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THERMODIFFUSION IN MICROPOLAR ELASTIC PLATES

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The fact of the mutual interaction of separate thermodynamic phenomena has been emphasised recently. For example, Soret's effect is known, which describes the phenomena of diffusion under the influence of inhomogenous temperature field. Dufovi's effect is opposite. It is connected with the occurence of temperature flux in the body as the result of gradients of the volume concentration of atoms. Coupled effects are described by adding supplementary terms to phenomenological laws which describe irreversible processes in the form of proportionality of certain quantities.

The problem of theory of plates being of essential importance from the theoretical as well as from practical point of view, it is pretty much considered in literature and the results, in case of classical theory, are in accordance with the experimental ones. However, in many cases, especially dynamical ones connected with materials of grain structure, these results do not agree completely. Because of that the inner structure of material has become the subject of studying and the results are new theories which adequately describe such materials. The theory if micropolar materials is an example.

However, since the coupling effects have become the subject of many investigations in this paper we will try to apply the results of these investigations to plates, i.e. the results obtained in [1] which refer to thermodiffusion of elastic micropolar materials are used.

The problem of thermodiffusion in classical materials was considered in [2] and [3] the problem of thermodiffusion of classical plates in [4] while this work treats the micropolar plates.

Basic Equations

Equations of balance of linear momentum, and moment of momentum, can be written according to [1] in the form of

$$(1.1)\qquad \begin{aligned} &t_{kl,\,l}+\rho\,(f_k-\ddot{u}_k)=0\\ &m_{kl,\,l}+\varepsilon_{kij}\,t_{ij}+\rho\,(l_k-j\ddot{\varphi}_k)=0 \end{aligned}$$

where t_{kl}, ρ, f_k, u_k, m_{kl}, l_i j are known quantities.

Besides the equations motion we also have constitutive equations for stress and stress couples according to [1] in the form of

$$(1.2)\qquad \begin{aligned} &t_{kl}=(\lambda e_l+\gamma_T T+\gamma_C C)\,\delta_{kl}+2\,\mu e_{kl}+K\varepsilon_{klm}(r_m-\varphi_m)\\ &m_{kl}=\alpha\varphi_l\delta_{kl}+\beta\varphi_{k,\,l}+\gamma\varphi_{l,\,k} \end{aligned}$$

where λ, γ_T, γ_C, μ, k, α, β and γ are material constants.

Deformation tensors and microdeformation tensors are given in the form of

$$2\,e_{kl}=u_{k,\,l}+u_{l,\,k} \qquad \varepsilon_{kl}=u_{k,\,l}-\varepsilon_{klm}\,\varphi_m \tag{1.3}$$

Using the results in [1] we can write the equations of temperature field and concentration field in the form

$$\begin{aligned} AT,_{kk}+BC,_{kk}+De_{l,\,kk} &= -a_1\,\dot{T}-a_2\,\dot{C}-a_3\,\dot{e}_l-\rho\,h \\ A_1\,T,_{kk}+BC,_{kk}+De_{l,\,kk} &= a_4\,\dot{C} \end{aligned} \tag{1.4}$$

The equations (1.1) combined with the equations (1.2) and (1.3), together with the equations (1.4) give eight partial differential equations for determination of unknown functions, that is: state parameters U_k, Φ_k, T and C. State parameters have some initial values. If the system is taken out of the balance state by being exposed outside to some action of mechanical, thermal, or chemical influence the state parameters start to change from their initial values. The uniqueness theorem shows that the solution with the following initial and mixed boundary conditions

$$\begin{gathered} u_k\,(x,0)=u_{k0}\,(x) \qquad \dot{u}_k\,(x,0)=V_{k0}\,(x) \\ \varphi_k\,(x,0)=\Phi_{k0}\,(x) \qquad \dot{\varphi}_k\,(x,0)=\nu_{k0}\,(x) \\ t_{kl}\,n_l=t_{0k} \qquad m_{kl}\,n_l=m_{0l} \qquad S_L \\ u_\alpha=u_{\alpha 0} \qquad \varphi_\alpha=\varphi_{0\alpha} \qquad T=T_0 \qquad C=C_0 \qquad S-S_L \end{gathered} \tag{1.5}$$

in unique [5].

Equations of motion for micropolar plates

We shall limit our analysis to the theory of thin plates Let us consider one plate of thickness of $2h$ which has $x=0$ as the middle plane. In the given coordinate systems the upper and lower surface of the plate are defined with $x_3=h$ and $x_3=-h$. Let C be the boundary curve of the middle plane.

The theory of thin plates is constructed on the following assumptions [5].

a) the thickness of the plate $2h$ is small when compared with any characteristic lenght in the middle plane.

b) stress, displacement field, temperature and concentration do not change much across the thickness of the plate.

According to these assumptions we can use already known (averaged) values and the first moments of various quantities, by integration of the equations of motion, temperature field and concentration field with respect to x_3.

By integrating equations (1.1) and (1.4) with respect to x_3 from h to $-h$ we get:

$$\text{(1.6)}\qquad \begin{aligned} &\bar{t}_{\alpha\beta,\beta}+\frac{\tau_{\alpha 3}}{2h}+\rho(\bar{f}_\alpha-\ddot{\bar{u}}_\alpha)=0\\ &\bar{t}_{3\beta,\beta}+\frac{\tau_{33}}{2h}+\rho(\bar{f}_3-\ddot{\bar{u}}_3)=0\end{aligned}$$

$$\text{(1.7)}\qquad \begin{aligned} &\bar{m}_{\alpha\beta,\beta}+\varepsilon_{\alpha\beta}(\bar{t}_{\beta,3}-\bar{t}_{3,\beta})+\frac{\mu_{\alpha 3}}{2h}+\rho(\bar{l}_\alpha-j\ddot{\varphi}_\alpha)=0\\ &\bar{m}_{3\beta,\beta}+\varepsilon_{\alpha\beta}\bar{t}_{\alpha\beta}+\frac{\mu_{33}}{2h}+\rho(\bar{l}_3-j\ddot{\varphi}_3)=0\end{aligned}$$

$$\text{(1.8)}\qquad \begin{aligned} &A\bar{T}_{,\alpha\alpha}+B\bar{C}_{,\alpha\alpha}+D\bar{e}_{I,\alpha\alpha}+\frac{1}{2h}(Ar_T+Br_C)=-a_1\dot{\bar{T}}-a_2\dot{\bar{C}}-a_3\dot{\bar{e}}_I-\rho h\\ &A_1\bar{T}_{,\alpha\alpha}+B\bar{C}_{,\alpha\alpha}+D\bar{e}_{I,\alpha\alpha}+\frac{1}{2h}(A_1 r_T+Br_C)=a_4\dot{\bar{C}}\end{aligned}$$

where bared quantities are averaged across the thickness of the plate:

$$2h\,\bar{t}_{\alpha\beta}=\int_{-h}^{h}t_{\alpha\beta}\,dx_3\quad 2h\,\bar{m}_{\alpha\beta}=\int_{-h}^{h}m_{\alpha\beta}\,dx_3\quad 2h\,\bar{T}=\int_{-h}^{h}T\,dx_3\quad 2h\,\bar{C}=\int_{-h}^{h}C\,dx_3$$

where we use the following notation

$$\begin{aligned} &\tau_{\alpha 3}=t_{\alpha 3}(x_1,x_2,h)-t_{\alpha 3}(x_1,x_2,-h)\\ &\tau_{33}=t_{33}(x_1,x_2,h)-t_{33}(x_1,x_2,-h)\\ &\mu_{\alpha 3}=m_{\alpha 3}(x_1,x_2,h)-m_{\alpha 3}(x_1,x_2,-h)\\ &\mu_{33}=m_{33}(x_1,x_2,h)-m_3(x_1,x_2,-h)\end{aligned}\qquad r_T=\left.\frac{\partial T}{\partial x_3}\right|_{-h}^{+h}\qquad r_C=\left.\frac{\partial C}{\partial x_3}\right|_{-h}^{+h}$$

Equations (1.6, 1.7, 1.8) are supplemented by the equations of stress couples, temperature and concentration moments, which we get by multiplying equations (1.1 and 1.4) by x_3 and by integration from $x_3=h$ to $x_3=-h$. Thus we get:

$$\text{(1.9)}\qquad \begin{aligned} &M_{\alpha\beta,\beta}-2h\,\bar{t}_{\alpha 3}+h\tau_{\alpha 3}+\rho(L_\alpha-I\ddot{v}_\alpha)=0\\ &A\tilde{T}_{,\alpha\alpha}+B\tilde{C}_{,\alpha\alpha}+D\tilde{E}_{I,\alpha\alpha}+h(Ar_T+Br_C)-(S_T+S_C)=-a_1\dot{\tilde{T}}-a_2\dot{\tilde{C}}-a_3\dot{\tilde{E}}_1-\rho h\end{aligned}$$

$$\text{(1.10)}\qquad A_1\tilde{T}_{,\alpha\alpha}+B\tilde{C}_{,\alpha\alpha}+D\tilde{E}_{I,\alpha\alpha}+h(A_1 r_T+Br_C)-(S_T+S_C)=a_4\dot{\tilde{C}}$$

where $M_{\alpha\beta}$, L_α and v_α stress couples, moment couple and rotation angle are given with

$$M_{\alpha\beta}=\int_{-h}^{+h}t_{\alpha\beta}\,x_3\,dx_3\quad L_\alpha=\int_{-h}^{+h}f_\alpha\,x_3\,dx_3\quad Iv_\alpha=\int_{-h}^{+h}u_\alpha\,x_3\,dx_3$$

and I is surface moment of the normal section of the plate that has thickness $2h$ and unit length with respect to the middle plane. T and C are temperature and concentration moments given by

$$\tilde{T}=\int_{-h}^{+h} Tx_3\,dx_3 \quad \tilde{C}=\int_{-h}^{+h} Cx_3\,dx_3 \quad S_T=2\,hA\bar{T} \quad S_C=2\,hB\bar{C}$$

A group of equations (1.6, 1.7, 1.8) with equations (1.9, 1.10) give balance equation of micropolar plates in temperature and concentration field.

Displacement, rotation, temperatures, concentration

Basic assumptions used in the plate theory make it possible to decompose along x_3, direction displacement vector $\underset{\sim}{u}$, microrotation vector $\underset{\sim}{\varphi}$, temperature T and concentration C.

The usual way of decomposition takes into account the extension of the plate, which is done in the following way:

$$\begin{aligned}
&\underset{\sim}{u}=[u_\alpha(x_1,x_2,t)+x_3 v_\alpha(x_1,x_2,t)]\,\underset{\sim}{e}_\alpha+w(x_1,x_2,t)\,\underset{\sim}{e}_3\\
(1.10)\qquad &\underset{\sim}{\varphi}=\varphi_\alpha(x_1,x_2,t)\,e_\alpha+\varphi(x_1,x_2,t)\,\underset{\sim}{e}_3\\
&T=\bar{T}(x,x_2,t)+x_3\,\tilde{T}(x,x_2,t)\\
&C=\bar{C}(x_1,x_2,t)+x_3\,\tilde{C}(x_1,x_2,t)
\end{aligned}$$

from which it can be seen that u_k is the displacement field in the middle plane, v_k is twodimensional rotation field and w is transfersal displacement of the middle plane of the plate, φ_k is the microrotation field in the middle plane of the plate, φ is the component in the x_3 direction. $\bar{T}$ and $\bar{C}$ are temperature and concentration fields in the middle plane and $\tilde{T}$ and $\tilde{C}$ are temperature and concentration distribution across the thickness of the plate.

Using the group of equations (1.10) and substituting them in (1.3) we get:

$$\begin{aligned}
&e_{\alpha\beta}=\bar{e}_{\alpha\beta}+x_3\,\tilde{e}_{\alpha\beta} \quad 2\,\bar{e}_{\alpha\beta}=\bar{u}_{\alpha,\beta}+\bar{u}_{\beta,\alpha} \quad 2\,\tilde{e}_{\alpha\beta}=v_{\alpha,\beta}+v_{\beta,\alpha}\\
&2\,e_{\alpha3}=2\,e_{3\alpha}=v_\alpha+w,_\alpha \quad e_{33}=0\\
(1.11)\qquad &r_{\alpha\beta}=\bar{r}_{\alpha\beta}+x_3\,\tilde{r}_{\alpha\beta} \quad 2\,\bar{r}_{\alpha\beta}=\bar{u}_{\alpha,\beta}-\bar{u}_{\beta,\alpha} \quad 2\,\tilde{r}_{\alpha\beta}=v_{\alpha,\beta}-v_{\beta,\alpha}\\
&2\,r_{\alpha3}=-2\,r_{3\alpha}=v_\alpha-w,_\alpha \quad r_{33}=0
\end{aligned}$$

Substituting (1.13) in (1.2) we can write constitutive equations for stresses in terms of displacement, temperature and concentration in the form

$$\begin{aligned}
&t_{\alpha\beta}=t_{\alpha\beta}+x_3/I\,M_{\alpha\beta}\\
&t_{\alpha3}=\bar{t}_{3\alpha}=\mu\,(v_\alpha+w,_\alpha)+\frac{k}{2}(v_\alpha-w,_\alpha)+k\,\varepsilon_{\alpha\beta}\,\varphi_\beta\\
(1.12)\qquad &t_{3\alpha}=\bar{t}_{3\alpha}=\mu\,(v_\alpha+w,_\alpha)-\frac{k}{2}(v_\alpha-w,_\alpha)-k\,\varepsilon_{\alpha\beta}\,\varphi_\beta\\
&t_{33}=(\lambda\,\bar{u}_{\alpha,\alpha}+\gamma_T\,\bar{T}+\gamma_C\,\bar{C})\,\delta_{\alpha\beta}+x_3(\lambda\,v_{\alpha,\alpha}+\gamma_T\,\tilde{T}+\gamma_C\,\tilde{C})\,\delta_{\alpha\beta}
\end{aligned}$$

where $\bar{t}_{\alpha\beta}$ is the stress tensor in the middle plane and the stress couples in the middle plane $M_{\alpha\beta}$ are given as

$$\bar{t}_{\alpha\beta}=(\lambda\bar{u}_{r,r}+\gamma_T\bar{T}+\gamma_C\bar{C})\,\delta_{\alpha\beta}+\mu(\bar{u}_{\alpha,\beta}+\bar{u}_{\beta,\alpha})+\frac{k}{2}(\bar{u}_{\alpha,\beta}-\bar{u}_{\beta,\alpha})-k\,\varepsilon_{\alpha\beta}\varphi$$

(1.13)

$$\tilde{t}_{\alpha\beta}=\frac{M_{\alpha\beta}}{I}=(\lambda v_{r,r}+\gamma_T\tilde{T}+\gamma_C\tilde{C})\,\delta_{\alpha\beta}+\mu(v_{\alpha,\beta}+v_{\beta,\alpha})+\frac{k}{2}(v_{\alpha,\beta}-v_{\beta,\alpha})$$

We also get

(1.14)

$$m_{\alpha\beta}=\alpha\varphi_{r,r}\,\delta_{\alpha\beta}+\beta\varphi_{\alpha,\beta}+\gamma\varphi_{\beta,\alpha}$$

$$m_{\alpha 3}=\gamma\varphi_{,\alpha}\qquad m_{3\alpha}=\beta\varphi_{,\alpha}\qquad m_{33}=0$$

The two dimensional theory which uses $\bar{t}_{\alpha\beta}$ is called twodimensional stress state. In the plate theory however, the condition $t_{33}=0$ is usually used. From (1.2) we get the solution for e_{33} in the form of:

$$e_{33}=-\frac{\lambda\bar{e}_I+\gamma_T T+\gamma_C C}{\lambda+2\mu}$$

If instead of material constants λ and μ we introduce new constants i.e. E-Yung moduo, ν-Poisson ratio and G shear moduo of the classical elasticity theory, the constitutive equations (1.2) become:

(1.15)

$$\bar{t}_{\alpha\beta}=\frac{E}{1-\nu^2}\left[(\nu\,e_{rr}+(1+\nu)\,\alpha_T T+(1+\nu)\,\alpha_C C)\,\delta_{\alpha\beta}+(1-\nu)\,e_{\alpha\beta}\right]$$

$$+k\,e_{\alpha\beta\gamma}(r_\gamma-\varphi_\gamma)$$

$$t_{\alpha 3}=2\,Ge_{\alpha 3}+kr_{\alpha 3}+k\,\varepsilon_{\alpha\beta}\varphi_\beta\qquad t_{3\alpha}=2\,Ge_{3\alpha}+kr_{3\alpha}-k\,\varepsilon_{\alpha\beta}\varphi_\beta$$

where

$$\lambda=\frac{\nu E}{(1+\nu)(1-2\nu)}\qquad 2\mu=2G=\frac{E}{1+\nu}$$

Substituting the equations (1.11) into (1.15) we get constitutive equations for stresses $\bar{t}_{\alpha\beta}$ in the middle plane and stress couples for the case $t_{33}=0$.

(1.16)

$$\bar{t}_{\alpha\beta}=\frac{E}{1-\nu^2}\left[(\nu\bar{u}_{r,r}+(1+\nu)\,\alpha_T\bar{T}+(1+\nu)\,\alpha_C\bar{C})\,\delta_{\alpha\beta}+\frac{1-\nu}{2}(\bar{u}_{\alpha,\beta}+\bar{u}_{\beta,\alpha})\right.$$

$$\left.+\frac{k}{2}(\bar{u}_{\alpha,\beta}-\bar{u}_{\beta,\alpha}-k\,\varepsilon_{\alpha\beta}\varphi)\right.$$

$$M_{\alpha\beta}=\frac{EI}{1-\nu^2}\left[(\nu\,v_{r,r}+(1+\nu)\,\alpha_T\tilde{T}+(1+\nu)\,\alpha_C\tilde{C})\,\delta_{\alpha\beta}+\frac{1-\nu}{2}(v_{\alpha,\beta}+v_{\beta,\alpha})+\right.$$

$$+\frac{kI}{2}(v_{\alpha,\beta}-v_{\beta,\alpha})$$

In that way we can get field equations of extension and bending of plate using either equations (1.13) or equations (1.16).

Field equations

Partial differential equations that describe fields of displacement, microdisplacement, temperature and concentration are obtained by substituting constitutive equations (1.12) and (1.14) in motion equations (1.6), (1.7) and (1.8).

The system of so obtained equations is naturally grouped into two sets of equations. One set of equations describes symmetric distribution of stresses, temperature and concentration across the middle plane $x_3=0$, and the other represents nonsymmetric distribution. The first set refers to the extension of the plane and the other to bending.

1. We require that the state of deformation be $e_{33}=0$

In this case using (1.12) and (1.14) we get a set of equations for twodimensional problem of thermodifusion of micropolar plates for extension of the middle plane

$$\left(\lambda+\mu-\frac{k}{2}\right)\bar{u}_{\gamma,\gamma\alpha}+\left(\mu+\frac{k}{2}\right)\bar{u}_{\alpha,\beta\beta}+\gamma_T\bar{T},_{\alpha}+\gamma_C\bar{C},_{\alpha}-k\,\varepsilon_{\alpha\beta}\varphi,_{\beta}+$$

$$+\frac{\tau_{\alpha 3}}{2h}-\rho(\bar{f}_\alpha-\ddot{\bar{u}}_\alpha)=0$$

$$(1.17)\quad \beta\varphi,_{\beta\beta}+k\,\varepsilon_{\alpha\beta}\bar{u}_{\alpha,\beta}-2k\varphi+\frac{\mu_{33}}{2h}+\rho(\bar{l}_3-j\ddot{\varphi})=0$$

$$A\bar{T},_{\alpha\alpha}+B\bar{C},_{\alpha\alpha}+\bar{D}e_{I,\alpha\alpha}+\frac{1}{2h}(Ar_T+Br_C)=-a_1\dot{\bar{T}}-a_2\dot{C}-a_3\dot{\bar{u}}_{\alpha,\alpha}-\rho\bar{h}$$

$$A_1\bar{T},_{\alpha\alpha}+BC,_{\alpha\alpha}+\bar{D}e_I,_{\alpha\alpha}+\frac{1}{2h}(A_1r_T+Br_C)=a_4\dot{\bar{C}}$$

and for bending

$$I\left(\lambda+\mu-\frac{k}{2}\right)v_{\beta,\alpha\beta}+I\left(\mu+\frac{k}{2}\right)v_{\alpha,\beta\beta}+I(\gamma_T\bar{T},_\alpha+\gamma_C\bar{C},_\alpha)-2h\left(\mu+\frac{k}{2}\right)v_\alpha+$$

$$+\left(2h-\frac{k}{2}\right)w,_\alpha-2hk\,\varepsilon_{\alpha\beta}\varphi_\beta+h\,\tau_{\alpha 3}-\rho(L_\alpha-I\ddot{v}_\alpha)=0$$

$$\left(\mu-\frac{k}{2}\right)v_{\alpha,\alpha}+\left(\mu+\frac{k}{2}\right)w,_{\alpha\alpha}+k\,\varepsilon_{\alpha\beta}\varphi_{\alpha,\beta}+\frac{\tau_{33}}{2h}+\rho(f_3-\ddot{w})=0$$

$$(1.18)\quad (\alpha+\beta)\varphi_{\beta,\alpha\beta}+\gamma\varphi_{\alpha,\beta\beta}+k\,\varepsilon_{\alpha\beta}(v_\beta-w,_\beta)+2k\varphi_\alpha+\frac{\mu_{\alpha 3}}{2h}+\rho(\bar{l}_\alpha-j\ddot{\varphi}_\alpha)=0$$

$$A\tilde{T},_{\alpha\alpha}+B\tilde{C},_{\alpha\alpha}+\tilde{D}e_I,_{\alpha\alpha}+h(Ar_T+Br_C)-(S_T+S_C)=$$

$$=-a_1\dot{\tilde{T}}-a_2\dot{\tilde{C}}-a_3\dot{\tilde{u}}_{\alpha,\alpha}-\rho h$$

$$A_1\tilde{T},_{\alpha\alpha}+B\tilde{C},_{\alpha\alpha}+D\tilde{e}_{I,\alpha\alpha}+h(A_1r_T+Br_C)-(S_T+S_C)=a_4\dot{\tilde{C}}$$

From the set of equations (1.17) we can determine the displacement field $\bar{u}_\alpha$, microrotation field φ, temperature field $\bar{T}$ and the concentration field $\bar{C}$ of the middle plane. The set of equations (1.18) is used to determine the unknown v_α, φ_α, w, $\tilde{T}$ and $\tilde{C}$. However, since the boundary conditions are not coupled the problem of extension of thermo-diffusive plates can be considered separately from bending.

2. For plane stress state $t_{33}=0$

In this case the equations of motion (1.6, 1.7, 1.8) substitute equations (1.16) and we again get a similar set of equations for extension and for bending of plate.

The equations for extension are:

$$\frac{1}{2}\left(\frac{E}{1-\nu}-k\right)\bar{u}_{\beta,\alpha\beta}+\frac{1}{2}\left(\frac{E}{1+\nu}+k\right)\bar{u}_{\alpha,\beta\beta}+\frac{E}{1-\nu}(\alpha_T\bar{T},_\alpha+\alpha_C\bar{C},_\alpha)-$$

$$-k\,\varepsilon_{\alpha\beta}\varphi_\beta+\frac{\tau_{\alpha 3}}{2h}+\rho(\bar{f}_\alpha-\ddot{\bar{u}}_\alpha)=0$$

$$\beta\varphi,_{\beta\beta}+k\,\varepsilon_{\alpha\beta}\bar{u}_\alpha,_\beta-2k\varphi+\frac{\mu_{33}}{2h}+\rho(l_3-j\ddot{\varphi})=0 \tag{1.19}$$

$$A\bar{T},_{\alpha\alpha}+B\bar{C},_{\alpha\alpha}+D\bar{e}_I,_{\alpha\alpha}+\frac{1}{2h}(Ar_T+Br_C)=-a_1\dot{\bar{T}}-a_2\dot{\bar{C}}-a_3\dot{\bar{u}}_\alpha,_\alpha-\rho h$$

$$A_1\bar{T},_{\alpha\alpha}+B\bar{C},_{\alpha\alpha}+D\bar{e}_I,_{\alpha\alpha}+\frac{1}{2h}(A_1r_T+Br_C)=a_4\dot{\bar{C}}$$

and for bending

$$\frac{I}{2}\left(\frac{E}{1-\nu}-k\right)v_\beta,_{\alpha\beta}+\frac{I}{2}\left(\frac{E}{1+\nu}+k\right)v_\alpha,_{\beta\beta}+\frac{EI}{1-\nu}(\alpha_T\dot{\tilde{T}},_\alpha+\alpha_C\tilde{C},_\alpha)-$$

$$-2h\left(G+\frac{k}{2}\right)v_\alpha-2h\left(G-\frac{k}{2}\right)w,_\alpha-2hk\,\varepsilon_{\alpha\beta}\varphi_\beta+h\,\tau_{\alpha 3}+\rho(L_\alpha-I\ddot{v}_\alpha)=0$$

$$\left(G+\frac{k}{2}\right)v_\alpha,_\alpha+\left(G-\frac{k}{2}\right)w,_{\alpha\alpha}+k\,\varepsilon_{\alpha\beta}\varphi_\alpha,_\beta+\frac{\tau_{33}}{2h}+\rho(f_3-\ddot{w})=0 \tag{1.20}$$

$$(\alpha+\beta)\varphi_\beta,_{\alpha\beta}+\gamma\varphi_\alpha,_{\beta\beta}+k\,\varepsilon_{\alpha\beta}(v_\beta-w,_\beta)+2k\varphi_\alpha+\frac{\mu_{\alpha 3}}{2h}+\rho(l_\alpha-\rho\ddot{\varphi}_\alpha)=0$$

$$A\tilde{T},_{\alpha\alpha}+B\tilde{C},_{\alpha\alpha}+D\tilde{e}_I,_{\alpha\alpha}+h(Ar_T+Br_C)-(S_T+S_C)=-a_1\dot{\tilde{T}}-a_2\dot{\tilde{C}}-a_3\dot{\tilde{v}}_\alpha,_\alpha-\rho\tilde{h}$$

$$A_1\tilde{T},_{\alpha\alpha}+B\tilde{C},_{\alpha\alpha}+D\tilde{e}_I,_{\alpha\alpha}+h(A_1r_T+Br_C)-(S_T+S_C)=a_4\dot{\tilde{C}}$$

The boundary conditions we had in (1.5) are reduced to the following set of equations by the process of averaging

$$\bar{t}_{\alpha\beta}n_\beta=\bar{t}_\alpha \quad \bar{m}_{\alpha 3}n_\alpha=\bar{m}_3 \qquad C_L$$

$$\bar{u}_\alpha=\bar{u}_{0\alpha} \quad \varphi=\varphi_0 \quad \bar{T}=\bar{T}_0 \quad \bar{C}=\bar{C}_0 \qquad C-C_L$$

and for bending

$$\overline{M}_{\alpha\beta} n_\alpha = \overline{M}_\beta \qquad \overline{t}_{\alpha 3} n_\alpha = \overline{t}_3 \qquad m_{\alpha\beta} n_\alpha = m_\beta \qquad C_L$$

$$v_\alpha = v_{0\alpha} \qquad w = w_0 \qquad \varphi_\alpha = \varphi_{0\alpha} \qquad \tilde{T} = \tilde{T}_0 \qquad \tilde{C} = \tilde{C}_0$$

where the quantities $\overline{t}_\alpha$, m_3, $u_{0\alpha}$, Φ_0, M_β, $\overline{t}_3$, m_3, $v_{0\beta}$, w_0, $\Phi_{0\beta}$, $\overline{T}_0$, $\overline{C}_0$, $\tilde{T}_0$ and $\tilde{C}_0$ are given on the parts C_1 and $C—C_1$ of the contour C of the middle plane of the plate.

In the same way from (1.5) we get initial conditions for the extension of the middle plane:

$$\overline{u}_\alpha (x_1, x_2, 0) = \overline{u}_{0\alpha} (x_1, x_2) \qquad \varphi (x_1, x_2, 0) = \Phi_0 (x_1, x_2)$$

$$\dot{u}_\alpha (x_1, x_2, 0) = V_{0\alpha} (x_1, x_2) \qquad \dot{\varphi} (x_1, x_2, 0) = v_0 (x_1, x_2)$$

$$\overline{T} (x_1, x_2, 0) = T_0 (x_1, x_2) \qquad \overline{C} (x_1, x_2, 0) = \overline{C}_0 (x_1, x_2)$$

and for bending

$$v_\alpha (x_1, x_2, 0) = \hat{v}_\alpha (x_1, x_2) \qquad w (x_1, x_2, 0) = \hat{w} (x_1, x_2)$$

$$\varphi_\alpha (x_1, x_2, 0) = \hat{\varphi}_\alpha (x_1, x_2) \qquad \dot{v}_\alpha (x_1, x_2, 0) = \hat{\dot{v}}_\alpha (x_1, x_2)$$

$$\dot{w} (x_1, x_2, 0) = \hat{\dot{w}} (x_1, x_2) \qquad \dot{\varphi}_\alpha (x_1, x_2, 0) = \hat{\dot{\varphi}}_\alpha (x_1, x_2)$$

$$\tilde{T} (x_1, x_2, 0) = \tilde{T} (x_1, x_2) \qquad \tilde{C} (x_1, x_2, 0) = \tilde{C} (x_1, x_2)$$

The functions appearing on the right side of these equations are given functions on the surfaces of the middle plane. From this it can be seen that it is possible to treat the problems of extension and bending separately, because the initial and boundary conditions are independent.

REFERENCES

[1] M. Plavšić and N. Naerlović; *Thermodiffusion in micropolar elastic materials*, Archives of Mechanics 28, 4, Warszawa, 1976

[2] W. Nowacki; *Certain problems of thermodiffusion in solids*, Arch. Mech. Stosowanej, 23, 6, Warszawa, 1971

[3] J. S. Podstrigač; *Diferencijalni rawnania zadaczi termodyfuzii w twierdomu deformowanemu izotropnomu tili*, D. A. N. USSR, 2, 1961

[4] R. Čukić; *Diffusion in Thermoelastic Plates*, xii Jugoslovenski kongres racionalne i primenjene mehanike

[5] A. C. Eringen; *Theory of Micropolar Plates*, J. App. Math., 18, 1967.

THERMODIFUSION IN DEN MIKROPOLARISCHEN PLATTEN

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Zusammenfassung

Von den Grundgleichungen des Gleichgewichtes der mikropolarischen, thermodiffusionischen Materialen ausgehend und die konstitutiven Gleichungen für die Spannungen und Zusammenspannungen der gleichen Materialen benutzend, wurden die Gleichungen des Feldes für den Gleichzustand (Geradezustand) der Deformationen und für den Gleichzustand der Spannungen der mikropolarischen, thermodifusionischen Platten abgeleitet.

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М. Вукобрат:

ТЕРМОДИФУЗИЈА У МИКРОПОЛАРНИМ ЕЛАСТИЧНИМ ПЛОЧАМА

Користећи основне једначине баланса за микрополарне материале, конститутивне једначине за напоне и напонске спрегове, једначине поља температуре и поља концентрације као и предпоставке које се уводе за танке плоче, изводе се једначине напрезања и савијања у случају равног стања деформације и равног стања напона за микрополарне термодифузионе плоче.