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THE CONTRIBUTION TO THE SOLVING OF THE FUNDAMENTAL BOUNDARY-VALUE PROBLEMS OF BENDING MODERATELY THICK PLATES

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1. Introduction

It is well known [1] that it is possible to transform the boundary-value problems in two-dimensional isotropic elastostatics into the problem of seeking for two analytic functions $\varphi(z)$ and $\psi(z)$, which are holomorphic in every point of the domain D under discussion. If besides the boundary loading other loadings too in some points inside the domain D are given, then $\varphi(z)$ and $\psi(z)$ are not analytical any more. In this paper the bending of a moderately thick plate by an improved theory will be treated.

Let z_0 be a point inside the domain D, where a concentrated force F_z and a concentrated couple M_x and M_y are acting. In this case the behaviour of the plate around z_0 is given by the following functions [2]

(1—1) $$\varphi(z)=[(z-z_0)F+M]\ln(z-z_0)+\varphi_0(z)$$

(1—2) $$\psi(z)=(\overline{M}-\overline{z}_0 F)\ln(z-z_0)-M\overline{z}_0(z-z_0)^{-1}+\alpha\varphi''(z)-\psi_0(z)$$

where

(1—3) $$F=\frac{3}{2\pi h^2}F_z$$

(1—4) $$M=-\frac{3i}{2\pi h^2}(M_x+iM_y)$$

(1—5) $$\alpha=\frac{4h^2}{1-\nu}$$

(1—6) $$2h=\text{thickness of the plate}$$

(1—7) $$\nu=\text{Poisson's number}$$

$\varphi_0(z)$ and $\psi_0(z)$ are in z_0 holomorphic functions.

Further on

$$w=-\frac{1-\nu}{16\mu}\{[\overline{z}\varphi(z)+z\overline{\varphi(z)}+\chi(z)+\overline{\chi(z)}]-\alpha[\varphi'(z)+\overline{\varphi'(z)}]\}=$$

$$=-\frac{1-\nu}{16\mu}\{\ln[(z-z_0)\overline{(z-z_0)}]\cdot[(z-z_0)\overline{(z-z_0)}F+$$

$$+(z+z_0)\overline{M}+\overline{(z-z_0)}M]-(z-z_0)(\overline{M}-\bar{z}_0F)-$$

(1—8)

$$-\overline{(z-z_0)}(M-z_0)F+[\bar{z}\varphi_0(z)+z\overline{\varphi_0(z)}+\chi_0(z)+\overline{\chi_0(z)}\}$$

$$\frac{\partial w}{\partial \bar{z}}=-\frac{1-\nu}{16\mu}\Big\{z\cdot F+\ln[(z-z_0)(\bar{z}-\bar{z}_0)]\cdot[(z-z_0)F+M]+$$

(1—8′)

$$+\frac{z-z_0}{\bar{z}-\bar{z}_0}\overline{M}\Big\}-\frac{1-\nu}{16\mu}[\varphi_0(z)-z\overline{\varphi_0'(z)}+\overline{\psi_0(z)}]$$

where w means a bending of the middle plane of the plate and $\chi'(z)=\psi(z)$.

If in a given point $z_0\in D$ (or in several such points) M and F are prescribed and if on the boundary curve C of the domain D the plate is clamped, then the problem of the bending of the plate is uniquely solvable [3], [4].

Now let the force and couple act in every point $\zeta=\xi+i\eta$ of the domain D and let $M(\xi)$ and $F(\zeta)$ mean densities of the couple and force on the element $d\xi\,d\eta$ respectively.

It follows from (8) that:

$$w=-\frac{1-\nu}{16\mu}\iint_D\{\ln[(z-\zeta)(\bar{z}-\bar{\zeta})]\cdot[(z-\zeta)(\bar{z}-\bar{\zeta})F(\zeta)+$$

$$+(z-\zeta)\overline{M(\zeta)}+(\bar{z}-\bar{\zeta})M(\zeta)]-(z-\zeta)[\overline{M(\zeta)}-\bar{\zeta}F(\zeta)]-$$

$$-(\bar{z}-\bar{\zeta})[M(\zeta)-\zeta F(\zeta)]\}\,d\xi\,d\eta-\frac{1-\nu}{16\mu}\{\bar{z}\varphi_0(z)+z\overline{\varphi_0(z)}+$$

(1—9)

$$+\chi_0(z)+\overline{\chi_0(z)}\}$$

(1—9′)

$$\chi_0'(z)=\psi_0(z)$$

$$\frac{\partial w}{\partial \bar{z}}=-\frac{1-\nu}{16\mu}\iint_D\Big\{(zF(\zeta)+\ln[(z-\zeta)(\bar{z}-\bar{\zeta})]\cdot[(z-\zeta)F(\zeta)+$$

(1—10)

$$+M(\zeta)]+\frac{z-\zeta}{\bar{z}-\bar{\zeta}}\overline{M(\zeta)}\Big\}\,d\zeta\,d\eta-\frac{1-\nu}{16\mu}\{\varphi_0(z)+z\overline{\varphi_0'(z)}+\overline{\psi_0(z)}\}.$$

If we introduce

(1—11)
$$\tilde{\varphi}(z)=\iint_D[(z-\zeta)F(\zeta)+M(\zeta)]\ln(z-\zeta)\,d\xi\,d\eta$$

(1—11′)
$$\tilde{\varphi}_1(z)=\iint_D\left[F(\zeta)\ln(z-\zeta)+F(\zeta)+\frac{M(\zeta)}{z-\zeta}\right]d\xi\,d\eta$$

(1—11″)
$$\tilde{\psi}(z)=\iint_D\left\{[\overline{M(\zeta)}-\bar{\zeta}F(\zeta)]\ln(z-\zeta)-\frac{M(\zeta)\bar{\zeta}}{z-\zeta}\right\}d\xi\,d\eta$$

$$\bar{\chi}(z)=\iint_D \{[\underline{M(\zeta)}-\underline{\zeta}F(\zeta)][(z-\zeta)\ln(z-\zeta)-(z-\zeta)]- \tag{1—11'''}$$
$$-M(\zeta)\bar{\zeta}\ln(z-\zeta)\}\,d\xi\,d\eta$$

then the equations (1—9) and (1—10) can be expressed in a clear way

$$w=-\frac{1-\nu}{16\mu}\{[z\overline{\tilde{\varphi}(z)}+\bar{z}\tilde{\varphi}(z)+\tilde{\chi}(z)+\overline{\tilde{\chi}(z)}]+ \tag{1—12}$$
$$+[z\overline{\varphi_0(z)}+\bar{z}\varphi_0(z)+\chi_0(z)+\overline{\chi_0(z)}$$

$$\frac{\partial w}{\partial\bar{z}}=-\frac{1-\nu}{16\mu}\{[\tilde{\varphi}(z)+z\overline{\tilde{\varphi}_1(z)}+\overline{\tilde{\psi}(z)}]+[\varphi_0(z)+z\overline{\varphi_0'(z)}+\overline{\psi_0(z)}]\}. \tag{1—13}$$

The functions $\tilde{\varphi}(z)$, $\varphi_1(z)$, $\tilde{\psi}(z)$ and $\tilde{\chi}(z)$ in D are not holomorphic any more. But they are holomorphic outside the domain D, what will be of essential importance in solving of the discussed problem. Each boundary-value problem can be thus reduced to the establishment of functions $\varphi_0(z)$ and $\psi_0(z)$, which are analytic in D.

2. Bending of a round plate with a clamped boundary

Let the domain D be a circle with the centre in $z=0$ and radius $r=1$. For loading let the following equations

$$\iint_D F(\zeta)\,d\xi\,d\eta=0 \tag{2—1}$$

$$\iint_D [M(\zeta)-\zeta F(\zeta)]\,d\xi\,d\eta=0 \tag{2—2}$$

be fulfilled, which requires a separate fulfilment of balance conditions for the loading in the domain D and on the boundary curve C. Then $\tilde{\varphi}(z)$ becomes a holomorphic function outside D. For each $|z|>1$ holds

$$\Big[\tilde{\varphi}(z)\Big]_z^z=2\pi i\iint_D \{zF(\zeta)+[M(\zeta)-\zeta F(\zeta)]\}\,d\xi\,d\eta=0 \tag{2—3}$$

and also in the same way

$$\Big[\tilde{\psi}(z)\Big]_z^z=2\pi i\iint_D [\overline{M(\zeta)}-\bar{\zeta}F(\zeta)]\,d\xi\,d\eta=0 \tag{2—3'}$$

and

$$\tilde{\varphi}_1(z)=\tilde{\varphi}'(z) \tag{2—3''}$$

$$\tilde{\psi}(z)=\tilde{\chi}'(z),\qquad z\notin D. \tag{2—3'''}$$

For the simplicity of expression the following notations will be introduced:

$$(2\text{—}4) \qquad f_1(z) = \iint\limits_D F(\zeta) \ln\left(1 - \frac{\zeta}{z}\right) d\xi\, d\eta$$

$$(2\text{—}5) \qquad f_2(z) = \iint\limits_D [M(\zeta) - \zeta F(\zeta)] \ln\left(1 - \frac{\zeta}{z}\right) d\xi\, d\eta$$

$$(2\text{—}6) \qquad f_3(z) = \iint\limits_D [\overline{M(\zeta)} - \overline{\zeta} F(\zeta)] \ln\left(1 - \frac{\zeta}{z}\right) d\xi\, d\eta$$

$$(2\text{—}7) \qquad f_4(z) = \iint\limits_D \frac{\overline{\zeta} M(\zeta)}{z - \zeta} d\xi\, d\eta.$$

All these functions are holomorphic outside D. For large values of z the equations

$$(2\text{—}8) \qquad f_1(z) = -z^{-1} \iint\limits_D \zeta F(\zeta)\, d\xi\, d\eta + O(z^{-2})$$

$$(2\text{—}9) \qquad f_2(z) = -z^{-1} \iint\limits_D M(\zeta) - \zeta F(\zeta)]\, \zeta\, d\xi\, d\eta + O(z^{-2})$$

$$(2\text{—}10) \qquad f_3(z) = -z^{-1} \iint\limits_D [\overline{M(\zeta)} - \overline{\zeta} F(\zeta)]\, \zeta\, d\xi\, d\eta + O(z^{-2})$$

$$(2\text{—}11) \qquad f_4(z) = -z^{-1} \iint\limits_D \overline{\zeta} M(\zeta)\, d\xi\, d\eta = O(z^{-2})$$

$$(2\text{—}11'') \qquad f_4(z) - f_3(z) = z^{-1} \iint\limits_D [\overline{\zeta} M(\zeta) + \zeta \overline{M(\zeta)} - \zeta\overline{\zeta} F(\zeta)]\, d\xi\, d\eta + O(z^{-2})$$

and

$$(2\text{—}12) \qquad f_1'(z) = O(z^{-2})$$

$$(2\text{—}13) \qquad f_2'(z) = O(z^{-2})$$

hold.

Now if follows for $z \notin \overline{D}$ and $z_0 \in C$ (circle)

$$(2\text{—}14) \qquad \tilde{\varphi}(z) = z f_1(z) + f_2(z)$$

$$(2\text{—}14') \qquad \tilde{\varphi}^+(z_0) = z_0 f_{\overline{1}}(z_0) + f_{\overline{2}}(z_0)$$

$$(2\text{—}15) \qquad \tilde{\varphi}'(z) = f_1(z) + z f_1'(z) + f_2'(z)$$

$$(2\text{—}15') \qquad \overline{\tilde{\varphi}'}\left(\frac{1}{z}\right) = \overline{f_1}\left(\frac{1}{z}\right) + \frac{1}{z}\overline{f_1'}\left(\frac{1}{z}\right) + \overline{f_2'}\left(\frac{1}{z}\right)$$

$$\overline{\overline{\varphi}_1(z_0)^+} = \overline{f}_1^+\left(\frac{1}{z_0}\right) + \frac{1}{z_0}\overline{f}_1'^+\left(\frac{1}{z_0}\right) + \overline{f}_2'^+\left(\frac{1}{z_0}\right) \tag{2—15'}$$

$$\tilde{\psi}(z) = f_3(z) - f_4(z) \tag{2—16}$$

$$\overline{\tilde{\psi}(z_0)^+} = \overline{f}_3^+\left(\frac{1}{z_0}\right) - \overline{f}_4^+\left(\frac{1}{z_0}\right) \tag{2—17}$$

$$\overline{\varphi_0'(z_0)^+} = \overline{\varphi}_0'^-\left(\frac{1}{z_0}\right) \tag{2—18}$$

$$\overline{\psi_0(z_0)^+} = \overline{\psi}_0^-\left(\frac{1}{z_0}\right). \tag{2—19}$$

So the boundary condition

$$\left(\frac{\partial w}{\partial \overline{z}}\right)^+ = 0, \qquad z \in C \tag{2—20}$$

can be expressed in the following form

$$\left[z f_1(z) + f_2(z) + z\overline{\varphi}_0'\left(\frac{1}{z}\right) + \overline{\psi}_0\left(\frac{1}{z}\right)\right]^- -$$

$$-\left[-z\overline{f}_1\left(\frac{1}{z}\right) - \overline{f}_1'\left(\frac{1}{z}\right) - z\overline{f}_2'\left(\frac{1}{z}\right) + \overline{f}_4\left(\frac{1}{z}\right) - \overline{f}_3\left(\frac{1}{z}\right) \tag{2—21}\right.$$

$$\left. - \varphi_0(z)\right]^+ = 0, \qquad z \in C.$$

In both brackets of the above equation there are boundary values of holomorphic functions only. This methods was first used in solving the boundary-value problem of a circular wall [5] in a somewhat changed form.

If we write

$$\varphi_0(z) = \alpha_0 + \alpha_s z + O(z^2) \tag{2—22}$$

$$\varphi_0'(z) = \alpha_1 + O(z) \tag{2—22'}$$

$$\overline{\varphi}_0'\left(\frac{1}{z}\right) = \overline{\alpha}_1 + O\left(\frac{1}{z}\right) \tag{2—22''}$$

then the function

$$F_1(z) = z f_1(z) + f_2(z) + z\overline{\varphi}_0'\left(\frac{1}{z}\right) + \overline{\psi}_0\left(\frac{1}{z}\right) - \overline{\alpha}_1 z \tag{2—23}$$

is holomorphic for $|z| > 1$ and the function

$$F_2(z) = -z\overline{f}_1\left(\frac{1}{z}\right) - \overline{f}_1'\left(\frac{1}{z}\right) - z\overline{f}_2'\left(\frac{1}{z}\right) + \overline{f}_4\left(\frac{1}{z}\right) -$$

$$-\overline{f}_3\left(\frac{1}{z}\right) - \varphi_0(z) - \overline{\alpha}_1 z \tag{2—24}$$

is holomorphic for $|z|<1$ and

$$F_1^-(z) - F_2^+(z) = 0, \qquad z \in C \tag{2—25}$$

$$F_2(0) = \varphi_0(0) = -\alpha_0. \tag{2—25'}$$

It immediately follows from the above equations that

$$F(z) = F_1(z) = -\alpha_0, \qquad |z| \geqslant 1 \tag{2—26}$$

$$F(z) = F_2(z) = -\alpha_0 \qquad |z| \leqslant 1. \tag{2—26'}$$

Now the equation (2—24) yields

$$\varphi_0(z) = \alpha_0 - \overline{\alpha}_1 z - z\overline{f}_1\left(\frac{1}{z}\right) - \overline{f}_1'\left(\frac{1}{z}\right) - z\overline{f}_2'\left(\frac{1}{z}\right) + \overline{f}_4\left(\frac{1}{z}\right) - \overline{f}_3\left(\frac{1}{z}\right). \tag{2—27}$$

From (2—23) then follows

$$\psi_0(z) = -\frac{1}{z}\overline{f}_1\left(\frac{1}{z}\right) - \overline{f}_2\left(\frac{1}{z}\right) - \frac{\varphi_0'(z) - \alpha_1}{z} - \overline{\alpha}_0. \tag{2—28}$$

It is obvious that $\psi_0(z)$ is regular at $z=0$ with regard to (2—22). It still remains to determine α_1. From (2—11'), (2—22') and (2—27) we obtain

$$\varphi_0(z) = \alpha_0 - \overline{\alpha}_1 z + \iint_D [\overline{\zeta} M(\zeta) + \zeta \overline{M(\zeta)} - \zeta\overline{\zeta} F(\zeta)]\, d\zeta\, d\eta +$$

$$+ O(z^2) = \alpha_0 + \alpha_1 z + O(z^2)$$

where from

$$\alpha_1 + \overline{\alpha}_1 = \iint_D [\overline{\zeta} M(\zeta) + \zeta \overline{M(\zeta)} - \zeta\overline{\zeta} F(\zeta)]\, d\xi\, d\eta. \tag{2—29}$$

From the above equation it is possible to determine the real part of α_1 only, because the right side is real. The imaginary part of α_1 and the whole α_0 remain an arbitrarily real and arbitrarily complex number respectively, which corresponds to the known degree of determination of the complex functions $\varphi_0(z)$ and $\psi_0(z)$.

3. Bending of a circular plate with a free boundary

Here the already known notations (1—1), (2—1), (2—2), (1—11)—(1—11''') will be retained. There is only a nonessential change at the expression of w and at the value of constant α. The boundary condition can be expressed in the form

$$\begin{aligned}\{[-\varkappa\tilde{\varphi}(z_0) + z_0\overline{\tilde{\varphi}_1(z_0)} + \overline{\tilde{\psi}(z_0)}] + [-\varkappa\varphi_0(z_0) + \\ + z_0\overline{\varphi_0'(z_0)} + \overline{\psi_0(z_0)}]\}^+ = 0, \qquad z \in C.\end{aligned} \tag{3—1}$$

From (2—14) and (2—19) is obtained

$$\left\{-\varkappa[zf_1(z)+f_2(z)]+z\overline{\varphi}_0'\left(\frac{1}{z}\right)+\overline{\varphi}_0\left(\frac{1}{z}\right)\right\}^- -$$

(3—2)
$$-\left\{-z\overline{f}_1\left(\frac{1}{z}\right)-\overline{f}_1'\left(\frac{1}{z}\right)-z\overline{f}_2'\left(\frac{1}{z}\right)+\overline{f}_4\left(\frac{1}{z}\right)-\overline{f}_3\left(\frac{1}{z}\right)+\right.$$

$$\left.+\varkappa\varphi_0(z)\right\}^+=0, \qquad z\in C.$$

If we introduce the notations

(3—3)
$$F_1(z)=-\varkappa[zf_1(z)+f_2(z)]+z\overline{\varphi}_0'\left(\frac{1}{z}\right)+\overline{\psi}_0\left(\frac{1}{z}\right)-\overline{\alpha}_1 z$$

(3—4)
$$F_2(z)=-z\overline{f}_1\left(\frac{1}{z}\right)-\overline{f}_1'\left(\frac{1}{z}\right)-z\overline{f}_2'\left(\frac{1}{z}\right)+\overline{f}_4\left(\frac{1}{z}\right)-\overline{f}_3\left(\frac{1}{z}\right)+$$

$$+\varkappa\varphi_0(z)-\overline{\alpha}_1 z$$

then by analogical deduction it follows

(3—5)
$$-\varkappa\varphi_0(z)=-z\overline{f}_1\left(\frac{1}{z}\right)-\overline{f}_1'\left(\frac{1}{z}\right)-z\overline{f}_2'\left(\frac{1}{z}\right)+\overline{f}_4\left(\frac{1}{z}\right)-$$

$$-\overline{f}_3\left(\frac{1}{z}\right)-\overline{\alpha}_1 z-\varkappa\alpha_0$$

(3—6)
$$\psi_0(z)=\varkappa\alpha_0+\varkappa\left[\frac{1}{z}\overline{f}_1\left(\frac{1}{z}\right)+\overline{f}_2\left(\frac{1}{z}\right)\right]-\frac{\varphi_0'(z)-\alpha_1}{z}$$

and

(3—7)
$$\overline{\alpha}_1-\varkappa\alpha_1=\iint_D[\overline{\zeta}M(\zeta)+\zeta\overline{M(\zeta)}-\zeta\overline{\zeta}F(\zeta)]\,d\xi\,d\eta.$$

From the equation (3—7) α_1 is uniquely determined, but α_0 remains an arbitrarily complex number.

4. Generalization of the boundary-value problem of the bending of a clamped circular plate

Let the generalization of this problem be expressed with regard to (2—1) and (2—2) in the equations:

(4—1)
$$\iint_D F(\zeta)\,d\xi\,d\eta=A$$

(4—2)
$$\iint_D[M(\zeta)-\zeta F(\zeta)]\,d\xi\,d\eta=B.$$

Here A is an arbitrarily real and B an arbitrarily complex number. Instead of equations (2—4)—(2—7) we have now

$$F_1(z) = A \ln z + f_1(z) \tag{4—3}$$

$$F_2(z) = B \ln z + f_2(z) \tag{4—4}$$

$$F_3(z) = \overline{B} \ln z + f_3(z) \tag{4—5}$$

$$F_4(z) = f_4(z). \tag{4—6}$$

If we denote

$$F_1^0(z) = A \ln z \tag{4—7}$$

$$F_2^0(z) = B \ln z \tag{4—8}$$

$$F_3^0(z) = \overline{B} \ln z \tag{4—9}$$

$$F_4^0(z) = 0 \tag{4—10}$$

we get

$$\{z F_1^0(z) + F_2^0(z) + z[\overline{F_1^0(z)} + \overline{z}\,\overline{F_1^{0\prime}(z)} + \overline{F_2^{0\prime}(z)}] + \overline{F_3^0(z)} - \overline{F_4^0(z)}\} =$$
$$= (Az + B) \ln (z\overline{z}) + Az + \overline{B} z \overline{z}^{-1}$$

and

$$\{\cdot\cdot\cdot\}^+ = Az_0 + \overline{B} z_0^2, \qquad z_0 \in \boldsymbol{C}. \tag{4—11}$$

If we now add the terms $-Az - \overline{B}z^2$ to $F_2(z)$ in (2 – 24), then

$$\varphi_0(z) = \alpha_0 - (\overline{\alpha}_1 + A) z - Bz^2 - z\overline{f}_1\left(\frac{1}{z}\right) - \overline{f}_1'\left(\frac{1}{z}\right) - \tag{4—12}$$
$$- z\overline{f}_2'\left(\frac{1}{z}\right) + \overline{f}_4\left(\frac{1}{z}\right) - \overline{f}_3\left(\frac{1}{z}\right)$$

$\psi_0(z)$ is expressed in the same way as in (2—28) and for α_1

$$\alpha_1 + \overline{\alpha}_1 = -A + \iint_D [\overline{\zeta} M(\zeta) + \zeta \overline{M(\zeta)} - \zeta\overline{\zeta} F(\zeta)]\, d\xi\, d\eta \tag{4.13}$$

holds.

From the above equation only $\mathrm{Re}[\alpha_1]$ can be determined, because the right side is real and α_0 remains arbitrary.

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BEITRAG ZUR LÖSUNG DER FUNDAMENTALEN RANDWERTAUFGABEN DER BIEGUNG EINER MITTELMÄSSIG DICKEN PLATTE

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Zusammenfassung

In diesem Aufsatz wird eine Randwertaufgabe der Biegung einer kreisförmigen Platte bei der verbesserten Theorie behandelt. Die Platte is im Bereich *D* mit stetiger Kraft and stetigen Moment belastet, der Rand ist im ersten Fall eingespannt und im zweiten Fall frei. Die ganze Randwertaufgabe ist auf die Bestimmung zweier holomorphen Funktionen $\varphi_0(z)$ und $\psi_0(z)$ im Bereich *D* reduziert. In der Gleichung, woraus diese Funktionen bestimmt werden, treten nur die Randwerte der holomorphen Funktionen auf.

PRISPEVEK K OSNOVNIM ROBNIM PROBLEMOM UPOGIBA SREDNJE DEBELIH PLOŠČ

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Povzetek

V tem članku je obravnavan robni problem upogiba krožne plošče po izboljšani teoriji. Plošča je v območju *D* zvezno obremenjena s silo in momentom, rob pa je enkrat togo vpet, drugič pa prost. Celoten robni problem je preveden na iskanje dveh holomorfnih funkcij $\varphi_0(z)$ in $\psi_0(z)$ v območju *D*. V enačbi, iz katere funkciji dokončamo, nastopajo robne vrednosti samih holomorfnih funkcij.