

A MICROCONTINUUM MECHANICS APPROACH TO THE CHARACTERIZATION OF TWO-COMPOSITE LINEAR ELASTIC MATERIAL

P. Marinov

1. Introduction

Various methods have been used in the characterization of two-composite elastic material. The characteristic approach to such problems in the past has been to model the composite material by a single continuum, occasionally anisotropic, whose constitutive constants are chosen in such a way that the behaviour of the single continuum provides an approximation to the behaviour of the composite [5], [6], [7]. Although this approach has been found adequate for static and quasi-static problems, it loses completely any characterization of the macroscopic structure. The structural effects give rise to dispersion and dissipation in dynamical processes in the composite material which the usual single continuum theories are incapable of describing [15].

However, the mathematical models which describe the composite structural effects are limited. The composite structural effects were investigated in [2], [3], [10]. Bedford and Stern [10] developed a thermomechanical theory for composite materials in which the composite constituents are modeled by individual superimposed continua. Recently we presented [11], [12] a nonlinear theory in the case of two and three constituents. We utilized a mean continuum and a polarization vector \bar{u}_i .

The present paper is an extension of our works [11], [12]. Field equations, non-isothermal and isothermal constitutive equations for two-composite linear elastic material are developed. It is shown that under conditions of arbitrary temporal and spatial variation of the temperature field the stress tensor, the internal forces, the couple stress tensor and the entropy are derivable from a potential function.

Finally it is shown that the present results generalize some theorems of the classical micropolar theory.

2. Kinematics

We consider a collection of two materials which occupy the material volume V bounded by the surface S . From the mathematical point of view such two-composite material may be considered as two continua, coincident at an initial reference configuration V , provided that they interact with each other by internal forces p_i .

Let us consider at time t_0 a material point P in underformed body with uniform temperature. After the deformation, at time t , the material point P occupies po-

sition p while the temperature is in general no longer uniform. Denoted by x_j ($j=1, 2, 3$) and x_i ($i=1, 2, 3$) the coordinates of P and p in rectangular systems used to describe the undeformed and deformed states respectively, fig. 1.

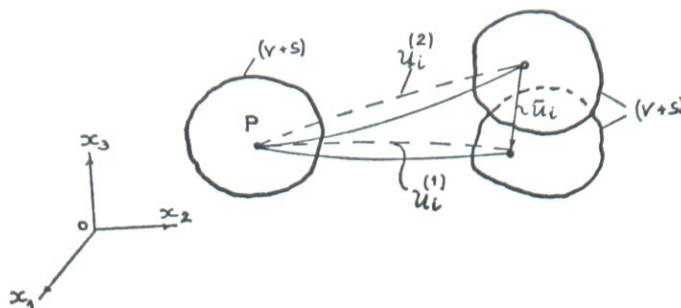


Fig. 1

The individual motion of each constituent is represented by

$$(1) \quad x_i^{(l)} = x_i^{(l)}(x_j^{(l)}, t) \quad (l=1, 2)$$

Displacement vectors of each constituent in a small deformation field are

$$(2) \quad u_i^{(l)} = x_i^{(l)}(x_j, t) \quad (l=1, 2)$$

Also the motion of two constituents may be described if we consider a mean continuum with a motion x_i and a polarization vector \bar{u}_i . The motion of the centre of masses of constituents and the polarization vector are determined by

$$(3) \quad x_i = \sum_l c^{(l)} x_i^{(l)}(x_j, t) \quad (l=1, 2)$$

$$(4) \quad \bar{u}_i = u_i^{(1)} - u_i^{(2)} = x_i^{(1)}(x_j, t) - x_i^{(2)}(x_j, t)$$

where $c^{(l)} = \rho^{(l)}/\rho$, $\rho = \sum_l \rho^{(l)}$; $\rho^{(l)}$ and ρ are the partial and total mass densities.

The strain and the polarization measures are

$$(5) \quad u_{i,j}^{(l)} = x_{i,j}^{(l)} - \bar{u}_i, \quad \bar{u}_{i,j} = x_{i,j}^{(1)} - x_{i,j}^{(2)}$$

where $x_{i,j}^{(l)} = \frac{\partial x_i^{(l)}}{\partial x_j}$.

Following [11], [12] we assume that mass transfer processes such as chemical reactions or phase changes are excluded and only particles which coincide before deformation are interacting. When the internal forces p_i are central then we have

$$(6) \quad \sum_l \rho^{(l)} p_i^{(l)} = 0, \quad \sum_l \rho^{(l)} e_{ijk} \bar{u}_j p_k^{(l)} = 0 \quad (l=1, 2)$$

where $p_i^{(l)}$ is the partial specific internal force and e_{ijk} are permutation symbols.

3. Field Equations

The Momentum Balance Equation. For each constituent the momentum balance equation is given by

$$(7) \quad \frac{\partial}{\partial t} \int_V \rho^{(l)} \dot{x}_i^{(l)} dV = \int_S t_{ji}^{(l)} dS_j + \int_V \rho^{(l)} (F_i^{(l)} + p_i^{(l)}) dV, \quad (l=1, 2)$$

where $t_{ji}^{(l)}$ is the partial stress tensor and $F_i^{(l)}$ is the partial specific body force.

Introducing eq. (3) into relation (7), summing up with respect to (l) and making use of Green's transformation theorem we get

$$(8) \quad \int_V \rho \ddot{x}_i dV = \int_V t_{ji,j} dV + \int_V \rho F_i dV$$

where $t_{ji} = \sum_l t_{ji}^{(l)}$, $F_i = \sum_l c^{(l)} F_i^{(l)}$ ($l=1, 2$) are the total stress tensor ($t_{ij} \neq t_{ji}$) and the total body force.

Equation (8) is the global form of the momentum balance equation. As this expression have to be valid for any arbitrary volume V we have locally

$$(9) \quad \rho \dot{v}_i = t_{ji,j} + \rho F_i$$

where $v_i = \dot{x}_i = \dot{u}_i$.

The Balance of Moment of Momentum Equation. We may write the balance of moment of momentum for each of the constituent as

$$(10) \quad \begin{aligned} \frac{\partial}{\partial t} \int_V \rho^{(l)} e_{mij} x_i^{(l)} \dot{x}_j^{(l)} dV &= \int_S e_{mij} x_i^{(l)} t_{kj}^{(l)} dS_k \\ &+ \int_V \rho^{(l)} e_{mij} x_i^{(l)} (F_j^{(l)} + p_j^{(l)}) dV \quad (l=1, 2) \end{aligned}$$

From eqs. (3),(4) and taking account **fig. 1** we obtain following system of equations

$$(11) \quad x_i^{(1)} = x_i + c^{(2)} \bar{u}_i, \quad x_i^{(2)} = x_i - c^{(1)} \bar{u}_i$$

Substituting relations (11) into eq. (10), summing up with respect to (l) and making use of Green's transformation theorem we get

$$(12) \quad \begin{aligned} \int_V \rho c^{(1)} c^{(2)} e_{mij} \bar{u}_i \ddot{\bar{u}}_j dV &= \\ &= \int_V e_{mij} x_{i,k} t_{kj} dV + \int_V M_{mk,m} dV + \int_V \rho \mathcal{L}_m dV. \end{aligned}$$

The local form of the balance of moment of momentum becomes

$$(13) \quad \rho c^{(1)} c^{(2)} e_{mij} \bar{u}_i \ddot{\bar{u}}_j = e_{mij} x_{i,k} t_{kj} + M_{mk,m} + \rho \mathcal{L}_m$$

where

$$(14) \quad \mathcal{M}_{mk} = e_{mij} \bar{u}_i (c^{(2)} t_{kj}^{(1)} - c^{(1)} t_{kj}^{(2)}) = e_{mij} \bar{u}_i g_{kj}$$

$$(15) \quad \mathcal{L}_m = c^{(1)} c^{(2)} e_{mij} \bar{u}_i (F_j^{(1)} - F_j^{(2)}) = e_{mij} \bar{u}_i P_j$$

are the total mechanical tensor and the total mechanical force which result from the internal polarization \bar{u}_i . The tensor g_{ij} may be considered as couple stress tensor ($g_{ij} \neq g_{ji}$).

The Energy Balance Equation. For each constituent the balance of energy is valid

$$(16) \quad \frac{\partial}{\partial t} \int_V \rho^{(l)} \left(\epsilon^{(l)} + \frac{1}{2} \dot{x}_i^{(l)} \dot{x}_i^{(l)} \right) dV = \\ = \int_V [\rho^{(l)} e^{(l)} + \rho^{(l)} (F_i^{(l)} + p_i^{(l)}) \dot{x}_i^{(l)}] dV + \int_S (t_{ji}^{(l)} \dot{x}_i^{(l)} - q_j^{(l)}) dS_j, \quad (l = 1, 2)$$

where $\epsilon^{(l)}$ is the partial specific internal energy; $e^{(l)}$ is the thermal energy interaction due to the presence of the other constituent; $q_j^{(l)}$ is the partial heat flux vector. Heat sources absence.

It must be observed that the partial specific internal energy is an unknown function.

Now let us give the conservation equation for the total energy. We assume that the time rate of change of the total energy is equal to the sum of the rate of work done upon the body by the external and the surface forces, the heat flux through the surface, i.e.

$$(17) \quad \frac{\partial}{\partial t} \int_V \rho E dV = \sum_{l=1}^2 \int_V \rho^{(l)} F_i^{(l)} \dot{x}_i^{(l)} dV + \sum_{l=1}^2 \int_S (t_{ji}^{(l)} \dot{x}_i^{(l)} - q_j^{(l)}) dS_j$$

Summing up eq. (16) with respect to (b) and introducing eq. (16) into eq. (17) we have locally

$$(18) \quad \rho \dot{E} = \sum_{l=1}^2 \rho^{(l)} (\dot{\epsilon}^{(l)} + \dot{x}_i^{(l)} \ddot{x}_i^{(l)}) - \sum_{l=1}^2 \rho^{(l)} (e^{(l)} + p_i^{(l)} \dot{x}_i^{(l)})$$

Using eq.(11), equation (18) may be written as

$$(19) \quad \dot{E} = \dot{K} + \dot{\epsilon}$$

where

$$(20) \quad \dot{K} = \dot{x}_i \ddot{x}_i + c^{(1)} c^{(2)} \dot{\bar{u}}_i \ddot{\bar{u}}_i$$

$$(21) \quad \dot{\epsilon} = \sum_{l=1}^2 c^{(l)} (\dot{\epsilon}^{(l)} - e^{(l)}) + c^{(2)} p_i^{(2)} \dot{\bar{u}}_i$$

are the time rate of change of the total specific kinetic energy and the time rate of change of the total specific internal energy.

It must be observed by eq. (19) that the time rate of change of the total specific internal energy $\dot{\epsilon}$ is a function of the time rate of change of the total specific energy \dot{E} and of the member $c^{(1)} c^{(2)} \dot{\bar{u}}_i \ddot{u}_i$.

To obtain \dot{E} we have introduced relations (11) into eq. (18), i.e

$$(22) \quad \frac{\partial}{\partial t} \int_V \rho E dV = \int_S (t_{ji} \dot{x}_i + g_{ji} \dot{\bar{u}}_i - q_i) dS_j + \int_V \rho (F_i \dot{x}_i + P_i \dot{\bar{u}}_i) dV.$$

Making use of Green's transformation in eq. (22), the local form of \dot{E} we get

$$(23) \quad \rho \dot{E} = t_{ji,j} \dot{x}_i + t_{ji} \dot{x}_{i,j} + g_{ji,j} \dot{\bar{u}}_i + g_{ji} \dot{\bar{u}}_{i,j} - q_{j,j} + \rho (F_i \dot{x}_i + P_i \dot{\bar{u}}_i)$$

To obtain the member $c^{(1)} c^{(2)} \dot{\bar{u}}_i \ddot{u}_i$ we have added and subtracted the local form of the eq. (7), i.e

$$(24) \quad \rho c^{(1)} c^{(2)} \ddot{u}_i = g_{ji,j} + \rho P_i + \rho c^{(1)} p_i^{(1)}.$$

Then we have multiplied eq. (24) with the vector $\dot{\bar{u}}_i$, i.e

$$(25) \quad \rho c^{(1)} c^{(2)} \dot{\bar{u}}_i \ddot{u}_i = g_{ji,j} \dot{\bar{u}}_i + \rho P_i \dot{\bar{u}}_i + \rho c^{(1)} p_i^{(1)} \dot{\bar{u}}_i.$$

Introducing eqs. (20), (23) and (25) into eq. (19), the local form of the balance of energy yields

$$(26) \quad \rho \dot{\epsilon} = t_{ji} \dot{x}_{i,j} + p_i \dot{\bar{u}}_i + g_{ji} \dot{\bar{u}}_{i,j} - q_{j,j}$$

where $p_i = \rho c^{(1)} p_i^{(1)}$ is the specific internal force.

In a small deformation field

$$v_{i,j} = \dot{x}_{i,j} = \dot{\bar{u}}_{i,j}.$$

Then the energy balance equation becomes

$$(27) \quad \rho \dot{\epsilon} = t_{ij} \dot{\bar{u}}_{j,i} + p_i \dot{\bar{u}}_i + g_{ij} \dot{\bar{u}}_{j,i} - q_{j,j}.$$

If we assume that the polarization vector \bar{u}_i absence, then the eq. (27) transforms into the classical energy equation.

The Entropy Production Inequality. Following [10] we assume that the present theory is applicable to bonded composite materials in which the constituents do not diffuse but interact only, at the interfaces. Then the Clausius-Duhem inequality for each constituent may be written

$$(28) \quad \frac{\partial}{\partial t} \int_V \rho^{(l)} \eta^{(l)} dV + \int_S q_i^{(l)} / \theta^{(l)} dS_i - \int_V \rho^{(l)} / \theta^{(l)} e^{(l)} dV \geq 0$$

where $\eta^{(l)}$ is the partial specific entropy and $\theta^{(l)}$ is the local absolute temperature.

Consider the case of polar thermal equilibrium, i.e

$$(29) \quad \theta^{(1)} = \theta^{(2)} = \theta$$

After summing up eq. (28) with respect to (l) we obtain

$$(30) \quad \frac{\partial}{\partial t} \int_V \rho \eta dV + \int_V \left(\frac{1}{\theta} q_i \right)_{,i} dV - \int_V \frac{1}{\theta} (\rho^{(1)} e^{(1)} + \rho^{(2)} e^{(2)}) dV \geq 0$$

where $\eta = c^{(1)} \eta^{(1)} + c^{(2)} \eta^{(2)}$ is the total specific entropy production.

In order that the Clausius-Duhem inequality be correct for the mean continuum it is necessary that

$$(31) \quad \rho^{(1)} e^{(1)} + \rho^{(2)} e^{(2)} = 0,$$

because the thermal energy interactions are not external supplies [8].

The local form of the entropy production inequality for the mean continuum becomes

$$(32) \quad \rho \dot{\eta} - q_i / \theta^2 \theta_{,i} + \frac{1}{\theta} q_{i,i} \geq 0$$

where $q_i = \sum q_i^{(l)}$, $(l = 1, 2)$

4. Non-Isothermal Constitutive Equations

In this section, linear theory appropriate for thermal and mechanical processes in fiber reinforced elastic composite is developed under the assumption that constituent displacement gradients $u_{i,j}^{(l)}$ relative displacements and velocity gradients $v_{i,j}^{(l)}$ are small, but the temperature perturbations are sensitive.

The theory we seek is to be applicable to perfectly bonded, simple thermoelastic materials which interact at the interfaces.

Therefore, we assume that the constitutive equations for the total stress tensor t_{ij} , specific internal forces p_i , the couple stress tensor g_{ij} , the total specific entropy η , specific free energy ψ and heat transfer vector q_i are identical to those for a simple thermoelastic continuum with the presence of geometrically independent polarization vector \bar{u}_i . The independent variables in constitutive equations for such a material include the displacement gradient of mean continuum $u_{j,i} = u_{ij}$, the polarization vector \bar{u}_i , the polarization gradient $\bar{u}_{j,i} = \gamma_{ij}$ the temperature θ and the temperature gradient $\theta_{,i}$. Consequently, we adopt the following constitutive functions

$$(33) \quad t_{ij} = t_{ij}(u_{ij}, \bar{u}_i, \gamma_{ij}, \theta; \theta_{,i}), \quad p_i = p_i(\dots), \\ g_{ij} = g_{ij}(\dots), \quad \varepsilon = \varepsilon(\dots), \quad \eta = \eta(\dots) \text{ and } q_i = q_i(\dots)$$

For the Helmholtz free energy ψ per unit mass defined by

$$(34) \quad \psi = \varepsilon - \eta \theta$$

we have

$$(35) \quad \psi = \psi(u_{ij}, \bar{u}_i, \gamma_{ij}, \theta; \theta_{,i})$$

Substituting eq. (27) and eq. (34) into eq. (32) and bearing in mind the eq. (35), we conclude

$$(36) \quad \left(t_{ij} - \rho \frac{\partial \psi}{\partial u_{ij}}\right) \dot{u}_{ij} + \left(p_i - \rho \frac{\partial \psi}{\partial u_i}\right) \dot{u}_i + \left(g_{ij} - \rho \frac{\partial \psi}{\partial \gamma_{ij}}\right) \dot{\gamma}_{ij} - \rho \left(\eta + \frac{\partial \psi}{\partial \theta}\right) \dot{\theta} - \rho \frac{\partial \psi}{\partial \theta_{,i}} \dot{\theta}_{,i} - \frac{q_i}{\theta} \dot{\theta}_{,i} \geq 0$$

This inequality is linear in $\dot{u}_{ij}, \dot{u}_i, \dot{\gamma}_{ij}, \dot{\theta}, \dot{\theta}_{,i}, \dot{\theta}_{,i}$ since according to eq. (33) $t_{ij}, p_i, g_{ij}, \epsilon, \eta$ and q_i are independent of these quantities. Moreover these variables can be varied independently. For all independent variations of $\dot{u}_{ij}, \dot{u}_i, \dot{\gamma}_{ij}, \dot{\theta}, \dot{\theta}_{,i}, \dot{\theta}_{,i}$ the inequality (36) cannot be maintained unless

$$(37) \quad t_{ij} = \rho \frac{\partial \psi}{\partial u_{ij}}, \quad p_i = \rho \frac{\partial \psi}{\partial u_i}, \quad g_{ij} = \rho \frac{\partial \psi}{\partial \gamma_{ij}} \\ \eta = -\frac{\partial \psi}{\partial \theta}, \quad \frac{\partial \psi}{\partial \theta_{,i}} = 0 \quad \text{and} \quad q_i \theta_{,i} \geq 0.$$

Equations (9), (13), (27) together with the constitutive eq. (37) form an incomplete set of 32 equations for 33 unknown functions

$$u_i, \bar{u}_i, t_{ij}, p_i, g_{ij}, \psi, \eta, \theta \text{ and } q_i$$

To obtain a restriction on the form of Helmholtz free energy function we have multiplied eq. (13) with vector u_i and eq. (24) with vector \bar{u}_i too, i.e

$$(38) \quad \rho c^{(1)} c^{(2)} \bar{u}_i \ddot{u}_i = x_{i,j} t_{ji} + \bar{u}_{i,j} g_{ji} + \bar{u}_i g_{ji,j} + \rho \bar{u}_i P_i$$

$$(39) \quad \rho c^{(1)} c^{(2)} \bar{u}_i \ddot{u}_i = g_{ji,j} \bar{u}_i + \rho P_i \bar{u}_i + p_i \bar{u}_i$$

From eqs. (38), (39) it follows

$$(40) \quad p_i \bar{u}_i = t_{ij} u_{ij} + g_{ij} \gamma_{ij}$$

The eq. (40) may be written as

$$(41) \quad p_i \bar{u}_i = t_{(ij)} u_{(ij)} + g_{(ij)} \gamma_{(ij)}$$

or

$$(42) \quad \frac{\partial \psi}{\partial u_i} \bar{u}_i = \frac{\partial \psi}{\partial d_{ij}} d_{ij} + \frac{\partial \psi}{\partial \gamma_{(ij)}} \gamma_{(ij)}$$

where d_{ij} is the infinitesimal strain tensor and $\gamma_{(ij)}$ is the symmetric part of the tensor γ_{ij} .

The eq. (42) is in addition to the constitutive equations (37).

5. Isothermal Constitutive Equations

In the case of isothermal conditions, equations (35) and (37) become

$$(43) \quad \psi = \psi(u_{ij}, \bar{u}_i, \gamma_{ij}; \theta_0)$$

$$(44) \quad t_{ij} = \rho \frac{\partial \psi}{\partial u_{ij}}, \quad p_i = \rho \frac{\partial \psi}{\partial u_i}, \quad g_{ij} = \rho \frac{\partial \psi}{\partial \gamma_{ij}}$$

We postulate now that the free energy ψ is a smooth function of the equilibrium state at which $u_{ij} = u_i = \gamma_{ij} = 0$. Then ψ may be expanded in a Taylor's series in the neighborhood of equilibrium state in u_{ij} , \bar{u}_i and γ_{ij} . The terms higher than the second order may be ignored and linear terms on u_{ij} , u_i and γ_{ij} are absent on the basis that the material is unstressed in the equilibrium state. Thus

$$(45) \quad \begin{aligned} \psi = & \frac{1}{2} a_{ijkl} u_{ij} u_{kl} + b_{ijkl} u_{ij} \gamma_{kl} + \frac{1}{2} c_{ijkl} \gamma_{ij} \gamma_{kl} \\ & + a_{ijk} u_{ij} \bar{u}_k + b_{ijk} \gamma_{ij} \bar{u}_k + \frac{1}{2} b_{ij} \bar{u}_i \bar{u}_j \end{aligned}$$

where the coefficients a_{ijkl} , b_{ijkl} , c_{ijkl} , a_{ijk} , b_{ijk} and b_{ij} are elastic constants.

The tensor u_{ij} is a true tensor; the tensors γ_{ij} and \bar{u}_i are pseudotensors.

Further simplification of eq. (45) may be obtained if we consider centrosymmetric two-composite linear elastic material (in such continuum all the material properties must be invariant with a central inversion and arbitrary rotation of coordinates). In this case the coefficients b_{ijkl} , b_{ijk} and a_{ijk} must be equal to zero ($\psi > 0$ and we consider with the true tensors and pseudotensors).

Hence for a centrosymmetric isotropic two-composite elastic material we get

$$(46) \quad t_{ij} = A_{ijkl} d_{kl}; \quad p_k = B_{km} \bar{u}_m; \quad g_{ij} = C_{ijkl} \gamma_{kl}$$

where the coefficients A_{ijkl} , B_{km} and C_{ijkl} are elastic constants and $t_{ij} = t_{ji}$, $g_{ij} = g_{ji}$.

6. Relation to the Micropolar Theory

If we introduce into eq. (13) the notations which are used in the micropolar theory then eq. (13) may be rewritten in a form of the equation of balance of moment of momentum (micropolar theory)

$$(i) \quad \frac{\partial}{\partial t} (j_0 \dot{\Omega}_i) = e_{mij} x_{i,k} t_{kj} + \mathcal{M}_{mk,m} + \rho \mathcal{L}_m$$

where

$$j_0 = \rho c^{(1)} c^{(2)} \bar{u}_i \bar{u}_i, \quad \dot{\Omega}_i = e_{ijk} n_j \dot{n}_k$$

$$n_i = \frac{\bar{u}_i}{|\bar{u}_i|}$$

j_0 is the microinertia and $\dot{\Omega}_i$ is the time rate of change of the microinertia of the director n_i .

If we assume $|\dot{\bar{u}}_i| = 0$ and express eq. (26) in terms of j_0 and $\dot{\Omega}_i$, then eq. (26) becomes

$$(ii) \quad \rho \dot{\varepsilon} = t_{ji} \dot{x}_{i,j} - e_{mij} x_{i,k} t_{kj} \dot{\Omega}_m + M_{mk} \dot{\Omega}_{m,k} - q_{j,j}$$

which is an other form of the micropolar energy equation.

References

- [1] Седов, Л. И., *Успехи матем. наук*, т. 20, вып. 5, Москва, 1965
- [2] Ивлев, Д. Д. Доклады, *АН СССР*, 176, 1037, 1967
- [3] Быковцев, Г. И., Нго Тхань Фонг, *Сборник мех. спл. ср. 80 л. ак. Н. И. Мухелишвили, изд. наука физмат-литер.*, Москва, 1972
- [4] Кунин, И. А., *Изд. наука физико-мат. лит.*, Москва, 1975
- [5] Болотин, В. В. *Механика полимеров*, 2, изд. Зинатне, Рига, 1965
- [6] Hashin, Z., Shtrikman, S. J., *Mech. Phys. Solids*, 2, 127, 1963
- [7] Hill, R. J., *Mech. Phys. Solids*, 2, 357, 1963
- [8] Truesdell, C., Toupin, R., *Encyclopedia of Physics* III/I., Springer, 1965
- [9] Green, A. E., Nachdi, P., *Quart. J. Mech. Appl. Math.*, vol. XXII. pt. 4, 1969
- [10] Bedford, A., Stern, M., *Acta Mech.* 14, Springer-Verlag, 1972
- [11] Brancov, G., Petrov, N., Ivanov, Z., Marinov, P., *Second Nat. Congr. Theor Appl. Mech. Varna*, October 1973. Publ. House of the Bulg. Acad. Sci. Sofia, 1975
- [12] Marinov, P., *Bulletin Société Royale des Sciences de Liège*, 43^e année, № 1—2, pp 106. 1974
- [13] Eringen, A. C., *Mech. Micromorphic Continua*, IUTAM Symposium. Freudenstadt Stuttgart. Germany 1967, Springer-Verlag, Berlin, 1968
- [14] Eringen, A. C., *Micropolar Elastic Solids with Stretch*, Ari Kitabevi Matbaasi, Istanbul, 1971
- [15] Hermann, G., Achenbach, J. D., *Applications of Theories of Generalized Cosserat Continua*, IUTAM Symposium, Freudenstadt-Stuttgart, Germany 1967, Springer-Verlag, Berlin. 1968

ХАРАКТЕРИЗАЦИЯ ДВУХКОМПОНЕНТНОГО ЛИНЕЙНОГО ЭЛАСТИЧНОГО МАТЕРИАЛА В МЕХАНИКЕ МИКРОКОНТИНУУМА

П. Маринов

Резюме

В работе рассматривается термоупругое поведение двухкомпонентного материала в поле инфинитезимальной деформации. Двухкомпонентный материал представлен введением средней сплошной среды и вектора поляризации. Выведены кинематические связи, полевые уравнения и не-изотермические и изотермические определяющие уравнения. Форма потенциальной функции определена с учетом материальных симметрий. Полученные результаты представляют вообщение классической микрополярной теории.

КАРАКТЕРИЗАЦИЈА ДВОКОМПОНЕНТНОГ ЛИНЕАРНОГ
ЕЛАСТИЧНОГ МАТЕРИЈАЛА У МЕХАНИЦИ
МИКРОКОНТИНУУМА

П. Маринов

Резиме

Разматра се термоеластично понашање двокомпонентног материјала у пољу инфинитезималне деформације. Двокомпонентни материјал представљен је увођењем средње деформабилне средине и вектора поларизације. Изведене су кинематичке везе и једначине. Потенцијална јункција одређена је при урачунавању материјалне симетрије. Добијени резултати представљају уопштења класичне микрополарне теорије.

Dr. P. Marinov. Bulgarian
Academy of Sciences, Inst. Math. Mech.,
acad. G. Bonchev str. bl. 8
Sofia 13, Bulgaria.