

## SIMILARITY SOLUTIONS BY GROUP TRANSFORMS FOR UNSTEADY HYDROMAGNETIC FREE CONVECTION FLOW

G. S. Brar

(Received August 30, 1976)

**S**ummary. Similarity solutions of the equations by group transforms have been investigated for the unsteady hydromagnetic free convection flow past an infinite flat plate. The expressions for the specific heat flow from the plate and skin friction are obtained. It is found that by increasing the magnetic field we can decrease the skin friction at the wall.

### 1. Introduction

Nanda and Sharma [1] have studied the similarity solutions of the equations which describe the unsteady free convections flow past a vertical flat plate with suction and obtained closed form expressions for the velocity and temperature distributions. Hosimoto [2] obtained exact solutions of the Navier-Stokes equations for the boundary layer growth on an infinite flat plate with uniform suction or injection. Gupta and Suryaprakasaraao [3] solved the boundary layer equations by the momentum integral method and determined the velocity and temperature distribution inside the free convection boundary layer. In the present paper group transforms have been used to obtain the similarity solutions and expressions for the heat transfer and skin friction obtained.

### 2. Basic Equations

Let the origin be at the lowest point of the flat plate,  $\bar{x}$ -axis being along the plate and the  $\bar{y}$ -axis perpendicular to it. Let  $\bar{H}_0$  be the intensity of the magnetic field acting perpendicular to the plate. Under this condition, the equations which describe the unsteady hydromagnetic free convection flow of a viscous, incompressible fluid past an infinite flat plate are:

$$(1) \quad \frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \alpha \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}$$

$$(2) \quad \frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T} - \bar{T}_\infty) - \frac{\sigma_1 B_0^2}{\rho} \bar{u} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$

where  $\bar{t}$  is the time variable;  $\bar{u}$  - the velocity component along the plate;  $\bar{v}$  - the velocity component perpendicular to the plate such that  $\bar{v} = v(t) \cdot \bar{v}_s$ ;  $\bar{v}_s$  - a non-zero negative constant suction velocity;  $\bar{T}$  - the temperature in the boundary

layer;  $\bar{T}_\infty$  — the temperature at a large distance from the plate;  $\bar{B}_0 = \mu_c \bar{H}_0$  is the magnetic induction;  $\sigma_1$  — the electrical conductivity of the fluid;  $\rho$  — the density;  $\nu$  — the kinematic viscosity;  $\mu_c$  — the magnetic permeability of the fluid;  $\beta$  — the coefficient of volume expansion;  $g$  — the acceleration due to gravity;  $\alpha$  — the thermal diffusivity. We have assumed that the fluid has a very small electrical conductivity so that the perturbation in the magnetic field due to the electric current flowing in the fluid may be neglected and this assumption simplifies mathematical analysis.

We introduce non-dimensional quantities defined in the following manner:

$$(3) \quad \left[ \begin{array}{l} u = \frac{\bar{u}}{|\bar{v}_s|}, \quad y = \frac{y |\bar{v}_s|}{\nu}, \quad t = \frac{\bar{v}_s^2 t}{\nu}, \\ T = \frac{\bar{T}_s - \bar{T}_\infty}{\bar{T}_s - \bar{T}_\infty}, \quad G = \frac{g \nu \beta (\bar{T}_s - \bar{T}_\infty)}{|\bar{v}_s|^3}, \\ \sigma = \frac{\nu}{\alpha}, \quad M = \frac{\sigma_1 B_0^2 \nu}{\rho \bar{v}_s^2} \end{array} \right]$$

where  $\bar{T}_s$  is the mean wall temperature;  $G$  — the Grashof number;  $\sigma$  — the Prandtl number and  $M$  — the hydromagnetic parameter. Using (3) in (1) and (2), we have

$$(4) \quad \left[ \begin{array}{l} \frac{\partial^2 T}{\partial y^2} - \nu \frac{\partial T}{\partial y} - \sigma \frac{\partial T}{\partial t} = 0 \\ \frac{\partial^2 u}{\partial y^2} - \nu \frac{\partial u}{\partial y} - \frac{\partial u}{\partial t} = -GT + Mu \end{array} \right]$$

with boundary conditions

$$(5) \quad \left[ \begin{array}{l} T = 1 + \varepsilon e^{i \omega t} \quad \varepsilon \ll 1, \quad u = 0 \text{ at } y = 0 \\ T \rightarrow 0, \quad u \rightarrow 0 \text{ as } y \rightarrow \infty \end{array} \right]$$

where  $\varepsilon$  is a constant small parameter and  $\omega$  is the non-dimensional frequency given by  $\frac{\nu \omega}{\bar{v}_s}$ .

### 3. Solution of Equations

A one-parameter group of transformation is chosen in the form

$$(6) \quad [ \quad t = \bar{t} + \beta_1 b; \quad u = e^{\beta_2 b} \bar{u}; \quad v = e^{\beta_3 b} \bar{v}; \quad y = e^{\beta_4 b} \bar{y}; \quad T = e^{\beta_5 b} \bar{T} ]$$

where  $\beta_i$  ( $i = 1, 2, 3, 4, 5$ ) and  $b$  are certain constants. Substituting (6) in (4), we have for invariance of the equations

$$(7) \quad \beta_3 = \beta_4 = 0, \quad \beta_2 = \beta_5$$

Thus we have

$$(8) \quad \begin{cases} t = \bar{t} + \beta_1 b; & u = e^{\beta_2 b} \bar{u}; & v = \bar{v} \\ y = \bar{y}; & T = e^{\beta_5 b} \bar{T} \end{cases}$$

Putting  $\beta_2/\beta_1 = p$ , we have

$$(9) \quad \frac{u}{e^{pt}} = \frac{\bar{u}}{e^{\bar{p}\bar{t}}}, \quad \frac{T}{e^{pt}} = \frac{\bar{T}}{e^{\bar{p}\bar{t}}}.$$

Hence the absolute invariants are

$$(10) \quad f_p(y) = \frac{u}{e^{pt}}; \quad \theta_p(y) = \frac{T}{e^{pt}}.$$

In the case of the constant suction velocity

$$(11) \quad v = -v_0.$$

Substituting (10) and (11) in (4) we have

$$(12) \quad \frac{d^2 \theta}{dy^2} + v_0 \frac{d\theta}{dy} - \sigma p \theta = 0$$

$$(13) \quad \frac{d^2 f}{dy^2} + v_0 \frac{df}{dy} - pf = -G\theta + Mf$$

Solving equation (12) for  $\theta$ , we have

$$(14) \quad \begin{cases} \theta = C_1 \exp(-h_1 y) \\ \text{where} \\ h_1 = \{v_0 + (v_0^2 + 4\sigma p)^{1/2}\}/2 \end{cases}$$

To calculate  $C_1$ , let  $\theta = \theta_0$  on the plate  $y = 0$ . Thus

$$(15) \quad \theta = \theta_0 e^{-h_1 y}.$$

Hence the specific heat flow from the plate

$$(16) \quad q = -k \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = k \theta_0 [v_0 + (v_0^2 + 4\sigma p)^{1/2}]/2.$$

(17) Putting  $p = i\omega$  (for oscillatory motion)

$$(18) \quad q = k \theta_0 [v_0 + (v_0^2 + 4i\sigma\omega)^{1/2}]/2.$$

Separating into real and imaginary parts we have

$$(19) \quad q = D_1 + iD_2$$

where

$$(20) \quad \left\{ \begin{array}{l} D_1 = (kv_0 \theta_0)/2 + k \theta_0 \left[ \frac{1}{2} \{v_0^2 + (v_0^4 + 16 \sigma^2 \omega^2)^{1/2}\} \right]^{1/2} \\ D_2 = \frac{k \theta_0}{2} \left[ \frac{1}{2} \{v_0^4 + 16 \sigma^2 \omega^2\}^{1/2} - v_0^2 \right]^{1/2} \\ D = \sqrt{D_1^2 + D_2^2} \\ \phi = \tan^{-1} D_2/D_1 \\ k = \text{Thermal conductivity} \end{array} \right.$$

The variations of phase angle  $\phi$  and  $\omega$  have been shown graphically.

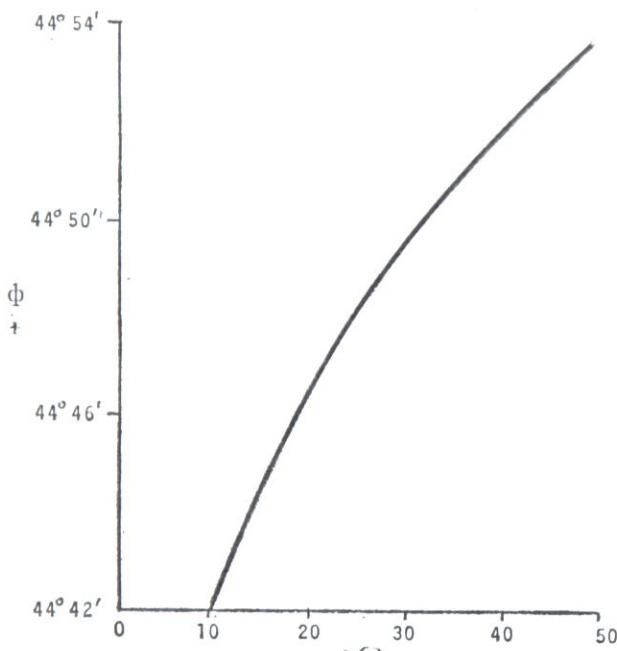


Fig. 1

From equation (13) we have

$$(21) \quad \frac{d^2 f}{dy^2} + v_0 \frac{df}{dy} - (p + M)f = -G \theta_0 e^{-h_1 y}$$

whose solution satisfying the boundary conditions is

$$(22) \quad f(y) = \frac{G \theta_0}{h_1^2 - v_0 h_1 - (p + M)} \cdot (e^{-h_2 y} - e^{-h_1 y})$$

where

$$(23) \quad h_2 = \frac{1}{2} [v_0 + (v_0^2 + 4p + 4M)^{1/2}]$$

Thus the non-dimensional form of the skin friction at the wall

$$(24) \quad \tau = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} = \mu e^{pt} \left[ \frac{G \theta_0}{h_1^2 - v_0 h_1 - (p + M)} (h_1 - h_2) \right]$$

The oscillatory motion is easily obtained by putting  $p = i\omega$ .

The frequency of oscillation must be large to neglect the effect of induced magnetic field. Assuming  $v_0$  to be small, then we have

$$(25) \quad \left\{ \begin{array}{l} h_1 \approx 2\sqrt{i\omega\sigma} \\ h_2 \approx 2\sqrt{i\omega + M} \\ h_1^2 - v_0 h_1 - (p + M) \approx i\omega (4\sigma - 1) - M \end{array} \right.$$

Thus

$$(26) \quad \tau = \mu e^{pt} G \theta_0 \left[ \frac{2\sqrt{i\omega\sigma} - 2\sqrt{i\omega + M}}{i\omega(4\sigma - 1) - M} \right].$$

We see that by increasing the magnetic field we can decrease the skin friction at the wall.

*Acknowledgement.* The author wishes to express his thanks to Professor K. Lal for his valuable suggestions.

#### R E F E R E N C E S

- [1] Nanda, R. S. and Sharma, V. P., *Possible similarity solutions of unsteady free convection flow past a vertical plate with suction*, J. Phys. Soc. Japan, 17 (1962), 1651.
- [2] Hosimoto, H., *Boundary layer growth on a flat plate with suction or injection*, J. Phys. Soc. Japan, 12 (1957), 68.
- [3] Gupta, A. S., *Hydromagnetic free convection past a vertical porous flat plate subjected to suction or injection*, J. Phys. Soc. Japan, 20 (1965), 1936.

Department of Electrical Engineering  
University of Calgary  
Calgary, Alberta, Canada

#### SOLUTIONS SIMILAIRES DANS LE CAS DE LA CONVECTION LIBRE D'UN FLUIDE CONDUCTEUR EN RÉGIME NON STATIONNAIRE

*G. S. Brar*

#### Résumé

On traite dans ce travail le problème des solutions similaires dans le cas d'une convection libre non stationnaire d'un fluide conducteur sur une plaque plane, le champ magnétique étant normal à la paroi, tandis que le champ électrique est négligeable. En appliquant la méthode des transformations des groupes, on trouve les solutions analytiques pour la quantité spécifique de chaleur et la contrainte tangentielle pariétale. En étudiant plus en détail numériquement ce problème de la contrainte tangentielle on démontre que la contrainte tangentielle pariétale diminue avec une intensification du champ magnétique extérieur.

## SLIČNA REŠENJA NESTACIONARNOG SLOBODNO-KONVEKTIVNOG STRUJANJA METODOM GRUPNIH TRANSFORMACIJA

*G. S. Brar*

### Rezime

U ovom radu se tretira problem sličnih rešenja pri nestacionarnom magnetohidrodinamičkom slobodno-konvektivnom strujanju na beskonačnoj ravnoj ploči u bezinduktivnom približenju. Spoljašnje magnetno polje je upravno na ravnu ploču, a električno polje se zanemaruje.

Primenjujući metodu grupnih transformacija, određena su analitička rešenja za specifičnu toplotu i tangencijalni napon na zidu. Posebno analizirajući problem tangencijalnog napona na zidu dokazuje se da on opada sa pojačavanjem spoljašnjeg magnetnog polja.