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THERMOELASTIC THEORY OF MICROPOLAR PLATES

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Introduction

The plate problem is important from theoretical as well as practical point of view. By studying plate Theory within the framework of classical Elasticity Theory, one gets results that are close to the experimental observations. However, in lot of problems, especially dynamical problems of granular materials, the Classical Theory of Elasticity gives results that are not close to the experimental observations. For example, it is well known fact that the plane infinitesimal waves propagate in elastic materials with dispersion. Classical continuum theory does not give dispersion, because it does not take into account motion of the microstructure of material. Thus, in the case of dunbell plates rotation of dunbell moleculs gives rise to the new type of wave that does not exist in classical Theory.

Lately continuum mechanics developed new theories in which motion of the inner structure of the material is taken into account. Few different models were developed that describe behavior of such materials.

One of this theories is Theory of micropolar continuum developed by Eringen and Suhubi [1]. In the Theory developed in [1] influence of the temperature was not taken into account. This was done in later works.

Thermodynamical Theory of micropolar continuum with microstructure was developed by Grot [2], who assumed that microelements have different temperatures. In other words, he assumed that there exists field of temperature (macrotemperature) in macrocontinuum, and the field of microtemperature in microcontinuum. Thus, he introduces notion of microtemperature as a continuous function of microcoordinates of microelements i.e., of the coordinates of a particle inside macroelement that we call microelement.

Microtemperature was taken as a known function of macrotemperature and the gradient of the macrotemperature, by Vožnjak. However, Vožnjak did not give balance law for the determination of microtemperature as did Grot [2].

Using the results of the theory of micropolar continuum Eringen [1] gave the theory of micropolar plates [3], without taking into account influence of the temperature. Thus applicability of the theory developed in [3] was greatly reduced.

In this work we will consider micropolar plate theory taking into account influence of the temperature and microtemperature. Our method is similar to the one used in [3]. We will also made assumptions about the thickness of the plate and the distribution of stresses and deformations across the thickness, which will enable us to use average values of different quantities across the plate thickness.

Validity of different kind of approximations depends on the ratio between the thickness of the plate and other dimensions. Thus, one can in general, consider; thin plates with small deflection, thin plates with large deflection, thick plates ect. In this work we will consider only thin plates that in the deformed state have deflection small when compared to the thickness of the plate. For such a case we can use approximate theories of bending with the transversal loads.

1. Basic equations

According to the results given in [3] balance equations for micropolar elasticity are

$$\underset{\sim}{t}_{\alpha,\alpha}+\rho(\underset{\sim}{f}-\underset{\sim}{\ddot{u}})=0 \tag{1.1}$$

$$\underset{\sim}{m}_{\alpha,\alpha}+\underset{\sim}{e}_\alpha\times\underset{\sim}{t}_\alpha+\rho(\underset{\sim}{l}-j\ddot{\varphi})=0 \tag{1.2}$$

where $\underset{\sim}{t}_\alpha$, ρ, $\underset{\sim}{f}$, $\underset{\sim}{u}$, $\underset{\sim}{m}$, l_α and j are known quantities and $\underset{\sim}{e}_\alpha$ are basis vectors. Using usual notation we assume that Greek indicies are summed from 1 to 3 and Latin indicies from 1 to 2.

Stress tensor $t_{\alpha\beta}$ and couple $m_{\alpha\beta}$, are defined as

$$\underset{\sim}{t}_\alpha=t_{\alpha\beta}\underset{\sim}{e}_\beta \quad \underset{\sim}{m}_\alpha=m_{\alpha\beta}\underset{\sim}{e}_\beta \qquad \alpha,\ \beta=1,\ 2,\ 3. \tag{1.3}$$

In linear theory of micropolar elasticity together with (1.1) and (1.2) we need also constitutive equations one for stress tensor and one for couples. They are derived in [4] in the form

$$t_{\alpha\beta}=(\lambda e_{r,r}-\lambda_1 T)\delta_{\alpha\beta}+2(\mu+k)e_{\alpha\beta}-k\varepsilon_{\alpha\beta} \tag{1.4}$$

$$m_{\alpha\beta}=\alpha_1\varepsilon_{\alpha\beta\gamma}\theta_\gamma+\alpha\varphi_{r,r}\delta_{\alpha\beta}+\beta\varphi_{\alpha,\beta}+\gamma\varphi_{\beta,\gamma} \tag{1.5}$$

where $\lambda, \mu, \alpha, k, \beta, \alpha_1$, and γ are elastic constants.

Deformation tensor and microdeformation tensor are given in the form [4].

$$2e_{\alpha\beta}=u_{\alpha,\beta}+u_{\beta,\alpha} \tag{1.6}$$

$$\varepsilon_{\alpha\beta}=u_{\alpha\ \beta}+\varepsilon_{\alpha\beta\gamma}\varphi_\gamma \tag{1.7}$$

and the balance equations for the temperature and microtemperature are

$$C\dot{T}+\lambda_1\dot{e}_{rr}=k_0 T_{,\alpha\alpha}+k_1\theta_{\alpha,\alpha}+\rho_0\mathcal{H} \tag{1.8}$$

$$C_1\dot{\theta}_\alpha+\alpha_1\varepsilon_{\alpha\beta\gamma}\dot{\varphi}_{\gamma,\beta}=(k_4-k_5)\theta_{\beta,\alpha\beta}+k_6\theta_{\alpha,\beta\beta}-k_3T_{,\alpha}-k_2\theta_\alpha-\rho_0\mathcal{H} \tag{1.9}$$

Equations (1.1) and (1.2) when combined with (1.4) and (1.5) as well as with (1.8) and (1.9) give ten partial differential equations for the determination of ten unknown functions.

By the uniqueness theorem, it can be shown that solution to that system with the following mixed boundary conditions is unique [3]

$$t_{\alpha\beta}\underset{\sim}{n}_\alpha=\underset{\sim}{t}_{0\beta} \quad m_{\alpha\beta}\underset{\sim}{n}_\alpha=\underset{\sim}{m}_{0\beta} \quad S_L \tag{1.10}$$

$$u_\alpha=u_{0\alpha} \quad \varphi_\alpha=\varphi_{0\alpha}$$

(1.11) $$T = T_0 \quad \theta_\alpha = \theta_{0\alpha} \quad S - S_L$$

$$\underset{\sim}{u}(x, 0) = u_0(x) \quad \varphi(x, 0) = \underset{\sim}{\Phi}_0(x)$$

(1.12) $$\dot{\underset{\sim}{u}}(x, 0) = V_0(x) \quad \dot{\varphi}(x, 0) = \nu_0(x)$$

$$T(x, 0) = T_0(x) \quad \underset{\sim}{\theta}(x, 0) = \underset{\sim}{\theta}_0(x)$$

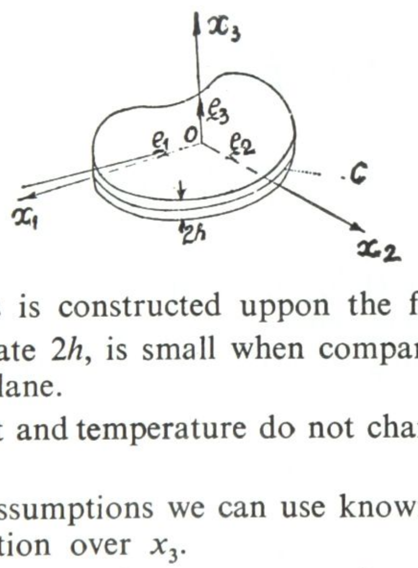
where $\underset{\sim}{t}_0$ and $\underset{\sim}{m}_0$ are given on the part of the boundary S_L of the body $V+S$, and $\underset{\sim}{u}_{0\alpha}$, $\varphi_{0\alpha}$, $\tilde{T}_0$ and $\theta_{0\alpha}$ are given on the rest of the boundary $S—S_L$.

Initial conditions (1.12) are given for the volume V. Now, we are in the position to develop theory of micropolar plates with the influence of temperature and micro-temperature.

Equations of motion for micropolar plates

As noted earlier we will limit our analysis to the case of thin plates. Let us consider a thin plate of the thickness $2h$ that has $x=0$ as a middle plane.

Upper and lower surface in the given coordinate system are $x_3=h$ and $x_3=—h$. Let C be boundary line of the middle plane i.e. a curve gotten from the intersection of the middle plane with the cylinder whose axis is x_3.



Theory of thin plates is constructed uppon the following assumptions [3]:

a) thickness of the plate $2h$, is small when compared with any characteristic dimension in the middle plane.

b) stress, displacement and temperature do not change much across the thickness of the plate.

According to above assumptions we can use known (averaged) values of different variables by integration over x_3.

Integrating (1.1) and (1.2) with respect to x_3 from $x_3=—h$ to $x_3=h$ we get

(1.13) $$\overline{\underset{\sim}{t}}_{k,k} + \frac{\underset{\sim}{\tau}}{2h} + \rho\,(\overline{\underset{\sim}{f}} - \ddot{\overline{\underset{\sim}{u}}}) = 0$$

(1.14) $$\overline{\underset{\sim}{m}}_{k,k} + \underset{\sim}{e}_k \times \underset{\sim}{t}_k + \underset{\sim}{e}_3 \times \underset{\sim}{t}_3 + \frac{\underset{\sim}{\mu}}{2k} + \rho\,(\underset{\sim}{l} - j\,\ddot{\mathscr{L}}) = 0$$

by the same method (1.8) and (1.9) become

(1.15) $$C\dot{\overline{T}} + \lambda_1 \dot{\overline{e}} = k_0\,\overline{T}_{,kk} + k_1\,\overline{\theta}_{k,k} + \frac{r}{2h} + \rho_0\,\overline{\mathscr{H}}$$

(1.16) $$C_1\,\dot{\overline{\theta}}_k + \varepsilon_{kl}\,\dot{\overline{\varphi}}_{,l} = (k_4 + k_5)\,\overline{\theta}_{l,kl} + k_0\,\overline{\theta}_{k,ll} - k_3\,\overline{T} - k_2\,\overline{\theta}_k - \rho_0\,\overline{\mathscr{H}_k}$$

where bared quntities are averaged across the plate thickness.

Introducting notation

$$\text{(1.17)}\qquad \begin{aligned} \underset{\sim}{\tau} &= \underset{\sim}{t}_3(x_1, x_2, h) - \underset{\sim}{t}_3(x_1, x_2, -h) = \tau_k \underset{\sim}{e}_k + p\underset{\sim}{e}_3 \\ \underset{\sim}{\mu} &= \underset{\sim}{m}_3(x_1, x_2, h) - \underset{\sim}{m}_3(x_1, x_2, -h) = \mu_k \underset{\sim}{e}_k + m\underset{\sim}{e}_3 \end{aligned}$$

and using (1.3) we have,

$$\text{(1.18)}\qquad \begin{aligned} \underset{\sim}{t}_k &= t_{kl}\underset{\sim}{e}_l + t_{k3}\underset{\sim}{e}_3 \quad \underset{\sim}{t}_3 = t_{3l}\underset{\sim}{e}_l + t_{33}\underset{\sim}{e}_3 \\ \underset{\sim}{m}_k &= m_{kl}\underset{\sim}{e}_l + m_{k3}\underset{\sim}{e}_3 \quad \underset{\sim}{m}_3 = m_{3l}\underset{\sim}{e}_l + m_{33}\underset{\sim}{e}_3 \end{aligned}$$

substituting (1.17) and (1.18) into (1.1) and (1.2) we get equations of motion in component form

$$\text{(1.19)}\qquad \begin{aligned} \overline{t}_{kl,k} + \frac{\tau_l}{2h} + (\overline{f}_l - \ddot{\overline{u}}_l) &= 0 \\ \overline{t}_{k3,k} + \frac{p}{2h} + \rho(\overline{f}_3 - \ddot{\overline{w}}) &= 0 \end{aligned}$$

$$\text{(1.20)}\qquad \begin{aligned} \overline{m}_{kl,k} + \varepsilon_{kl}(\overline{t}_{3k} - \overline{t}_{k3}) + \frac{\mu_l}{2h} + (\overline{l}_l - j\ddot{\overline{\varphi}}_l) &= 0 \\ \overline{m}_{k3,k} + \varepsilon_{kl}\overline{t}_{kl} + \frac{m}{2h} + \rho(l_3 - j\ddot{\varphi}_3) &= 0 \end{aligned}$$

Equations (1.19) and (1.20) are supplemented with the equations of stress couples and temperature moments, that we get by cross multiplication of (1.1) and (1.8) with $x_3 e_3$ and integration from $x_3 = h$ to $x_3 = -h$

Thus we get,

$$\text{(1.21)}\qquad M_{kl,k} - 2h\overline{t}_{3l} + h\tau_l + \rho(L_l - I\ddot{v}_l) = 0$$

$$\text{(1.22)}\qquad C\dot{\tilde{T}} + \lambda_1 \dot{\tilde{e}} = k_0 \tilde{T}_{,kk} + hr - S + \rho\mathscr{H}$$

where M_{k1} are components of the stress couple, L_l moment of the couple, v_l rotation angle, I surface moment of the normal section of the plate that has thickness $2h$ and unit length, $\tilde{T}$ distribution of temperature across the thickness of the plate and S averaged temperature.

Displacement, rotation, temperature and the constitutive equations

Basic assumptions of the plate theory give the possibility to decompose along x_3 direction displacement vector u, microrotation vector $\underset{\sim}{\varphi}$, temperature T and microtemperature $\underset{\sim}{\theta}$. Usual way of decomposition includes into consideration extension of the middle plane, as well as bending, and usually takes the form

$$(1.23)\qquad \begin{aligned} &\underset{\sim}{u}=[\bar{u}_k(x_1, x_2, t)+x_3 v_k(x_1, x_2, t)]\underset{\sim}{e}_k+w(x_1, x_2, t)\underset{\sim}{e}_3 \\ &\underset{\sim}{\varphi}=\varphi_k(x_1, x_2, t)\underset{\sim}{e}_k+\varphi(x_1, x_2, t)\underset{\sim}{e}_3 \end{aligned}$$

$$(1.24)\qquad \begin{aligned} &T=\bar{T}(x_1, x_2, t)+x_3\tilde{T}(x_1, x_2, t) \\ &\underset{\sim}{\theta}=\theta_k(x_1, x_2, t)\underset{\sim}{e}_k+\theta(x_1, x_2, t)\underset{\sim}{e}_3 \end{aligned}$$

It is obvious from (1.23) that u_k is twodimensional displacement field in the middle plane, v_k is twodimensional rotation field and W is transversal displacement of the middle plane of the plate.

Microrotational vector $\underset{\sim}{\varphi}$ is with (1.23_2) decomposed on microrotational vector in the middle plane φ_k and the component in the x_3 direction.

Temperature is decomposed into temperature of the middle plane $\bar{T}$ (1.24) and temperature distribution across the thickness of the plate $\tilde{T}$. Similarly as microrotation vector, vector of the microtemperature (1.24_2) is decomposed on the microtemperature in the middle plane θ_k and in the direction of x_3.

Substituting equations (1.23) and (1.24) into equations (1.6) and (1.7) and substituting this result again in (1.4) and (1.5) we get constitutive equations for stresses in terms of the displacement, temperature and microrotation in the form

$$(1.25)\qquad \begin{aligned} &t_{kl}=\bar{t}_{kl}+\frac{x_3}{I}M_{kl} \qquad t_{k3}=\bar{t}_{k3}=(\mu+k)(v_k+w,_k)-k(v_k-\varepsilon_{kl}\varphi) \\ &t_{3k}=\bar{t}_{3k}=(\mu+k)(v_k+w,_k)-k(w,_k+\varepsilon_{kl}\varphi) \\ &t_{33}=(\lambda\bar{u}_{rr}-\lambda_1\bar{T})\delta_{kl}+x_3(\lambda v_{rr}-\lambda_1\tilde{T})\delta_{kl} \end{aligned}$$

where the stress tensor in the middle plane $\bar{t}_{kl}$ and stress couples in the middle plane M_{kl} are given as

$$(1.26)\qquad \begin{aligned} &\bar{t}_{kl}=(\lambda\bar{u}_{rr}-\lambda_1\bar{T})\delta_{kl}+(\mu+k)(\bar{u}_{k,l}+\bar{u}_{l,k})-k(\bar{u}_{k,l}+\varepsilon_{kl}\varphi) \\ &M_{kl}/I=(\lambda v_{rr}-\lambda_1\tilde{T})\delta_{kl}+(\mu+k)(v_{k,l}+v_{l,k})-kv_{k,l} \end{aligned}$$

stress couples are expressed through microtemperature and microrotations as

$$(1.27)\qquad \begin{aligned} &m_{kl}=\alpha_1\varepsilon_{kl3}\theta+\alpha\varphi_{rr}\delta_{kl}+\beta\varphi_{k,l}+\gamma\varphi_{l,k} \\ &m_{k3}=\alpha_1\varepsilon_{kl3}\theta_l+\gamma\varphi,_k \\ &m_{3k}=\alpha_1\varepsilon_{kl3}\theta_l+\beta\varphi,_k \qquad m_{33}=0 \end{aligned}$$

Two dimensional theory that uses $\bar{t}_{kl}$ given as (1.26_1) is called twodimensional stress state. In the plate theory it is usually used condition $t_{33}=0$. Using that condition for determination of t from (1.4) and substituting into (1.26) we get constitutive equations of the plate theory for the stress in the middle plane t_{kl} and bending moment M_{kl}.

$$t_{kl}=\frac{E}{1-\nu^2}\left[(\nu\,\bar{u}_{r,r}-(1+\nu)\,\alpha_t\,\bar{T})\,\delta_{kl}+\right.$$

(1.28)
$$\left.+\frac{1-\nu}{2}\,(\bar{u}_{k,l}+\bar{u}_{l,k})\right]-\frac{k}{2}\,(\bar{u}_{k,l}-\bar{u}_{l,k}+2\,\varepsilon_{kl}\,\varphi)$$

$$M_{kl}=\frac{EI}{1-\nu^2}\left[(\nu\,v_{r,r}-(1+\nu)\,\alpha_t\,\tilde{T})\,\delta_{kl}+\frac{1-\nu}{2}\,(v_{k,l}+v_{l,k})\right]-\frac{kI}{2}\,(v_{k,l}+v_{l,k}).$$

Field equations-boundary and initial conditions

Partial differential equations that describe fields of displacement, microdisplacement, temperature and microtemperature we can get by substituting constitutive equations (1.25) and (1.22) into balance equations (1.1—1.2) and (1.8—1.9).

System of so gotten equations is grouped into two sets, one that represents symmetric distribution of stresses across the middle plane $x_3=0$ and one that represent nonsymmetric distribution. These two sets of equations describe two dimensional extension of the middle plane as well as its bending.

a) Case $e_{33}=0$. For this case using (1.25) and (1.27) we get a system of equations for twodimensional problem of micropolar elasticity with the influence of the temerature and microtemperature.

From this set of equations, we can determine displacement field u_k microotation field φ, and temperature field $\bar{T}$ and microtemperature field θ_k in the middle plane, as well as fields of rotation v_k, microrotation φ_k transversal displacement w, temperature $\tilde{T}$ and microtemperature θ.

Since boundary conditions are not coupled, the problems of extension and bending of micropolar plate could be considered separately.

b) Case $t_{33}=0$. Substituting equation (1.28) in the equations (1.20—1.22) and by using (1.15) we get a set of equations for extension and bending of the plate.

The equations for extension are

$$\frac{1}{2}\left(\frac{E}{1-\nu}-k\right)\bar{u}_{k,lk}+\frac{1}{2}\left(\frac{E}{1-\nu}+k\right)\bar{u}_{l,kk}-$$

$$-\frac{E\,\alpha_t}{1-\nu}\,\bar{T}_{,l}-k\,\varepsilon_{kl}\,\varphi_{,k}+\frac{\tau_l}{2h}+\rho\,(f_l-\ddot{\bar{u}}_l)=0$$

(1.29)
$$\gamma\varphi_{,kk}-2\,k\,\varphi+\alpha_1\,\varepsilon_{kl}\,\theta_{l,k}+k\,\varepsilon_{kl}\,\bar{u}_{k,l}+\rho\,(l_3-j\,\ddot{\varphi})=0$$

$$C\dot{\bar{T}}+\lambda_1\,\dot{\bar{e}}=k_0\,\bar{T}_{,kk}+k_1\,\theta_{k,k}+\frac{r}{2h}+\rho_0\,\overline{\mathcal{H}}$$

$$C_1\,\dot{\theta}_k+\alpha_1\,\varepsilon_{kl}\,\dot{\varphi}_{,l}=(k_4+k_5)\,\theta_{l,kl}+k_6\,\theta_{k,ll}-k_3\,\bar{T}_{,k}-k_2\,\theta_k-\rho_0\,\overline{\mathcal{H}}$$

and for bending

$$\frac{I}{2}\left(\frac{E}{1-\nu}-k\right)v_{k,lk}+2kh\,\varepsilon_{lk}\varphi_k+\frac{I}{2}\left(\frac{E}{1+\nu}+k\right)v_{l,kk}-$$

$$-\frac{IE\alpha_t}{1-\nu}\tilde{T}_{,l}-2h\left(G-\frac{k}{2}\right)w_{,l}-2h\left(G+\frac{k}{2}\right)v_l+h\tau_l+\rho(L_l-I\ddot{v}_l)=0;$$

$$\left(G-\frac{k}{2}\right)v_{k,k}+\left(G+\frac{k}{2}\right)w_{,kk}+k\,\varepsilon_{kl}\varphi_{l,k}+\frac{p}{2h}+\rho(f_3-\ddot{w})=0$$

(1.30)

$$(\alpha+\beta)\varphi_{k,lk}+\gamma\varphi_{l,kk}+\alpha_1\varepsilon_{kl}\theta_{,k}+k\,\varepsilon_{kl}(v_k-w_{,k})-2k\varphi_l+\frac{\mu_l}{2h}+\rho(l_l-j\ddot{\varphi}_l)=0$$

$$C_1\dot{\tilde{T}}+\lambda_1\dot{\tilde{e}}=k_0\tilde{T}_{,kk}+hr-S+\rho_0\bar{\varkappa}\cdot$$

$$\cdot C_1\dot{\theta}+\alpha_1\varepsilon_{kl}\dot{\varphi}_{l,k}=k_6\theta_{k,ll}-k_3\tilde{T}-k_2\theta-\rho_0\mathcal{H}$$

The set of boundary conditions, that we had (1.10, 1.11), by the process of averaging is reduced to the following: — for extension

$$\overline{t}_{kl}n_k=\overline{t}_l \quad m_{k3}n_k=\overline{m}_3 \qquad C_L$$

(1.31)
$$\overline{u}_k=\overline{u}_{0k} \qquad \varphi=\varphi_0$$

$$T=T_0 \quad \theta_k=\theta_{0k} \qquad C-C_2$$

— for bending

$$M_{kl}n_k=\overline{M}_l \quad \overline{t}_{k3}n_k=\overline{t}_3 \quad m_{kl}n_k=m_l$$

(1.32)
$$v_k=v_{0k} \quad w=w_0 \quad \varphi_k=\varphi_{0k} \quad \tilde{T}=\tilde{T} \quad \theta=\theta_0 \qquad C_L$$

The quantities $\overline{t}_l$, $\overline{m}_2$, $\overline{u}_{0k}$, φ_0, $\overline{M}_l$, $\overline{t}_3$, $\overline{m}_l$, v_{0k}, w_0 and φ_{0k} are given on the parts C_L and $C—C_L$ of the contour C of the middle plane.

Initial conditions that follow from (1.12) have the form: for extension

$$\overline{u}_k(x_1,x_2,0)=u_{0k}(x_1,x_2) \quad \varphi(x_1,x_2,0)=\Phi_0(x_1,x_2)$$

(1.33)
$$\dot{\overline{u}}_k(x_1,x_2,0)=V_{0k}(x_1,x_2) \quad \dot{\varphi}(x_1,x_2,0)=\nu_0(x_1,x_2)$$

$$\overline{T}(x_1,x_2,0)=\overline{T}_0(x_1,x_2) \quad \overline{\theta}_k(x_1,x_2)=\theta_{0k}(x_1,x_2)$$

for bending

$$v_k(x_1,x_2,0)=\tilde{V}_k(x_1,x_2) \quad w(x_1,x_2,0)=\tilde{w}_0(x_1,x_2)$$

$$\varphi_k(x_1,x_2,0)=\tilde{\varphi}_{0k}(x_1,x_2) \quad \dot{v}_k(x_1,x_2,0)=\dot{\tilde{v}}_{k0}(x_1,x_2)$$

(1.34)

$$\dot{w}(x_1,x_2,0)=\dot{\tilde{w}}_0(x_1,x_2) \quad \dot{\varphi}_k(x_1,x_2,0)=\dot{\varphi}_k(x_1,x_2)$$

$$\tilde{T}(x_1,x_2,0)=\tilde{T}_0(x_1,x_2) \quad \theta(x_1,x_2,0)=\theta_0(x,x_2)$$

Functions that appear on the right hand side of above equations are given functions on the surface S of the middle plane. From this we conclude that it is possible to treat problems of extension and bending separately ,because boundary conditions are separated.

The Theory of bending

Displacement and microrotation of thin micropolar plate subjected to the temperature and microtemperature field is determined by the equations (1.30) with $t_{33}=0$. In fact we have seven functions v_k, w, and φ_k and the values $\tilde{T}$ and θ. It is possible to get (by differentiation) uncoupled equations for every function. However, this equations are too complicated for the analytical work, therefore we decompose fields of the variables considered into the "gradient" and "rot" as

$$v_k = v_{,k} + \varepsilon_{kl} V_{,l} \quad \varphi_k = \varphi_{,k} + \varepsilon_{kl} \Phi_{,l} \tag{1.35}$$

where v, V, φ, Φ are functions of x_1, x_2 and t.

Substituting (1.35) into (1.30) and taking "div" and "rot" operation we get set of seven mutually coupled equations [4] that could be reduced to

$$D\nabla^4 W + \frac{4kGh}{G+\frac{k}{2}}\nabla^2 W + \rho\left(\frac{D}{G+\frac{k}{2}} + I\right)\nabla^2 \ddot{W} - \frac{\rho^2 I}{G+\frac{k}{2}}\ddot{\ddot{W}} - 2\rho h \ddot{W}$$

$$-\frac{G-\frac{k}{2}}{G+\frac{k}{2}} D(1+\nu)\alpha_t \nabla^2 \tilde{T} + \frac{kD}{G+\frac{k}{2}}\nabla^2\Phi - \frac{4kGh}{G+\frac{k}{2}}\nabla^2\Phi - \frac{k\rho I}{G+\frac{k}{2}}\nabla^2\ddot{\Phi}$$

$$-\frac{D}{2h\left(G+\frac{k}{2}\right)}\nabla^2 P + \frac{\rho I}{2h\left(G+\frac{k}{2}\right)}\ddot{P} + \ddot{P} + \frac{G+\frac{k}{2}}{G+\frac{k}{2}} F_{k,k} = 0$$

$$\gamma\nabla^4\Phi - \frac{2kG}{G-\frac{k}{2}}\nabla^2\Phi - \rho j \nabla^2\ddot{\Phi} + \frac{2kG}{G-\frac{k}{2}}\nabla^2 W - \frac{k\rho}{G-\frac{k}{2}}\ddot{W} - \alpha_1\nabla^2\theta$$

$$+\frac{k}{2h\left(G-\frac{k}{2}\right)} P + \varepsilon_{kl}\mathfrak{m}_{k,l} = 0$$

$$k\nabla^2 v = \gamma\nabla^4\Phi - 2k\nabla^2\Phi - \rho j\nabla^2\ddot{\Phi} + k\nabla^2 W - \alpha_1\nabla\theta + \varepsilon_{lk}\mathfrak{m}_{l,k}$$

$$k\nabla^2 V = -(\alpha+\beta+\gamma)\nabla^4\varphi + 2k\nabla^2\varphi + \rho j\nabla^2\ddot{\varphi} - \mathfrak{m}_{k,k}$$

$$-I(\alpha+\beta+\varphi)\nabla^6\varphi + 2[kI + h(\alpha+\beta+\gamma)]\nabla^2\ddot{\varphi} - \frac{\rho^2 jI}{G+\frac{k}{2}}\nabla^2\ddot{\ddot{\varphi}} - I\nabla^2\mathfrak{m}_{k,k} +$$

(1.36)

$$+2h\,\mathfrak{m}_{k,k} + \frac{\rho I}{G+\frac{k}{2}}\mathfrak{m}_{k,k} + \frac{k}{G+\frac{k}{2}}\varepsilon_{kl}F_{k,l} = 0$$

$$C\dot{\tilde{T}} + \Lambda_1\frac{G+\frac{k}{2}}{G-\frac{k}{2}}\nabla^2 W + \frac{k\Lambda_1}{G-\frac{k}{2}}\nabla^2\Phi + \frac{\rho\Lambda_1}{G-\frac{k}{2}}\ddot{W} - \frac{\Lambda_1 P}{2h\left(G-\frac{k}{2}\right)} =$$

$$= k_0\nabla^2\tilde{T} + hr - S + \rho_0\tilde{\mathcal{H}}$$

$$C_1\tilde{\theta} - \alpha_1\nabla_2\Phi = k_6\nabla^2\theta - k_3\tilde{T} - k_2\theta - \rho_1\tilde{\mathcal{H}}$$

Set of equations (1.35) we use for determination of v, V, φ, Φ, W, $\tilde{T}$ and θ. From the equation (1.36) we conclude that φ could be determined separately while for all other quantities we have to solve the system of coupled equations.

Thus, we conclude that this theory is coupled theory of mechanical and thermal effects. Set of equations (1.36) forms a complete set of equations and if we neglect effects of temperature it reduces to the micropolar theory of Eringen.

The plate theory presented here is complete. However, field equations (1.36) as well as boundary conditions (1.32) and (1.34) are rather complicated so that for analytical work one has to use some simplifications. Using ideas from the classical thin plate theory one can get different types of approximation.

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DIE THERMOELASTISCHE THEORIE DER MIKROPOLARISCHEN PLATTEN

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Zusammenfassung

In der Arbeit werden die thermoelastischen, dünnen Platten mit der Mikrostruktur betrachtet, dessen jedes Mikroelement verschiedene Temperatur hat, die uns den Begriff der Mikrotemperatur karakterisiert. Die Gleichungen des Gleichgewichts der mikropolarischen Platten und die Gleichungen der Temperaturfelder und der Mikrotemperatur nützend, werden dei Gleichungen der Anstrengungen und der Biegungen solcher Platten abgeleitet. Die Gleichungen werden in der Form geschrieben, dier zur annähernden Vereinfachung geeignet sind, mit der Hinsicht, dass sie in der Grundform sehr kompliziert und ungeeignet zur Anwendung sind.

ТЕРМОЕЛАСТИЧНА ТЕОРИЈА МИКРОПОЛАРНИХ ПЛОЧА

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Резиме

У раду се разматрајv термоелестичне танке плоче са микроструктуром чији сваки микроелемент има различиту температуру коју нам карактерише појам микротемпературе. Користећи једначине равнотеже микрополарних плоча и једначине поља температуре и микротемпературе изводе се једначине напрезања и савијања таквих плоча. Једначине се пишу у облику који је погодан за апроксимативна упрошћења с обзиром да су у основном облику доста компликоване и неподесне за примену.