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STATISTICAL ANALYSIS OF LINEAR SOLID BODY WITH CONTINUOUS DISTRIBUTION OF SCREW DISLOCATIONS

Milan Mićunović

Introduction

It is supposed that deformation and stress state of linear infinite solid body depends only on position and motion of great finite number N of parallel straight screw dislocations. In other words, the influence of body forces is neglected but could be taken into account as straightforward generalization of presented work.

By assumption, dislocations can move with arbitrary subsonic velocities staying during the motion parallel to their initial directions. Since dislocations perform relative motion with regard to particles of the body, the corresponding deformation is elasto-plastic.

For discrete defects the theory (cf. [1], [2], [3]) that properly describes shock waves, dislocation loops, point defects, cracks as well as vortex sheets is developed in rigorous way. In this theory defects are considered as material or immaterial surfaces on which there appears discontinuity of displacement, velocity or stress. However, the theory is rather complicated so that effective results can be obtained only if areas of defect surfaces are infinitesimal. The basic definition of dislocation in the theory follows Volterra's definition by which dislocation is boundary surface where jump of displacement exists.

On the other hand, each simply connected region of the body which does not contain dislocation lines deforms in a continuous way so that essential source of deformation is dislocation line by itself. Also, by means of experiments only dislocation lines are observed and not surfaces of displacement discontinuities. Dynamics of linear continuum with dislocations as line defects (cf. [4], [5], [6]) is as well developed. For this theory the size of dislocation line does not restrict the field of application.

However, first — because of enormous number (10^6—$10^{12}/cm^2$) practically arbitrarily distributed dislocations and second — owing to their complicated relative motion with respect to basic material, the continuum theory cannot succesfully explain the plastic deformation process (cf. [7], [8]). On account of this it is necessary, by means of statistical approach, to analyze what information from discrete defects is of essential value for continuum theory.

Model accepted in this paper is one of the simplest idealizations of real dislocation distribution and motion. Nevertheless, we believe that the results could be used for construction of more complicated models of dislocation networks.

1. Survey of discrete dislocation dynamics

In this section the most important results of discrete dislocation dynamics (cf. [2], [5], [6]), necessary for further analysis, are reviewed. Motion of particles as well as dislocations is observed in spatial Cartesian coordinate frame $x^i = x_i$.

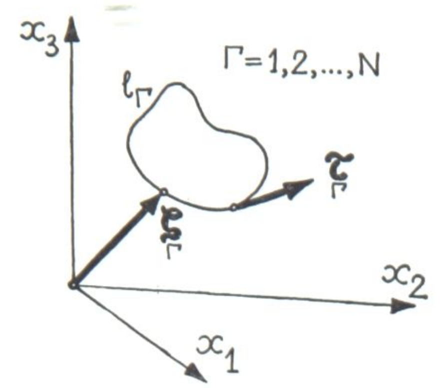

Fig. 1

Dislocation lines are supposed to be smooth and if of finite length then must be closed. On Figure 1 one dislocation loop is shown. $\underset{\Gamma}{\zeta_i}$ represents the place of arbitrary point of loop while $\underset{\Gamma}{\tau_i}$ is tangent unit vector.

In general, displacement gradient of arbitrary particle is (cf. [9])

$$u_{i,j} = \beta_{ij} + \overset{*}{\beta}_{ij}, \tag{1.1}$$

where β_{ij} is elastic while $\overset{*}{\beta}_{ij}$ is plastic distortion.

The corresponding strains are described by following tensors

$$\left\{\begin{aligned} E_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i}) = e_{ij} + \overset{*}{e}_{ij}, \\ e_{ij} &= \frac{1}{2}(\beta_{ij} + \beta_{ji}), \quad \overset{*}{e}_{ij} = \frac{1}{2}(\overset{*}{\beta}_{ij} + \overset{*}{\beta}_{ji}). \end{aligned}\right. \tag{1.2}$$

The fact that continuum is linear is described by the following response function

$$\sigma_{ij} = c_{ijkl}\, e_{kl} = c_{ijkl}\, \beta_{kl}, \tag{1.3}$$

where σ_{ij} is symmetric stress tensor and c_{ijkl} is tensor of material coefficients.

The elastic distortion tensor β_{ij} is connected with dislocation density tensor α_{kl} by the following relation

$$\alpha_{il} = \varepsilon_{lmk}\, \beta_{ik,m} \quad \text{or} \quad \vec{\alpha} = \operatorname{Rot} \vec{\beta}, \tag{1.4}$$

where ε_{lmk} is antisymmetric permutation tensor. By making use of Dirac delta function

$$\delta(\vec{x}) = \delta(x_1)\,\delta(x_2)\,\delta(x_3),$$

dislocation density is determined in the following form

$$\alpha_{il}(\vec{x}, t) = \sum_{\Gamma} \underset{\Gamma}{b_i} \oint_{l_\Gamma} d\underset{\Gamma}{\zeta_l}\, \delta[\vec{x} - \underset{\Gamma}{\zeta}(l, t)] \equiv \sum_{\Gamma} \underset{\Gamma}{\alpha_{il}}. \tag{1.5}$$

If $\vec{\alpha}$ is known in advance then $\vec{\beta}$ can be determined from (1.4) with accuracy only up to gradient of arbitrary vector function. Because of this we need the equation of balance of linear momentum in local form

$$\rho \frac{dv_i}{dt} - c_{ijkl}\, \beta_{kl,j} = 0, \quad \frac{dv_i}{dt} \approx \frac{\partial v_i}{\partial t}, \tag{1.6}$$

where ρ is mass density and v_i — velocity of displacement of the corresponding particle.

Finally, dynamical equation that connects velocity gradient and rate of elastic distortion on the basis of (1.1) may be written in the form

$$\frac{\partial}{\partial t}\beta_{ik}+\frac{\partial}{\partial t}\overset{*}{\beta}_{ik}=v_{i,k}. \tag{1.7}$$

Herein, the rate of plastic distortion can be connected with velocities and places of dislocation lines points in the following way (cf. [5], [6])

$$J_{ik}\equiv-\frac{\partial}{\partial t}\overset{*}{\beta}_{ik}=\varepsilon_{kmn}V_{mni}, \tag{1.8}$$

where V_{mni} is dislocation flux tensor which has the form

$$V_{mni}(\vec{x},t)=\sum_{\Gamma}\underset{\Gamma}{b_i}\oint_{l_\Gamma}d\underset{\Gamma}{\zeta_n}\underset{\Gamma}{\dot{\zeta}_m}\,\delta[\vec{x}-\underset{\Gamma}{\vec{\zeta}}(l,t)]\equiv\sum_{\Gamma}\underset{\Gamma}{V_{mni}}, \tag{1.9}$$

so that (1.7) transforms into

$$\frac{\partial}{\partial t}\beta_{ik}-v_{i,k}=\varepsilon_{kmn}V_{mni}. \tag{1.10}$$

Now, it is possible from (1.4), (1.6) and (1.10) to find elastic distortion field as well as velocity field if dislocation density and flux are known as functions of place and time.

However, in general case for complete set of field equations it is necessary to have the equations of motion for each dislocation. Such equations have the form (cf. [1], [2])

$$\frac{\partial}{\partial t}d\underset{\Gamma}{p_i}=d\underset{\circ\Gamma}{f_i}+\sum_{\substack{\Delta\\ \Delta\neq\Gamma}}d\underset{\Gamma\Delta}{f_i},\qquad \Gamma=1,2,\ldots,N, \tag{1.11}$$

where:

$d\underset{\circ\Gamma}{f_i}$ — the force on dislocation line element dl_Γ caused by external displacement field (this force we neglect in the paper as factor of secondary importance),

$d\underset{\Gamma\Delta}{f_i}$ — the force on dislocation line element dl_Γ caused by stress field of dislocation line l_Δ.

The last one equals to

$$d\underset{\Gamma\Delta}{f_i}=\varepsilon_{irs}[\rho\underset{\Gamma}{\dot{\zeta}_s}v_j(\underset{\Gamma}{\vec{\zeta}})+c_{sjpq}\underset{\Delta}{\beta_{pq}}(\underset{\Gamma}{\vec{\zeta}})]\underset{\Gamma}{b_j}\,d\underset{\Gamma}{\zeta_r}. \tag{1.12}$$

The linear momentum $d\underset{\Gamma}{p_i}$ of element dl_Γ is derived from the action of dislocation l_Γ on itself (cf. [1], [2]) and in the case of small velocities of dislocation lines has the form

$$d\underset{\Gamma}{p_i}=d\underset{\Gamma}{m_{ip}}\underset{\Gamma}{\dot{\zeta}_p},\qquad \underset{\Gamma}{\dot{\zeta}_p}\equiv\frac{\partial}{\partial t}\underset{\Gamma}{\zeta_p}, \tag{1.13}$$

where $\underset{\Gamma}{dm_{ip}}$ is tensorial mass of dislocation element dl_Γ given by means of the expression

$$\underset{\Gamma}{dm_{ir}} = \underset{\Gamma}{m_{iprs}} \underset{\Gamma}{b_r} \underset{\Gamma}{b_s} dl_\Gamma, \quad \underset{\Gamma}{m_{iprs}} = \text{const.} \tag{1.14}$$

Equation (1.11) obtains very simplified form if its left hand side is neglected (cf. [4], [6]).

Now, equations (1.4), (1.6), (1.10) and (1.11) represent the complete set of field equations. However, this set is difficult to be solved because of their great number corresponding to number of dislocations.

In our case we suppose that all the dislocations are straight and parallel to x_3-axis during their motion. They must be either closed or come out to external surface of the body. Owing to this the length of each dislocation line is infinite Therefore, velocities of all the points of any dislocation line are the same and projection of common velocity on x_3-axis is of no importance. Furthermore, our additional assumption is that Burgers vectors of all dislocations are the same

$$\underset{\Gamma}{b_i} \equiv b\,\delta_{i3} = b_i. \tag{1.15}$$

Taking into account homogeneity of material this assumption does not represent great restriction and makes statistics easier.

It is known that in an infinite continuum having one straight screw dislocation displacement vector is parallel to dislocation line i.e.

$$v_i(\vec{x}, t) = \delta_{i3} v_3. \tag{1.16}$$

Bearing in mind this and that all the quantities depend only on x_1 and x_2, equations (1.1), (1.5) and (1.9) take the form

$$u_{3,1} = \beta_{31} + \overset{*}{\beta}_{31}; \quad u_{3,2} = \beta_{32} + \overset{*}{\beta}_{32}; \tag{1.17}$$

$$\left\{\begin{aligned} \alpha_{il}(x_1, x_2, t) &= \delta_{i3}\delta_{l3} b \sum_{\Gamma} \int_{-\infty}^{\infty} d\underset{\Gamma}{\zeta_3}\, \delta(x_1 - \underset{\Gamma}{\zeta_1})\, \delta(x_2 - \underset{\Gamma}{\zeta_2})\, \delta(x_3 - \underset{\Gamma}{\zeta_3}) = \\ &= \delta_{i3}\delta_{l3} b \sum_{\Gamma} \delta(x_1 - \underset{\Gamma}{\zeta_1})\, \delta(x_2 - \underset{\Gamma}{\zeta_2}); \end{aligned}\right. \tag{1.18}$$

$$V_{mni}(x_1, x_2, t) = \delta_{i3}\delta_{n3} b \sum_{\Gamma} \delta(x_1 - \underset{\Gamma}{\zeta_1})\, \delta(x_2 - \underset{\Gamma}{\zeta_2}) \frac{\partial}{\partial t} \underset{\Gamma}{\zeta_m}. \tag{1.19}$$

Moreover, by integrating (1.11)—(1.14) we obtain per unit length of dislocation line

$$\frac{\partial}{\partial t} \underset{\Gamma}{p_i} = \sum_{\substack{\Delta \\ \Delta \neq \Gamma}} \underset{\Delta\Gamma}{f_i}, \tag{1.20}$$

$$\underset{\Delta\Gamma}{f_i} = \varepsilon_{3si} b [\rho \underset{\Gamma}{\dot{\zeta}_s} v_3(\underset{\Gamma}{\zeta_1}, \underset{\Gamma}{\zeta_2}) + c_{3spq} \underset{\Delta}{\beta_{pq}}(\underset{\Gamma}{\zeta_1}, \underset{\Gamma}{\zeta_2})], \tag{1.21}$$

$$\underset{\Gamma}{p_i} = m_{ip} \underset{\Gamma}{\dot{\zeta}_p}, \tag{1.22}$$

$$m_{ip} = \lim_{L \to \infty} \frac{b^2}{2L} \int_{-L}^{L} m_{iprs} \delta_{r3} \delta_{s3}\, d\zeta_3 = b^2 m_{ip33} = \text{const.} \tag{1.23}$$

2. Statistics

For statistical derivation of hydrodynamic field equations the following procedure (cf. [10], [11]) is applied. Places $\underset{1}{\vec{\zeta}}, \dots, \underset{N}{\vec{\zeta}}$ and linear momenta $\underset{1}{\vec{p}}, \dots, \underset{N}{\vec{p}}$ of N arbitrarily distributed molecules are defined by means of one representative point in $6N$-dimensional phase space. The probability distribution function (relative density of representative points in phase space) is denoted by

(2.1)
$$f(\underset{1}{\vec{\zeta}}, \dots, \underset{N}{\vec{\zeta}}, \underset{1}{\vec{p}}, \dots, \underset{N}{\vec{p}}, t) \equiv f(\underset{\Gamma}{\vec{\zeta}}, \underset{\Gamma}{\vec{p}}, t),$$

satisfying the normalization condition

(2.2)
$$\int f \, d\mathcal{H} = 1,$$

where

(2.3)
$$d\mathcal{H} \equiv d\underset{1}{\vec{\zeta}} \cdot d\underset{N}{\vec{\zeta}} \cdot d\underset{1}{\vec{p}} \cdots d\underset{N}{\vec{p}}$$

volume element in phase space. f changes with time in accordance with Liouville theorem

(2.4)
$$\frac{\partial f}{\partial t} + \sum_{\Gamma} \left(\frac{\partial f}{\partial \underset{\Gamma}{\zeta_i}} \underset{\Gamma}{\dot{\zeta}_i} + \frac{\partial f}{\partial \underset{\Gamma}{p_j}} \underset{\Gamma}{\dot{p}_j} \right) = 0.$$

Any dynamic entity (scalar, vector and tensor of arbitrary order) $\mathcal{P}$ depending on time and phase coordinates has at time t expectation value

(2.5)
$$\langle \mathcal{P} \rangle = \int \mathcal{P} f \, d\mathcal{H},$$

whose change with time is described by means of transport equation

(2.6)
$$\frac{\partial}{\partial t} \langle \mathcal{P} \rangle = \left\langle \frac{d\mathcal{P}}{dt} \right\rangle,$$

where

(2.7)
$$\frac{d\mathcal{P}}{dt} = \frac{\partial \mathcal{P}}{\partial t} + \sum_{\Gamma} \left(\frac{\partial \mathcal{P}}{\partial \underset{\Gamma}{\zeta_i}} \underset{\Gamma}{\dot{\zeta}_i} + \frac{\partial \mathcal{P}}{\partial \underset{\Gamma}{p_i}} \underset{\Gamma}{\dot{p}_i} \right).$$

Further derivation of hydrodynamic field equations consists of application of transport equation to statistically defined mass density, momentum and energy. The advantage of this method is a better understanding of stress tensor and other continuum variables on the basis of molecular interactions.

In this paper it is assumed that dislocations are randomly distributed in the similar way as molecules in kinetic theory of fluids, following the idea given in [3]. In consequence of our assumption about shape of dislocation lines all the variables depend only on x_1, x_2 and t so that our phase space is $4N$-dimensional. Now, velocity and elastic distortion depend on position of each dislocation and have expectation values

(2.8)
$$\bar{v}_i \equiv \langle v_i \rangle = \sum_{\Gamma} \langle \underset{\Gamma}{v_i} \rangle = \sum_{\Gamma} \int \underset{\Gamma}{v_i} f \, d\mathcal{H},$$

(2.9)
$$\bar{\beta}_{ij} \equiv \langle \beta_{ij} \rangle = \sum_{\Gamma} \langle \underset{\Gamma}{\beta_{ij}} \rangle = \sum_{\Gamma} \int \underset{\Gamma}{\beta_{ij}} f \, d\mathcal{H},$$

owing to their additive nature — continuum is linear. On the other hand, expectation values

$$\overline{\alpha}_{il} \equiv \langle \alpha_{il} \rangle = \sum_{\Gamma} \langle \underset{\Gamma}{\alpha_{il}} \rangle \equiv \delta_{i3}\, \delta_{l3}\, \alpha, \tag{2.10}$$

$$\alpha = \sum_{\Gamma} \langle \delta (x_1 - \underset{\Gamma}{\zeta_1})\, \delta (x_2 - \underset{\Gamma}{\zeta_2}) \rangle, \tag{2.11}$$

$$\overline{V}_{mni} \equiv \langle V_{mni} \rangle = \sum_{\Gamma} \langle \underset{\Gamma}{V_{mni}} \rangle \equiv b\, \alpha\, V_m\, \delta_{n3}\, \delta_{i3}, \tag{2.12}$$

$$V_m\, \alpha = \sum_{\Gamma} \langle \underset{\Gamma}{\dot{\zeta}_m}\, \delta (x_1 - \underset{\Gamma}{\zeta_1})\, \delta (x_2 - \underset{\Gamma}{\zeta_2}) \rangle, \tag{2.13}$$

obtained from (1.18) and (1.19), are obvious. On left sides of relations (2.8)—(2.13) all the variables are functions of x_1, x_2 and t. Also

$$\underset{\Gamma}{v_i} = \underset{\Gamma}{v_i} (x_1, x_2, t), \quad \underset{\Gamma}{\beta_{ij}} = \underset{\Gamma}{\beta_{ij}} (x_1, x_2, t), \tag{2.14}$$

while index Γ shows dislocation line, by influence of which partial velocity $\underset{\Gamma}{v_i}$ or partial elastic distortion $\underset{\Gamma}{\beta_{ij}}$ are caused.

3. Field equations

On the basis of dynamics of discrete dislocations and described statistical method we are able now to derive continuum equations.

Multiplying equation (1.4) by $f\, d\mathcal{H}$ and integrating over all the phase space we obtain

$$\overline{\alpha}_{il} = \varepsilon_{lmk}\, \overline{\beta}_{ik,m}, \tag{3.1$_1$}$$

or, explicitly, by means of (2.10) and (2.11)

$$\alpha = \frac{1}{b} (\overline{\beta}_{32,1} - \overline{\beta}_{31,2}). \tag{3.1$_2$}$$

Herein, the following transformation

$$\begin{aligned} \sum_{\Gamma} \langle \underset{\Gamma}{\beta_{ik,m}} \rangle &= \sum_{\Gamma} \int \left[\frac{\partial}{\partial x_m} \underset{\Gamma}{\beta_{ik}} (x_{n,t}) \right] f (\underset{\Gamma}{\zeta_p}, \underset{\Gamma}{p_q})\, d\mathcal{H} = \\ &= \frac{\partial}{\partial x_m} \sum_{\Gamma} \int \underset{\Gamma}{\beta_{ik}}\, f\, d\mathcal{H} = \frac{\partial}{\partial x_m} \langle \beta_{ik} \rangle \equiv \overline{\beta}_{ik,m} \end{aligned} \tag{3.2}$$

is used. In the same way, the relations

$$\rho \frac{\partial \overline{v}_i}{\partial t} - c_{ijkl}\, \overline{\beta}_{kl,j} = 0, \tag{3.3$_1$}$$

$$\rho \frac{\partial \overline{v}_3}{\partial t} - \mu (\overline{\beta}_{31,1} + \overline{\beta}_{32,2}) = 0, \tag{3.3$_2$}$$

follow directly from (1.6) and the transformation

$$\left\langle\frac{\partial v_i}{\partial t}\right\rangle \approx \left\langle\frac{dv_i}{dt}\right\rangle = \sum_{\Gamma} \int \frac{dv_i}{dt} f \, d\mathcal{H} = \frac{d}{dt}\langle v_i \rangle \equiv \frac{d\overline{v}_i}{dt} \approx \frac{\partial \overline{v}_i}{\partial t}. \tag{3.4}$$

Also, equation (1.10) by means of integration over all the phase space gives

$$\frac{\partial}{\partial t}\overline{\beta}_{ik} - \overline{v}_{i,k} = \varepsilon_{kmn} \overline{V}_{mni}, \tag{3.5$_1$}$$

or, according to (1.8) and (2.12), explicitly

$$\frac{\partial}{\partial t}\overline{\beta}_{31} - \overline{v}_{3,1} = b\,\alpha\, V_2, \quad \frac{\partial}{\partial t}\overline{\beta}_{32} - \overline{v}_{3,2} = -b\,\alpha\, V_1. \tag{3.5$_2$}$$

In order to derive conservation law for dislocation density Liouville theorem has to be used with one partial integration over the whole phase space. As a result, we obtain

$$\frac{\partial \overline{\alpha}_{il}}{\partial t} = -b\,\delta_{i3}\,\delta_{l3}\frac{\partial}{\partial x_p}(\alpha V_p), \tag{3.6$_1$}$$

or, after (2.10),

$$\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial x_1}(\alpha V_1) + \frac{\partial}{\partial x_2}(\alpha V_2) = 0. \tag{3.6$_2$}$$

For a complete set of field equations we still need equation of balance of dislocation momentum. Dislocation momentum per unit length is defined by

$$p_i = \sum_{\Gamma} \langle \underset{\Gamma}{p_i} \underset{\Gamma}{\delta} \rangle = m_{ip} \sum_{\Gamma} \langle \underset{\Gamma}{\dot{\zeta}_p} \underset{\Gamma}{\delta} \rangle = \alpha\, m_{ip} V_p. \tag{3.7}$$

Now, if we look for dp_i/dt and apply transport equation (2.4), the following relation

$$\frac{\partial p_i}{\partial t} = -\frac{\partial}{\partial x_k}(-\overset{K}{\sigma}_{ik} + p_i V_k) + \sum_{\Gamma}\left\langle \underset{\Gamma}{\delta}\frac{\partial}{\partial t}\underset{\Gamma}{p_i}\right\rangle \tag{3.8}$$

will be obtained, where

$$\overset{K}{\sigma}_{ij}(x_m, t) = -m_{ip}\sum_{\Gamma}\langle(\underset{\Gamma}{\dot{\zeta}_j} - V_j)(\underset{\Gamma}{\dot{\zeta}_p} - V_p)\underset{\Gamma}{\delta}\rangle. \tag{3.9}$$

After analogy with statistical theory of hydrodynamical field equations, $\overset{K}{\sigma}_{ik}$ is called kinetic stress tensor (cf. [3], [10], [11]). The last term of right hand side of (3.8) by means of (1.11), (1.12), (1.15) and (1.16) can be written in the form

$$\sum_{\Gamma}\left\langle \underset{\Gamma}{\delta}\frac{\partial}{\partial t}\underset{\Gamma}{p_i}\right\rangle = \mu\, b\, \varepsilon_{3si}\,\alpha\left(\frac{\rho}{\mu}\overline{v}_3 V_s + \overline{\beta}_{3s}\right) + F_i, \tag{3.10}$$

where

$$\begin{aligned} F_i = b\varepsilon_{3si}\rho \sum_{\Gamma,\Delta}\langle \underset{\Gamma}{\delta}\,\underset{\Gamma}{\dot{\zeta}_s}\{v_3(\underset{\Gamma}{\vec{\zeta}}) - \langle v_3(\underset{\Gamma}{\vec{\zeta}})\rangle\}\rangle + \\ + b\,\varepsilon_{3si}\mu\sum_{\Gamma,\Delta}\langle \underset{\Gamma}{\delta}\{\underset{\Delta}{\beta_{3s}}(\underset{\Gamma}{\vec{\zeta}}) - \langle \underset{\Delta}{\beta_{3s}}(\underset{\Gamma}{\vec{\zeta}})\rangle\}\rangle. \end{aligned} \tag{3.11}$$

Obviously, F_i represents the difference between the mean value of product and the product of mean values and could be neglected in first order approximation as small quantity of higher order (cf. [12]). Taking this into account we obtain from (3.7), (3.8) and (3.10) the relation

$$\frac{\partial}{\partial t}(\alpha V_l)+\frac{\partial}{\partial x_k}(\alpha V_k V_l)=\overset{-1}{m}_{li}\left[\frac{\partial}{\partial x_k}\overset{K}{\sigma}_{ik}+\alpha\, b\, \varepsilon_{3si}(\rho\, \bar{v}_3 V_s+\mu\bar{\beta}_{3s})\right]. \tag{3.12}$$

If $(3.6)_2$ is applied to (3.12) then the final form of equation for balance of dislocation momentum is obtained

$$\frac{\partial V_l}{\partial t}+\frac{\partial V_l}{\partial x_k}V_k=\overset{-1}{m}_{li}\left[\frac{1}{\alpha}\frac{\partial}{\partial x_k}\overset{K}{\sigma}_{ik}+b\,\varepsilon_{3si}(\rho\, V_s\bar{v}_3+\mu\bar{\beta}_{3s})\right]. \tag{3.13}$$

In order to have complete set of field equations we must have some constitutive relation for kinetic stress tensor. This is necessary due to fact that set the of phenomenological equations derived by statistical methods cannot be full set. For this constitutive relation we follow [3] where the analogy with hydrodynamics was explored in the way that kinetic stress tensor compared with stress caused by intermolecular forces (analogue for the second one in our case is Peach-Koehler force — the last term on right side of (3.13)) is important only for a fluid with small density. Because of this, by analogy with ideal barotropic fluid and with assumption about "adiabatic flow" we have

$$\overset{K}{\sigma}_{ij}=-m_{ij}\,p(\alpha),\quad p(\alpha)=p_0\left(\frac{\alpha}{\alpha_0}\right)^{\gamma}\equiv B\alpha^{\gamma}, \tag{3.14}$$

where B and γ are constants. By replacing (3.13) into (3.14) we finally obtain

$$\begin{gathered}\frac{\partial V_l}{\partial t}+\frac{\partial V_l}{\partial x_k}V_k=\frac{\gamma}{1-\gamma}\frac{\partial}{\partial x_l}(B\alpha^{\gamma-1})+b\,\overset{-1}{m}_{li}\,\varepsilon_{3si}(\rho\, V_s\bar{v}_3+\mu\bar{\beta}_{3s}),\\ l,\ k,\ i,\ s=1,\ 2.\end{gathered} \tag{3.15}$$

In this way the field equations are $(3.1)_2$, $(3.3)_2$, $(3.5)_2$, $(3.6)_2$ and (3.15) (7 in total) and they serve to determine the unknowns α, V_1, V_2, $\bar{v}_3$, $\bar{\beta}_{31}$, $\bar{\beta}_{32}$, all of them being functions of x_1, x_2 and t. Equation $(3.1)_2$ can be derived from $(3.5)_2$ and $(3.6)_2$ and owing to this the number of equations seems greater than the number of unknowns. If corresponding initial conditions are given the problem can be solved.

If dislocation density α and velocity V_k are given in advance then Green function can be used for determination of elastic distortion and velocity of displacement. Namely, from $(3.1)_1$, $(3.3)_1$ and $(3.5)_1$ we have

$$\begin{gathered}L_{ik}\bar{\beta}_{km}=b\,\delta_{i3}\,\varepsilon_{mp3}\left[\rho\frac{\partial}{\partial t}(\alpha V_p)+\mu\frac{\partial\alpha}{\partial x_p}\right],\\ L_{ik}\bar{v}_k=\mu\delta_{i3}\,\varepsilon_{mp3}\frac{\partial}{\partial x_m}(V_p\alpha),\end{gathered} \tag{3.16}$$

where

$$L_{ik}\equiv\rho\delta_{ik}\frac{\partial^2}{\partial t^2}-c_{ijkl}\frac{\partial}{\partial x^l}\frac{\partial}{\partial x^j} \tag{3.17}$$

is Lamé operator. As known, solutions for $\bar{\beta}_{km}$ and $\bar{v}_k$ are expressed in the form

$$\bar{\beta}_{km}(x_s, t) = \int_{-\infty}^{\infty} dt' \int d_3 x' \, G_{kn}(x_p - x_p', t - t') \, b \, \delta_{n3} \, \varepsilon_{mp3} \left[\frac{\partial}{\partial t} (\alpha V_p) \rho + \mu \frac{\partial \alpha}{\partial x_p} \right],$$

(3.18)

$$\bar{v}_k(x_l, t) = \int_{-\infty}^{\infty} dt' \int d_3 x' \, G_{kn}(x_p - x_p', t - t') \, \mu \delta_{n3} \, \varepsilon_{qs3} \frac{\partial}{\partial x_q} (\alpha V_s).$$

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REFERENCES

[1] H. Zorski: Arch. Mech. Stosow., **18**, 301 (1966)

[2] E. Kossecka, H. Zorski: Int. J. Solids Structures, **3**, 881 (1967)

[3] H. Zorski: Int. J. Solids Structures, **4**, 959 (1968)

[4] A. M. Kosevitch: Usp. Fiz. Nauk, **84**, 4, 579 (1964)

[5] T. Mura: *The continuum theory of dislocations*. In: Advances in Material Research, ed. H. Herman, Vol. 3, Interscience Publishers, New York, 1968

[6] E. Kossecka: Arch. Mech. Stosow., 2, **21**, 167 (1969)

[7] E. Kröner: Rheol. Acta **14**, 5 (1975)

[8] Fundamental aspects of dislocation theory (ed. J. A. Simmons, R. de Wit, R. Bullough) Nat. Bur. Stand. US, Spec. Publ. 317, Vols I, II, 1970

[9] E. Kröner: *Kontinuumstheorie der Versetzungen und Eigenspannungen*. Berlin. Springer 1958.

[10] J. H. Irwing, J. G. Kirkwood: J. Chem. Phys., 6, **18**, 817 (1950)

[11] D. Massignon: *Mécanique Statistique des Fluides*, Paris, Dunod 1957.

[12] I. Z. Fisher: *Statisticeskaya teoria zidkostei*, Moskva, Gos. Izd. Fiz. Mat. Lit. 1961

СТАТИСТИЧЕСКИЙ АНАЛИЗ ЛИНЕЙНОГО ТВЕРДОГО ТЕЛА С НЕПРЕРИВНЫМ РАСПРЕДЕЛЕНИЕМ ВИНТОВЫХ ДИСЛОКАЦИЙ

Милан Мичунович

Рассматриваемое твердое тело-бесконечное, однородное и имеет линейную конститутивную функцию для напряжения. Предполагается, что это твердое тело содержит конечное большое число прямих винтовюх взаимно параллельных дислокаций, которые могут произвольно двигаться в плоскости перпендикулярной к направлениям дислокационных линий. Динамика дискретных дислокаций сжато показана и к ней применен метод статистической гидродинамики. Полученная система уравнений поля ком-

плектная, так что проблема определения плотности дислокаций, скорости дислокаций, поля упругой дисторсии и скорости перемещения может быть решена, если даны начальные условия.

СТАТИСТИЧКА АНАЛИЗА ЛИНЕАРНОГ ЧВРСТОГ ТЕЛА СА НЕПРЕКИДНИМ РАСПОРЕДОМ ЗАВОЈНИХ ДИСЛОКАЦИЈА

М. Мићуновић

Резиме

Посматрано чврсто тело је бесконачно, хомогено и има линеарну конститутивну једначину за напон. Предпоставља се да ово тело садржи коначан врло велики број правик бесконачно дугих међусобно паралелних завојних дислокација. Затим је метод статистичке хидродинамике примењен на динамику дискретних дислокација. Добијени систем једначина поља је комплетан тако да проблем одређивања густине дислокација, брзине дислокација, поља еластичне дисторзије и поља брзине може да се реши ако су дати почетни услови. Дати поступак је могуће применити и на компликованије распореде дислокационих линија и омчи.