\right)^2 - 2,35 \left(\frac{\overline{T}}{1-f_0} \frac{T_1}{10\,000}\right)^4.$$

Taking into account the distributions of physical values determined by the expressions (14), (15), (16), (17) and (19) and the condition (13), the system of equations (12) is reduced to the following nondimensional form:

$$\frac{\partial}{\partial \eta}\left(N \frac{\partial^2 \Phi}{\partial \eta^2}\right) + \frac{F+2f}{2B_L^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \frac{f}{B_L^2}\left[\frac{1+\alpha}{1+\alpha_1} \frac{\overline{T}}{1-f_0} - \left(\frac{\partial \Phi}{\partial \eta}\right)^2\right] =$$
$$= \frac{u_e f}{u_e' B_L^2}\left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial s \partial \eta} - \frac{\partial \Phi}{\partial s} \frac{\partial^2 \Phi}{\partial \eta^2}\right),$$

$$\frac{\partial}{\partial \eta}\left(\frac{N}{S_m} \frac{\partial \alpha}{\partial \eta}\right) + \frac{F+2f}{2B_L^2} \Phi \frac{\partial \alpha}{\partial \eta} = \frac{u_e f}{u_e' B_L^2}\left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \alpha}{\partial s} - \frac{\partial \Phi}{\partial s} \frac{\partial \alpha}{\partial \eta}\right),$$

$$\frac{\partial}{\partial \eta}\left(\frac{NC^*}{P_r C_1^*} \frac{\partial \overline{T}}{\partial \eta}\right) + \frac{F+2f}{2B_L^2} \frac{C^*}{C_1^*} \Phi \frac{\partial \overline{T}}{\partial \eta} - \frac{2 f_0 \overline{T}}{1-f_0} \frac{1+\alpha}{1+\alpha_1} \frac{f}{B_L^2} \frac{\partial \Phi}{\partial \eta} +$$
$$+ 2 f_0 N \left(\frac{\partial^2 \Phi}{\partial \eta^2}\right)^2 + \frac{ND^*}{S_m C_1^*} \frac{\partial \alpha}{\partial \eta} \frac{\partial \overline{T}}{\partial \eta} = \tag{22}$$
$$= \frac{u_e f}{u_e' B_L^2} \frac{C^*}{C_1^*}\left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \overline{T}}{\partial s} - \frac{\partial \Phi}{\partial s} \frac{\partial \overline{T}}{\partial \eta}\right);$$

$$\Phi = \frac{\partial \Phi}{\partial \eta} = 0, \qquad \alpha = \alpha_w = 0, \qquad \overline{T} = \overline{T}_w = \text{const} \quad \text{at} \quad \eta = 0,$$

$$\frac{\partial \Phi}{\partial \eta} \to 1, \qquad \alpha \to \alpha_1, \qquad \overline{T} \to 1 - f_0 \quad \text{at} \quad \eta \to \infty,$$

$$\Phi = \Phi_0(\eta), \qquad \alpha = \alpha_0(\eta), \qquad \overline{T} = \overline{T}_0(\eta) \quad \text{at} \quad s = s_0.$$

4. Universalization of equations of the problem considered

With regard to the fact that the obtained system of equations (22) explicitly contains the velocity at the outer edge of the boundary layer $u_e(s)$, the appropriate solution will depend on the concrete form of the given function $u_e(s)$, i.e. on each particular case of the problem considered.

According to Lojcjanskij [1], i.e. according to Krivcova [3] the system of equations (22) can be made to be independent of the outer velocity distribution by means of the appropriate set of infinitely large number of mutually independent parameters of the following form:

$$f_0=\frac{u_e^2}{2\,c_{p1}\,T_1},\qquad f_k=u_e^{k-1}\,\frac{d^k u_e}{ds^k}\,Z^{**k}\qquad (k=1,\,2,\,\dots,\,\infty),\tag{23}$$

where f_0 represents the above cited compressibility parameter and the first parameter of the set f_k coincides with the parameter f (10). It should be noted that the parameters $f_k(s)$ satisfy [3] the following recurrent ordinary differential equations

$$\begin{aligned}&\frac{u_e}{u_e'}\,f_1 f_k'=[(k-1)f_1+k\,F]f_k+f_{k+1}=\theta_k,\\ &\frac{u_e}{u_e'}\,f_1 f_0'=2f_0 f_1=\theta_0,\end{aligned}\tag{24}$$

($'$ represents the derivative to s), on the basis of which and by means of the operator:

$$\frac{\partial}{\partial s}=\sum_{k=0}^{\infty}\frac{\partial}{\partial f_k}\,\frac{df_k}{ds}=\frac{u_e'}{u_e f_1}\sum_{k=0}^{\infty}\theta_k\,\frac{\partial}{\partial f_k},\tag{25}$$

the differentiation along the longitudinal coordinate is substituted.

In this way the system (22) is reduced to the following form:

$$\begin{aligned}&\frac{\partial}{\partial\eta}\left(N\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{F+2f_1}{2B_L^2}\,\Phi\,\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f_1}{B_L^2}\left[\frac{1+\alpha}{1+\alpha_1}\,\frac{\overline{T}}{1-f_0}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]=\\ &=\frac{1}{B_L^2}\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial f_k\,\partial\eta}-\frac{\partial\Phi}{\partial f_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right),\\ &\frac{\partial}{\partial\eta}\left(\frac{N}{S_m}\,\frac{\partial\alpha}{\partial\eta}\right)+\frac{F+2f_1}{2B_L^2}\,\Phi\,\frac{\partial\alpha}{\partial\eta}=\frac{1}{B_L^2}\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\alpha}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\,\frac{\partial\alpha}{\partial\eta}\right),\\ &\frac{\partial}{\partial\eta}\left(\frac{NC^*}{P_r C_1^*}\,\frac{\partial\overline{T}}{\partial\eta}\right)+\frac{F+2f_1}{2B_L^2}\,\frac{C^*}{C_1^*}\,\Phi\,\frac{\partial\overline{T}}{\partial\eta}-\frac{2f_0\overline{T}}{1-f_0}\,\frac{1+\alpha}{1+\alpha_1}\,\frac{f_1}{B_L^2}\,\frac{\partial\Phi}{\partial\eta}+2f_0 N\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+\\ &+\frac{ND^*}{S_m C_1^*}\,\frac{\partial\alpha}{\partial\eta}\,\frac{\partial\overline{T}}{\partial\eta}=\frac{1}{B_L^2}\,\frac{C^*}{C_1^*}\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\overline{T}}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\,\frac{\partial\overline{T}}{\partial\eta}\right);\end{aligned}\tag{26}$$

$$\Phi = \frac{\partial \Phi}{\partial \eta} = 0, \qquad \alpha = 0, \qquad \overline{T} = \overline{T}_w = \text{const} \quad \text{at} \quad \eta = 0,$$

$$\frac{\partial \Phi}{\partial \eta} \to 1, \qquad \alpha \to \alpha_1, \qquad \overline{T} \to 1 - f_0 \qquad \text{at} \quad \eta \to \infty,$$

$$\Phi = \Phi_0(\eta), \qquad \alpha = \alpha_0(\eta), \qquad \overline{T} = \overline{T}_0(\eta) \ \text{at} \ f_0 = \text{const}, \ f_1 = f_2 = \cdots = 0,$$

in which, as it is evident, the velocity at the outer edge of the boundary layer $u_e(s)$ does not exist and, therefore it becomes universal according to Lojcjanskij's definition [1].

In the result of the integration of the system (26) the universal solutions would be obtained on the basis of which the solution of any concrete case defined by the determined velocity distribution $u_e(s)$ at the outer edge of the boundary layer is reduced to the additional integration of the impulse equation (9). This, however, reduces the practical applicability of this method.

This lack could be eliminated, that is, the improvement of the previous parametric method of Lojcjanskij could be achieved in the way V. N. Saljnikov [5] did for the case of incompressible fluid flow.

5. Generalization of derived universal equations

Namely, according to Saljnikov the following purposeful transformations are applied to the system of equations (5):

$$\xi = \xi(s), \qquad \eta(s, z) = h(s) \frac{u_e^{b/2} z}{\nu_n \sqrt{2\xi}}, \tag{27}$$

where the exponent b is for the moment an arbitrary real number and $\xi(s)$ and $h(s)$ some arbitrary continuous function of the variable s.

By introducing the nondimensional stream function Φ and nondimensional temperature $\overline{T}$ through the following expressions:

$$\psi(s, z) = \frac{\nu_n}{h(s)} u_e^{1-\frac{b}{2}} \sqrt{2\xi}\, \Phi(\xi, \eta), \qquad \overline{T} = \frac{T}{T_1}, \tag{28}$$

and by using the relations (14), under the condition (13), the system of equations (5) is firstly transformed to this form:

$$\frac{\partial}{\partial \eta}\left(N \frac{\partial^2 \Phi}{\partial \eta^2}\right) + \frac{\sqrt{2\xi}}{h u_e^{b/2}} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} \frac{d}{ds}\left(\frac{\nu_n}{h} \sqrt{2\xi}\, u_e^{1-\frac{b}{2}}\right) + \frac{2\nu_n \xi}{h^2 u_e^b} \frac{du_e}{ds}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial \Phi}{\partial \eta}\right)^2\right] =$$

$$= \frac{2\nu_n \xi}{h^2 u_e^{b-1}} \frac{d\xi}{ds}\left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \xi \partial \eta} - \frac{\partial \Phi}{\partial \xi} \frac{\partial^2 \Phi}{\partial \eta^2}\right),$$

$$\frac{\partial}{\partial \eta}\left(\frac{N}{S_m}\frac{\partial \alpha}{\partial \eta}\right)+\frac{\sqrt{2\xi}}{hu_e^{b/2}}\Phi\frac{\partial \alpha}{\partial \eta}\frac{d}{ds}\left(\frac{\nu_n}{h}\sqrt{2\xi}u_e^{1-\frac{b}{2}}\right)=\frac{2\nu_n\xi}{h^2u_e^{b-1}}\frac{d\xi}{ds}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial\xi}-\frac{\partial\Phi}{\partial\xi}\frac{\partial\alpha}{\partial\eta}\right),$$

(29)

$$\frac{\partial}{\partial \eta}\left(\frac{NC^*}{P_r C_1^*}\frac{\partial \overline{T}}{\partial \eta}\right)+\frac{\sqrt{2\xi}}{hu_e^{b/2}}\frac{C^*}{C_1^*}\Phi\frac{\partial \overline{T}}{\partial \eta}\frac{d}{ds}\left(\frac{\nu_n}{h}\sqrt{2\xi}u_e^{1-\frac{b}{2}}\right)-\frac{\rho_e}{\rho}\frac{u_e^2}{c_{p1}T_1}\frac{2\nu_n\xi}{h^2u_e^b}\frac{du_e}{ds}\frac{\partial\Phi}{\partial\eta}+$$

$$+N\frac{u_e^2}{c_{p1}T_1}\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+\frac{ND^*}{S_m C_1^*}\frac{\partial\alpha}{\partial\eta}\frac{\partial\overline{T}}{\partial\eta}=\frac{2\nu_n\xi}{h^2u_e^{b-1}}\frac{C^*}{C_1^*}\frac{d\xi}{ds}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{T}}{\partial\xi}-\frac{\partial\Phi}{\partial\xi}\frac{\partial\overline{T}}{\partial\eta}\right);$$

with the boundary conditions:

(30)
$$\Phi=\frac{\partial\Phi}{\partial\eta}=0,\qquad \alpha=\alpha_w,\qquad \overline{T}=\overline{T}_w\quad \text{at}\quad \eta=0,$$
$$\frac{\partial\Phi}{\partial\eta}\to 1,\qquad \alpha\to\alpha_e\qquad \overline{T}\to\overline{T}_e\quad\text{at}\quad \eta\to\infty,$$
$$\Phi=\Phi_0(\eta),\qquad \alpha=\alpha_0(\eta)\qquad \overline{T}=\overline{T}_0(\eta)\ \text{at}\quad \xi=\xi_0(s_0).$$

By means of the new-introduced variables (27), the momentum thickness (8) is reduced to the following form:

(31)
$$\Delta^{**}(s)=\frac{\nu_n\sqrt{2\xi}}{hu_e^{b/2}}\int_0^\infty\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)d\eta=\frac{\nu_n\sqrt{2\xi}}{hu_e^{b/2}}\cdot B,$$

where the continuous function B is determined by the integral

(32)
$$B=\int_0^\infty\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)d\eta=B[\xi(s)]=B(s).$$

After introducing (31), the parameter f defined by the relation (10) is

(33)
$$f=\frac{2\nu_n\xi B^2}{h^2u_e^2}\frac{du_e}{ds}.$$

On the basis of (27) and (31) it can be concluded that this ratio is satisfied

(34)
$$\frac{\nu_n\sqrt{2\xi}}{h^2u_e^{b/2}}=\frac{\Delta^{**}(s)}{B(s)}=\frac{z}{\eta},$$

by means of which the displacement thickness is expressed as follows

(35)
$$\Delta^*(s)=\frac{\Delta^{**}(s)}{B(s)}\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{\partial\Phi}{\partial\eta}\right)d\eta.$$

The thickness ratio is now:

(36) $$H=\frac{\Delta^*}{\Delta^{**}}=\frac{1}{B(s)}\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{\partial\Phi}{\partial\eta}\right)d\eta=\frac{A(s)}{B(s)},$$

where

(37) $$A(s)=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{\partial\Phi}{\partial\eta}\right)d\eta.$$

All three forms of the impulse equation (9) remain unchanged and with regard to the new — introduced variables and ratio of characteristic thicknesses of the boundary layer, the function F existing in the impulse equation is transformed in the following form

(38) $$F=2\left[\zeta-\left(2+\frac{A}{B}\right)f\right],$$

where

(39) $$\zeta=\left[\frac{\partial(\partial\Phi/\partial n)}{\partial(\eta/B)}\right]_{\eta=0}=B\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)_{\eta=0}.$$

By using the impulse equation (9) and the expression (33), the system of equations (29) is reduced to the more suitable form:

$$\frac{\partial}{\partial\eta}\left(N\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{F+2f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]=$$
$$=\frac{u_e f}{u_e' B^2}\frac{d\xi}{ds}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial\xi}-\frac{\partial\Phi}{\partial\xi}\frac{\partial^2\Phi}{\partial\eta^2}+\frac{1}{B}\frac{dB}{d\xi}\Phi\frac{\partial^2\Phi}{\partial\eta^2}\right),$$

$$\frac{\partial}{\partial\eta}\left(\frac{N}{S_m}\frac{\partial\alpha}{\partial\eta}\right)+\frac{F+2f}{2B^2}\Phi\frac{\partial\alpha}{\partial\eta}=\frac{u_e f}{u_e' B^2}\frac{d\xi}{ds}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial\xi}-\frac{\partial\Phi}{\partial\xi}\frac{\partial\alpha}{\partial\eta}+\frac{1}{B}\frac{dB}{d\xi}\Phi\frac{\partial\alpha}{\partial\eta}\right),$$

(40)

$$\frac{\partial}{\partial\eta}\left(\frac{NC^*}{P_r C_1^*}\frac{\partial\overline{T}}{\partial\eta}\right)+\frac{F+2f}{2B^2}\frac{C^*}{C_1^*}\Phi\frac{\partial\overline{T}}{\partial\eta}-\frac{\rho_e}{\rho}\frac{u_e^2}{c_{p1}T_1}\frac{f}{B^2}\frac{\partial\Phi}{\partial\eta}+N\frac{u_e^2}{c_{p1}T_1}\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+$$
$$+\frac{ND^*}{S_m C_1^*}\frac{\partial\alpha}{\partial\eta}\frac{\partial\overline{T}}{\partial\eta}=\frac{u_e f}{u_e' B^2}\frac{C^*}{C_1^*}\frac{d\xi}{ds}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{T}}{\partial\xi}-\frac{\partial\Phi}{\partial\xi}\frac{\partial\overline{T}}{\partial\eta}+\frac{1}{B}\frac{dB}{d\xi}\Phi\frac{\partial\overline{T}}{\partial\eta}\right),$$

where the boundary conditions (30) remain unchanged.

It should be noted that the transformation of the longitudinal variable

(41) $$\xi(s)=s,$$

enables to introduce Lojcjanskij's infinite set of parameters (23). Namely, by adopting the set of parameters (23) for the new independent variables and taking into account the distribution of physical values, the system (40) is reduced to the following universal form:

$$\frac{\partial}{\partial \eta}\left(N\frac{\partial^2 \Phi}{\partial \eta^2}\right)+\frac{F+2f_1}{2B^2}\Phi\frac{\partial^2 \Phi}{\partial \eta^2}+\frac{f_1}{B^2}\left[\frac{1+\alpha}{1+\alpha_1}\frac{\overline{T}}{1-f_0}-\left(\frac{\partial \Phi}{\partial \eta}\right)^2\right]=$$

$$=\frac{1}{B^2}\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial^2 \Phi}{\partial \eta \partial f_k}-\frac{\partial \Phi}{\partial f_k}\frac{\partial^2 \Phi}{\partial \eta^2}+\frac{1}{B}\frac{\partial B}{\partial f_k}\Phi\frac{\partial^2 \Phi}{\partial \eta^2}\right),$$

$$\frac{\partial}{\partial \eta}\left(\frac{N}{S_m}\frac{\partial \alpha}{\partial \eta}\right)+\frac{F+2f_1}{2B^2}\Phi\frac{\partial \Phi}{\partial \eta}=\frac{1}{B^2}\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial \alpha}{\partial f_k}-\frac{\partial \Phi}{\partial f_k}\frac{\partial \alpha}{\partial \eta}+\frac{1}{B}\frac{\partial B}{\partial f_k}\Phi\frac{\partial \alpha}{\partial \eta}\right),$$

(42)

$$\frac{\partial}{\partial \eta}\left(\frac{NC^*}{P_r C_1^*}\frac{\partial \overline{T}}{\partial \eta}\right)+\frac{F+2f_1}{2B^2}\frac{C^*}{C_1^*}\Phi\frac{\partial \overline{T}}{\partial \eta}-\frac{2f_0\overline{T}}{1-f_0}\frac{1+\alpha}{1+\alpha_1}\frac{f_1}{B^2}\frac{\partial \Phi}{\partial \eta}+2f_0 N\left(\frac{\partial^2 \Phi}{\partial \eta^2}\right)^2+$$

$$+\frac{ND^*}{S_m C_1^*}\frac{\partial \alpha}{\partial \eta}\frac{\partial \overline{T}}{\partial \eta}=\frac{1}{B^2}\frac{C^*}{C_1^*}\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial \overline{T}}{\partial f_k}-\frac{\partial \Phi}{\partial f_k}\frac{\partial \overline{T}}{\partial \eta}+\frac{1}{B}\frac{\partial B}{\partial f_k}\Phi\frac{\partial \overline{T}}{\partial \eta}\right);$$

$$\Phi=\frac{\partial \Phi}{\partial \eta}=0, \qquad \alpha=0, \qquad \overline{T}=\overline{T}_w=\text{const} \quad \text{at} \quad \eta=0,$$

$$\frac{\partial \Phi}{\partial \eta}\to 1, \qquad \alpha\to\alpha_1, \qquad \overline{T}\to 1-f_0, \qquad \text{at} \quad \eta\to\infty,$$

$$\Phi=\Phi_0(\eta), \qquad \alpha=\alpha_0(\eta), \qquad \overline{T}=\overline{T}_0(\eta) \ \text{at} \ f_0=\text{const}, \ f_1=f_2=\cdots=0.$$

The system of equations (42) derived for the first time in this paper represents the generalization of the system (26). This system, however, for the particular case defined by

$$B(s)=\text{const}=B_L,$$

is reduced to the system of universal equations (26). It can be, therefore, called the system of generalized universal equations of laminar boundary layer for the case of "frozen" flow of the ideally dissociated gas.

With regard to the fact that the obtained system (42) does not explicitly contain the function $h(s)$ it could be concluded at the first sight that its solution is possible although the function $h(s)$ is not defined. As multivalued solutions could be expected in that case in accordance with the analysis in [5], it is necessary to firstly determine the form of the function $h(s)$.

6. Determination of the function form $h(s)$

It is advisable to choose the form of the function $h(s)$ for this case of flow in the way Saljnikov [5] did for the case of incompressible fluid. According to this methodology the form of the function saught is determined by means of the generalized coordinate of similar solutions, i.e. generalized Görtler's transversal variable [2]

$$\eta(s, z) = \frac{u_e^{b/2} z}{\sqrt{a \nu_n \int_0^s u_e^{b-1} ds}}. \tag{43}$$

Namely, by comparing (43) with the transformation of the transversal variable (27), the function $h(s)$ is obtained in the following form

$$h(s) = \sqrt{\frac{2 \nu_n \xi(s)}{a \int_0^s u_e^{b-1} ds}}. \tag{44}$$

In this way the stream function is reduced to the form

$$\psi(s, z) = u_e^{1-\frac{b}{2}} \sqrt{a \nu_n \int_0^s u_e^{b-1} ds}\, \Phi(\xi, \eta) \tag{45}$$

Taking into account (34) and (41), the initial transformations will be:

$$s = s, \; \eta(s, z) = \frac{B(s)}{\Delta^{**}(s)} z, \; \psi = \frac{u_e(s) \Delta^{**}(s)}{B(s)} \Phi(s, \eta), \tag{46}$$

which are more general than the corresponding values (7). For the special case $B(s) = \text{const} = B_L$, the variables (46) are reduced to the variables (7) representing in that way their generalization.

By substituting (44) in (33), the final expression is obtained for the parameter

$$f(s) = B^2 \frac{a u_e'}{u_e^b} \int_0^s u_e^{b-1} ds. \tag{47}$$

From (47), by differentiating this expression to s and by comparing with the corresponding form of impulse equation (9), the following expression is obtained for the characteristic function F

$$F(s) = aB^2 - bf + \frac{2 u_e B'}{B u_e'} f. \tag{48}$$

by means of which the system (42) is reduced to the final form:

$$\frac{\partial}{\partial \eta}\left(N \frac{\partial^2 \Phi}{\partial \eta^2}\right) + \frac{aB^2 + (2-b) f_1}{2B^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \frac{f_1}{B^2}\left[\frac{1+\alpha}{1+\alpha_1} \frac{\overline{T}}{1-f_0} - \left(\frac{\partial \Phi}{\partial \eta}\right)^2\right] = \frac{1}{B^2} \sum_{k=0}^{\infty} \theta_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial^2 \Phi}{\partial \eta^2}\right);$$

$$\frac{\partial}{\partial \eta}\left(\frac{N}{S_m} \frac{\partial \alpha}{\partial \eta}\right) + \frac{aB^2 + (2-b) f_1}{2B^2} \Phi \frac{\partial \alpha}{\partial \eta} = \frac{1}{B^2} \sum_{k=0}^{\infty} \theta_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \alpha}{\partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial \alpha}{\partial \eta}\right); \tag{49}$$

$$\frac{\partial}{\partial \eta}\left(\frac{NC^*}{P_r C_1^*}\frac{\partial \overline{T}}{\partial \eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\frac{C^*}{C_1^*}\Phi\frac{\partial \overline{T}}{\partial \eta}-\frac{2f_0\overline{T}}{1-f_0}\frac{1+\alpha}{1+\alpha_1}\frac{f_1}{B^2}\frac{\partial \Phi}{\partial \eta}+2f_0N\left(\frac{\partial^2\Phi}{\partial \eta^2}\right)^2+$$

$$+\frac{ND^*}{S_m C_1^*}\frac{\partial \alpha}{\partial \eta}\frac{\partial \overline{T}}{\partial \eta}=\frac{1}{B^2}\frac{C^*}{C_1^*}\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial \overline{T}}{\partial f_k}-\frac{\partial \Phi}{\partial f_k}\frac{\partial \overline{T}}{\partial \eta}\right),$$

with the respective boundary conditions:

$$\Phi=\frac{\partial \Phi}{\partial \eta}=0,\qquad \alpha=0,\qquad \overline{T}=\overline{T}_w=\text{const}\quad \text{at}\quad \eta=0,$$

$$\frac{\partial \Phi}{\partial \eta}\to 1,\qquad \alpha\to\alpha_1,\qquad \overline{T}\to 1-f_0\quad \text{at}\quad \eta\to\infty,$$

$$\Phi=\Phi_0(\eta),\qquad \alpha=\alpha_0(\eta)\qquad \overline{T}=\overline{T}_0(\eta)\ \text{at}\ f_0=\text{const},\ f_1=f_2=\cdots=0.$$

7. Parametric solutions of the system of generalized equations of the problem considered

With regard to the complexity of the system (49) and its dependence on the infinite number of mutually independent parameters like the set f_k, the numerical calculation is carried out for the case of the twoparametric once localized approximation, that is, under the following conditions: $f_0\neq 0$, $f_1\neq 0$, $f_2=f_3=\cdots=0$, $\partial/\partial f_0=0$, $\theta_1=F\cdot f_1$.

In that case (49) is reduced to the following system of equations:

$$\frac{\partial}{\partial \eta}\left(N\frac{\partial^2\Phi}{\partial \eta^2}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial \eta^2}+\frac{f_1}{B^2}\left[\frac{1+\alpha}{1+\alpha_1}\frac{\overline{T}}{1-f_0}-\left(\frac{\partial \Phi}{\partial \eta}\right)^2\right]=$$

$$=\frac{Ff_1}{B^2}\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial^2\Phi}{\partial \eta\,\partial f_1}-\frac{\partial \Phi}{\partial f_1}\frac{\partial^2\Phi}{\partial \eta^2}\right),$$

$$\frac{\partial}{\partial \eta}\left(\frac{N}{S_m}\frac{\partial \alpha}{\partial \eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial \alpha}{\partial \eta}=\frac{Ff_1}{B^2}\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial \alpha}{\partial f_1}-\frac{\partial \Phi}{\partial f_1}\frac{\partial \alpha}{\partial \eta}\right),$$

(50)

$$\frac{\partial}{\partial \eta}\left(\frac{NC^*}{P_r C_1^*}\frac{\partial \overline{T}}{\partial \eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\frac{C^*}{C_1^*}\Phi\frac{\partial \overline{T}}{\partial \eta}-\frac{2f_0\overline{T}}{1-f_0}\frac{1+\alpha}{1+\alpha_1}\frac{f_1}{B^2}\frac{\partial \Phi}{\partial \eta}+2f_0N\left(\frac{\partial^2\Phi}{\partial \eta^2}\right)^2+$$

$$+\frac{ND^*}{S_m C_1^*}\frac{\partial \alpha}{\partial \eta}\frac{\partial \overline{T}}{\partial \eta}=\frac{Ff_1}{B^2}\frac{C^*}{C_1^*}\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial \overline{T}}{\partial f_1}-\frac{\partial \Phi}{\partial f_1}\frac{\partial \overline{T}}{\partial \eta}\right);$$

$$\Phi=\frac{\partial \Phi}{\partial \eta}=0,\qquad \alpha=0,\qquad \overline{T}=\overline{T}_w=\text{const}\quad \text{at}\quad \eta=0,$$

$$\frac{\partial \Phi}{\partial \eta}\to 1,\qquad \alpha\to\alpha_1,\qquad \overline{T}\to 1-f_0\quad \text{at}\quad \eta\to\infty,$$

$$\Phi=\Phi_0(\eta),\qquad \alpha=\alpha_0(\eta),\qquad \overline{T}=\overline{T}_0(\eta)\quad \text{at}\quad f_0=\text{const},\ f_1=0.$$

It should be noted that at this approximation the function F, required for the numerical integration of the system (50) is reduced to:

$$F = \frac{aB^2 - bf_1}{1 - \frac{2}{B} f_1 \frac{dB}{df_1}}, \tag{51}$$

where the function B is determined by (32).

The numerical solution of the simultaneous system of partial differential equations (50) is accomplished according to the methodology presented in [2]. For this, Boričić's procedure [6] is applied which is based on the work of Simuni and Terentjev [7]. The calculation has been carried out in the Electronic Computer Center of Zavodi "Crvena zastava" in Kragujevac on the computer IBM 370/145. On the basis of the results of this calculation the diagrams of dimensionless velocity (fig. 1), temperature (fig. 2), concentration (fig. 3), and other characteristic values of the boundary layer (fig 4. and fig. 5) have been drawn.

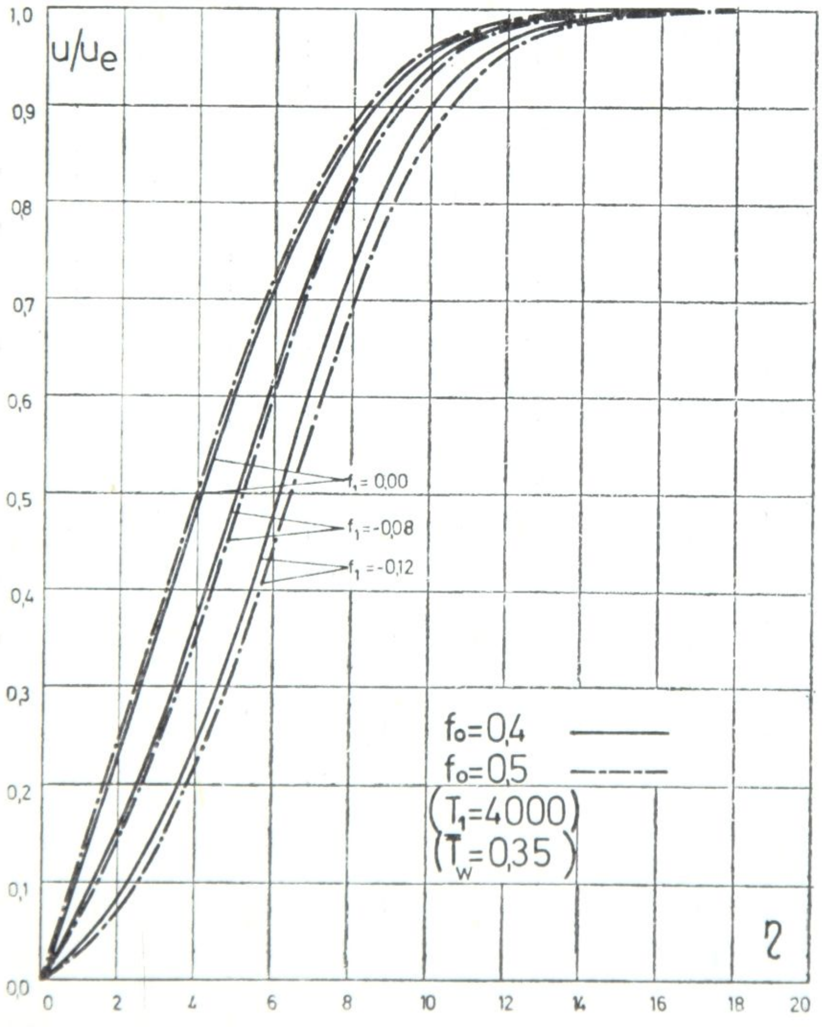

Fig. 1.

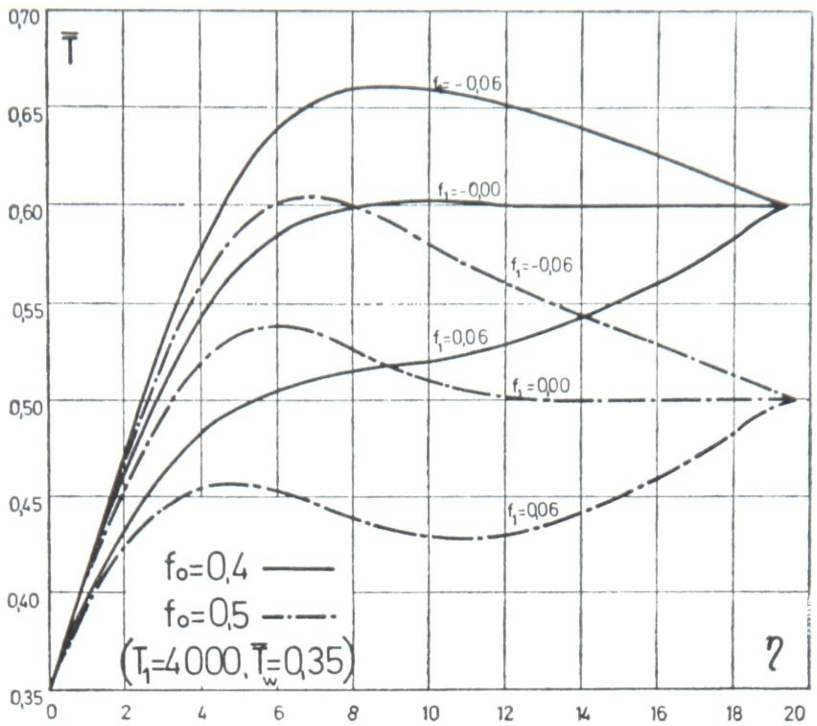
T̄
f₁ = -0,06
f₁ = -0,00
f₁ = -0,06
f₁ = 0,06
f₁ = 0,00
f₁ = 0,06
f₀=0,4
f₀=0,5
(T₁=4000, T̄w=0,35)
η
0,70
0,65
0,60
0,55
0,50
0,45
0,40
0,35
0 2 4 6 8 10 12 14 16 18 20

Fig. 2.

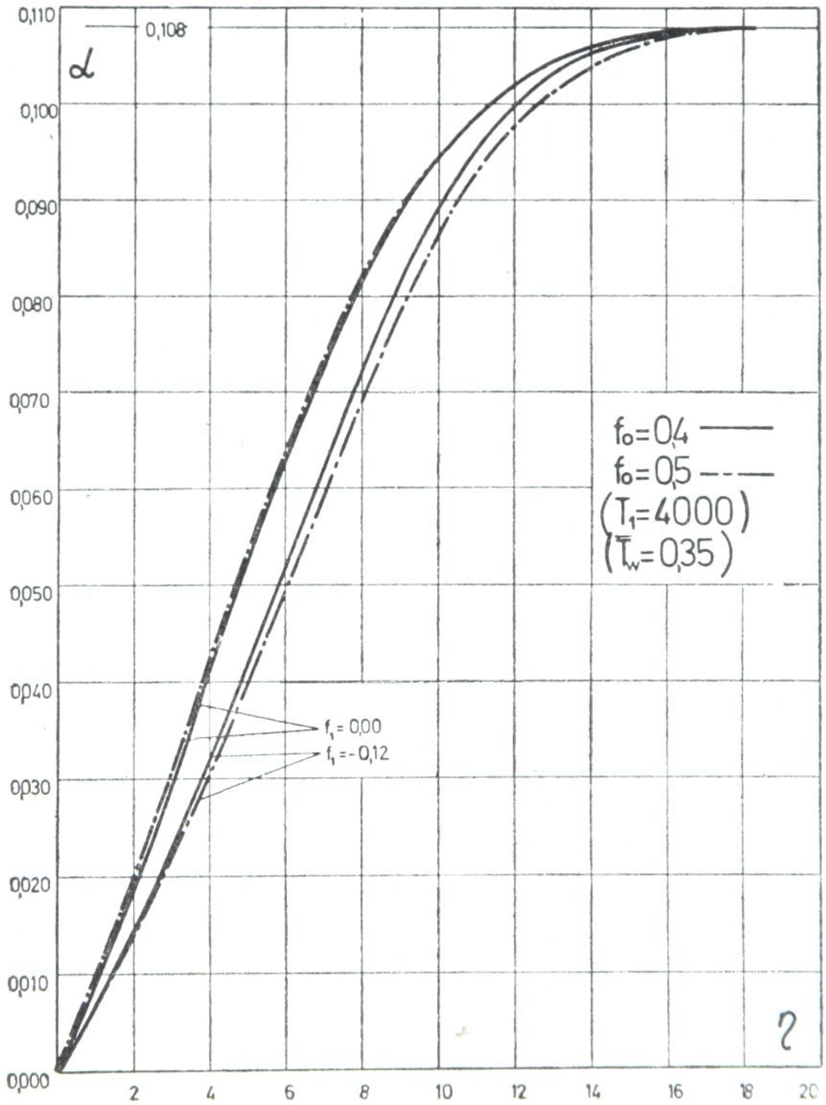
0,108
α
f₁ = 0,00
f₁ = -0,12
f₀=0,4
f₀=0,5
(T₁=4000)
(T̄w=0,35)
η
0,110
0,100
0,090
0,080
0,070
0,060
0,050
0,040
0,030
0,020
0,010
0,000
2 4 6 8 10 12 14 16 18 20

Fig. 3.

From the diagrams presented we come to the following conclusions:

a) Neither the dimensionless velocity u/u_e of the boundary layer (b. fig. 1), nor the concentration α of atoms (b. fig. 3) are considerably influenced by the compressibility parameter f_0.

b) The parameter f_0, however, essentially affects the dimensionless temperature $\overline{T}$ of the boundary layer, changing even the general character of the temperature behaviour (b. fig. 2).

c) While the influence of the compressibility parameter f_0 on the characteristic A of the boundary layer is considerable, it can not be concluded for the other characteristics of the layer.

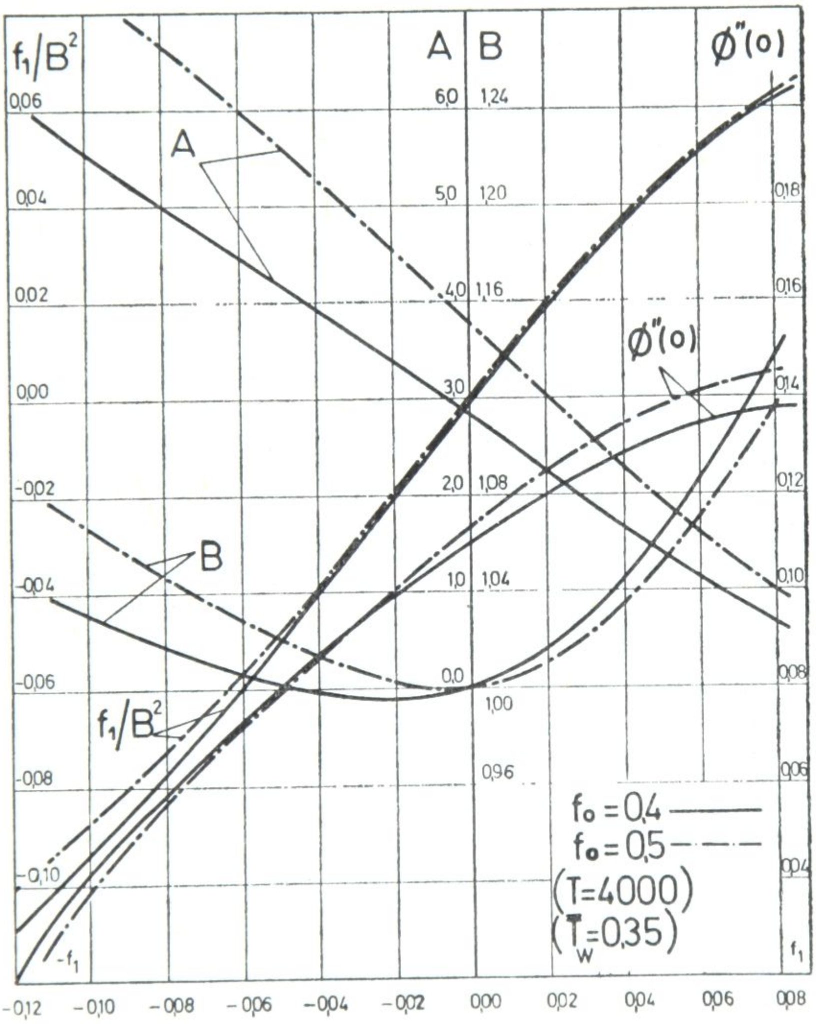

Fig. 4.

Therefore, in spite of the system (50) being simultaneous, various influences of the compressibility parameter on some equations of the system solved can be noted. With regard to the slight influence of the parameter f_0 on the dimensionless velocity and concentration of atoms in the boundary layer, the localization of momentum and diffusion equations of the boundary layer of the dissociated gas can be justified. As the influence of this parameter on the temperature distribution in the boundary layer is noticeable, the localization by this parameter is unjustifiable, that is, the energy equation is unjustifiable. This localization is due to mathematical difficulties [2]. It is, naturally, important

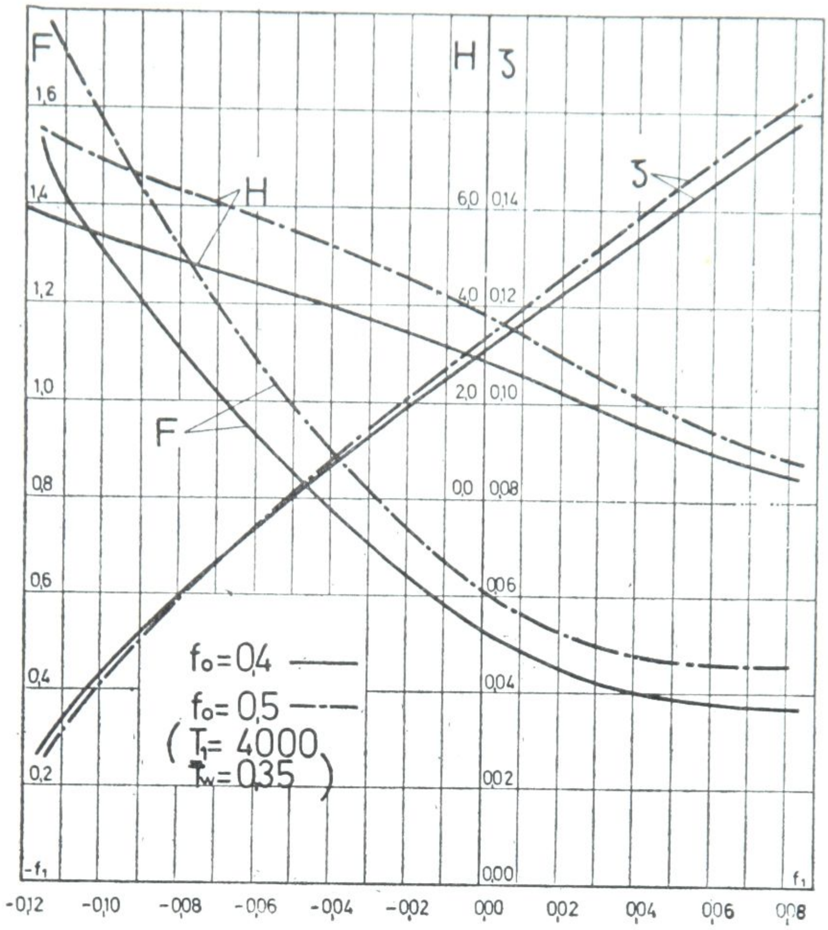

Fig. 5.

to solve the complete system of universal equations (49) in the twoparameter approximation — without the localization. In that way, the possibility would be realized to compare the solutions in order to determine the quantitative influence of the compressibility parameter f_0 on various values and characteristics of the boundary layer. This, however, could be the subject of further researches.

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УНИВЕРСАЛЬНЫ УРАВНЕНИЙ ЛАМИНАРНОГО ПОГРАНИЧНОГО СЛОЯ ДЛЯ СЛУЧАЯ „ЗАМОРОЖЕННОГО" ТЕЧЕНИЯ ИДЕАЛЬНОГО ДИССОЦИРОВАННОГО ГАЗА И ИХ ПАРАМЕТРИЧЄСКИЙ РЕШЕНИЙ

Бранко Р. Обрович

Р е з ю м е

В работе система уравнений ламинарного пограничного слоя для случая „замороженного" течения диссоцированного газа, с помощью целесообразной трансформации и соответствующих систем параметров, приведена в универсальную форму в смысле определєния Лойцянского.

Численное решение полученной системы унив.рсальных дифференциональных уравнений выполнено мєтодом конечных разностей.

На основании результатов вычислений, приведеных на приложенных диаграммах, сделаны заключения о воздействии параметра сжимаемости f_0 на характеристичные величины пограмичного слоя. На основании этого сделано заключение об оправданности локализации уравнений пограничного слоя по параметру сжимаемости f_0

УНИВЕРЗАЛНЕ ЈЕДНАЧИНЕ ЛАМИНАРНОГ ГРАНИЧНОГ СЛОЈА ЗА СЛУЧАЈ „ЗАМРЗНУТОГ" СТРУЈАЊА ИДЕАЛНО ДИСОЦИРАНОГ ГАСА И ЊИХОВА ПАРАМЕТАРСКА РЕШЕЊА

Бранко Р. Обровић

Р е з и м е

У раду је систем једначина ламинарног граничног слоја за случај „замрзнутог" струјања дисоцираног гаса доведен, уз помоћ сврсисходних трансформација и одговарајућег скупа параметара, на универзални облик у смислу дефиниције Лојцјанског.

Нумеричко решавање добијеног система универзалних диференцијалних једначина извршено је, према познатој методологији, методом коначних разлика. Резултати прорачуна су приказани на приложеним дијаграмима на основу којих су изведени закључци о утицају параметра стишљивости f_0 на карактеристичне величине граничног слоја. На основу тога је изведен и општи закључак о оправданости локализације једначина граничног слоја по параметру стишљивости f_0

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