TEORIJSKA I PRIMENJENA MEHANIKA 3, pp. 101—111, 1977.

MICROPOLAR ELASTIC DIELECTRICS

Milan Plavšić, *Slavko Đurić* and *Milan Gligorić*

1. Introduction

The first scientific writings dealing with the interaction between an electromagnetic field and an elastic dielectric referred to the classical model of a dielectric (with symmetric stress tensor), based on the assumption of infinitesimal deformations. Toupin was the first to formulate a nonlinear theory for the classical model of an elastic dielectric, both within the scope of electrostatics [1] and for a moving dielectric [2]. In his papers, Toupin introduced the first gradients of deformation and the polarization vector as the parameters of state. Furthemore, Mindlin [3, 4] introduced the polarization gradient as an independent variable of state within the linear theory of the classical model of elastic dielectric. Suhubi [5] also studied the classical model of elastic dielectric, taking in account of the polarization gradient as well.

The dynamic theory of the so-called polar elastic dielectric was formulated by Dixon and Eringen [6, 7]. In addition to the first and second gradients of deformation and the electric moment of dipole, they also introduced a quadratic electric moment as the state variable. Assuming the dependence of the specific internal energy of deformation and polarization on the first and second gradients of deformation, polarization vector and polarization vector gradient, Bauchert [8] studied the so-called dipolar elastic dielectric in the electromagnetic field. A dipolar continuum model was obtained as a special case of the multipolar continuum model, formulated by Green and Rivlin [9]. Bauchert's study has been generalized by a model of elastic dielectric with microstructure in Ref. [10] and [11].

The model of a micropolar elastic material is considered in this paper as a dielectric in an electromagnetic field. A mechanical model of micropolar continuum was introduced by Suhubi and Eringen [12], while Eringen alone developed linear theories of micropolar materials in a series of papers [13, 14, 15]. Later on, Kafadar and Eringen [16] formulated a nonlinear theory of micropolar elastic materials, and the same model was treated as an elastic Cosserat continuum in Ref. [17].

2. Kinematics of a micropolar continuum

If a micropolar continuum is considered as a simple Cosserat continuum [17], its motion is defined by the equations:

$$x^k = x^k (X^K, t), \quad d^k_{\cdot(\alpha)} = d^k_{\cdot(\alpha)} \left(X^K, D^K_{\cdot(\alpha)}, t\right), \tag{2.1}$$

where X^K are material coordinates, while x^k are spatial coordinates, and $D^K_{\cdot(\alpha)}$ and $d^k_{\cdot(\alpha)}$ $(\alpha=1, 2, 3)$ are triads of the orientation vectors (directors) in the undeformed and deformed configuration.

For simple materials we have

$$d^k_{\cdot(\alpha)}=\chi^k_{\cdot K}\,D^K_{\cdot(\alpha)}, \tag{2.2}$$

and the equations of motion therefore, may be written in the form

$$x^k=x^k(X^K, t), \quad \chi^k_{\cdot K}=\chi^k_{\cdot K}(X^L, t), \tag{2.3}$$

where $\chi^k_{\cdot K}$ is the orthogonal tensor determining the independent rigid rotations of the directors. Consequently,

$$|\chi^k_{\cdot K}|=+\sqrt{\frac{G}{g}}, \quad (G=|G_{KL}|,\ g=|g_{kl}|), \tag{2.4}$$

where G_{KL} and g_{kl} are metric tensor coordinates related to the system of material and spatial coordinates, respectively.

Since the director motion is a rigid one, we have

$$g_{kl}\,d^k_{\cdot(\alpha)}\,d^l_{\cdot(\beta)}=G_{KL}\,D^K_{\cdot(\alpha)}\,D^L_{\cdot(\beta)}=\text{const.}, \tag{2.5}$$

or rather, considering (2.2),

$$g_{kl}\,\chi^k_{\cdot K}\chi^l_{\cdot L}=G_{KL}, \tag{2.6}$$

which is a condition of orthogonality for the tensor $\chi^k_{\cdot K}$.

The velocity of an arbitrary point of the macroelement may be represented in the form

$$v'^k=v^k+\nu^k_{\cdot l}\,d^l, \tag{2.7}$$

where v^k is velocity of the center of the macroelement mass, d^l vector of the position of the point in the macroelement relatively to the macroelement center of mass, and

$$\nu_{kl}=\dot d_{k(\alpha)}\,d^{(\alpha)}_{\cdot l}=\dot\chi_{kK}\,\chi^K_{\cdot l} \tag{2.8}$$

is the gyration tensor. By differentiating the expression $(2.5)_2$ with respect to time, and using the expression (2.2), we obtain:

$$\nu_{kl}=\dot\chi_{kK}\,\chi^K_{\cdot l}=-\dot\chi_{lK}\,\chi^K_{\cdot k}=-\nu_{lk}. \tag{2.9}$$

It follows from this that ν_{kl} is a skew-symmetric tensor, i.e. it has three mutually independent coordinates. Tensors $\chi^k_{\cdot K}$ and $\chi^K_{\cdot k}$, as well as triads $d^k_{\cdot(\alpha)}$ and $d^{(\alpha)}_{\cdot k}$, that is $D^K_{\cdot(\alpha)}$ and $D^{(\alpha)}_{\cdot K}$, are mutually reciprocal.

3. Energy balance equation of deformation and polarization and differential equations of motion

Let us assume that the micropolar elastic dielectric is in an electromagnetic field subjected to the action of mechanical and electromagnetic forces, so that the equation of total energy balance takes the form

$$\int_v \rho\,(\dot{v}^k v_k + \Gamma^{ij}\nu_{ij} + \Gamma^i \dot{p}_i)\,dv + \int_v \rho\,\dot{w}\,dv =$$

(3.1)

$$= \int_v (\rho f^i v_i + F^i v_i + \varepsilon^i \dot{p}_i + \rho\, l^{ij}\nu_{ij})\,dv + \oint_s (T^i v_i + S^i v_i + M^{ij}\nu_{ij})\,ds.$$

In this equation:

$$\Gamma^{ij} = -\Gamma^{ji} = I^{\alpha\beta}\,\ddot{d}^{[i}_{\cdot(\alpha)} d^{j]}_{\cdot(\beta)} = I^{KL}\,\ddot{\chi}^{[i}_{\cdot K}\chi^{j]}_{\cdot L} \tag{3.2}$$

is a mechanical inertial spin, where $I^{\alpha\beta} = I^{\beta\alpha}$ are director coefficients of inertia, and $I^{KL} = I^{LK}$ are macroelement inertia coefficients with respect to its center of mass, while

$$\Gamma^i = \nu\,\ddot{p}^i \tag{3.3}$$

is the inertial spin of polarization, p^i being the polarization vector coordinates, and ν the polarization inertia coefficient. The other quantities in Eq. (3.1) are as follows:

f^i — mechanical body force per unit mass,
l^{ij} — mechanical body couples per unit mass,
T^i — mechanical surface force per unit area,
M^{ij} — mechanical surface couples per unit area,
w — specific internal energy deformation and polarization per unit mass,
F^i — electromagnetic force per unit volume, determined by the expression [8]

$$F^i = s^{ik}_{\ \ ,k} - \frac{1}{c}\,\dot{g}^i, \tag{3.4}$$

where c is the velocity of light, and

$$s^{ik} = \varepsilon^i p^k + E^i E^k + B^i B^k - \frac{1}{2}\,(E^l E_l + B^l B_l)\,g^{ik} \tag{3.5}$$

is the electromagnetic stress tensor, with E^i — electric field and B^i — magnetic induction, while

$$g^i = \varepsilon^{ijk} E_j B_k, \quad (\vec{g} = \vec{E} \times \vec{B}), \tag{3.6}$$

is electromagnetic impulse,

ε^i — dynamic electric field, determined by the expression [8]

$$\varepsilon^i = E^i + \frac{1}{c}\,\varepsilon^{ijk} v_j B_k, \tag{3.7}$$

where E^i is electric field at rest;

S^i — electromagnetic surface force, which is on the outer surface of the dielectric defined by the relation [8]

$$S^i = \left[\left| s^{ik} + \frac{1}{c} g^i v^k \right|\right] n_k, \tag{3.8}$$

where n_k is the outward unit normal vector of the dielectric boundary surface.

An identical transformation can be used to present Eq. (3.1) in the form

$$\begin{aligned}
&\int_v \rho\,(\dot{v}^k v_k + \Gamma^{ij} \dot{\nu}_{ij} + \Gamma^i \dot{p}_i)\, dv + \int_v \rho\, \dot{w}\, dv = \\
&= \int_v (\rho f^i v_i + F^i v_i + \varepsilon^i \dot{p}_i + \rho\, l^{ij} \nu_{ij})\, dv + \oint_s [(T^i ds - t^{ik} ds_k) v_i + \\
&+ (M^{ij} ds - m^{ijk} ds_k) \nu_{ij} + S^i v_i\, ds - r^{ik} \dot{p}_i\, ds_k] + \\
&+ \oint_s (t^{ik} v_i\, ds_k + m^{ijk} \nu_{ij}\, ds_k + r^{ik} \dot{p}_i\, ds_k),
\end{aligned} \tag{3.9}$$

where t^{ik} is nonsymmetric stress tensor, m^{ijk} — couple stress tensor, and r^{ik} — local electric tensor of the field. If the last surface integral from the preceding equation is transformed into a volume integral, we obtain

$$\begin{aligned}
&\int_v \rho\,(\dot{v}^k v_k + \Gamma^{ij} \dot{\nu}_{ij} + \Gamma^i \dot{p}_i)\, dv + \int_v \rho\, \dot{w}\, dv = \\
&= \int_v \left[\rho f^i v_i + \rho\, l^{ij} \nu_{ij} + \left(s^{ik}_{\ ,k} - \frac{1}{c} \dot{g}^i\right) v_i + \varepsilon^i \dot{p}_i\right] dv + \oint_s \left[(T^i ds - t^{ik} ds_k) v_i + \right. \\
&+ (M^{ij} ds - m^{ijk} ds_k) \nu_{ij} + \left(\left[\left| s^{ik} + \frac{1}{c} g^i v^k \right|\right]\right) v_i ds_k - \left. r^{ik} \dot{p}_i\, ds_k \right] + \\
&+ \int_v (t^{ik}_{\ ,k} v_i + t^{ik} v_{i,k} + m^{ijk}_{\ ,k} \nu_{ij} + m^{ijk} \nu_{ij,k} + r^{ik}_{\ ,k} \dot{p}_i + r^{ik} \dot{p}_{i,k})\, dv,
\end{aligned} \tag{3.10}$$

where use has been made of expressions (3.4) and (3.8).

By applying Eq. (3.10) to an elementary tetrahedron and allowing its volume to tend towards zero, retaining the orientations of its sides, we obtain boundary conditions on the dielectric's outer surface:

$$\begin{aligned}
T^i &= t^{ik} n_k - \left[\left| s^{ik} + \frac{1}{c} g^i v^k \right|\right] n_k, \\
M^{ij} &= m^{ijk} n_k, \\
0 &= r^{ik} n_k,
\end{aligned} \tag{3.11}$$

and in case these conditions are satisfied, Eq. (3.10) can be reduced to the form

$$\int_v \rho(\dot{v}^k v_k + \Gamma^{ij}\nu_{ij} + \Gamma^i \dot{p}_i)\,dv + \int_v \rho\,\dot{w}\,dv =$$

(3.12)
$$= \int_v \left[\rho f^i v_i + \rho\, l^{ij}\nu_{ij} + \left(s^{ik}_{\ ,k} - \frac{1}{c}\dot{g}^i\right)v_i + \varepsilon^i \dot{p}_i\right] dv +$$

$$+ \int_v \left(t^{ik}_{\ ,k} v_i + t^{ik} v_{i,k} + m^{ijk}_{\ ,k}\nu_{ij} + m^{ijk}\nu_{ij,k} + r^{ik}_{\ ,k}\dot{p}_i + r^{ik}\dot{p}_{i,k}\right) dv.$$

From the conditions of invariance of this equation with respect to the superposed rigid motions we obtain the differential equations of motion and the balance equation for the specific internal energy of deformation and polarization.

If the velocity of virtual translation is superposed on the real velocities in Eq. (3.12), all the quantities in the equation remain unchanged except v_i, which should be replaced by $v_i + a_i$, where a_i is superposed velocity of virtual translation. To render Eq. (3.12) invariant with respect to this superposition, the following equation must be satisfied:

(3.13)
$$\rho\,\dot{v}^i = t^{ik}_{\ ,k} + s^{ik}_{\ ,k} - \frac{1}{c}\dot{g}^i + \rho f^i,$$

which is Cauchy's first law of motion, i.e. a necessary and sufficient condition for the balance of momentum, made up of a system of three partial differential equations of motion.

When Cauchy's first law of motion (3.13) is satisfied, equation (3.12) may be reduced to the form

(3.14)
$$\int_v \rho(\Gamma^{ij}\nu_{ij} + \Gamma^i\dot{p}_i)\,dv + \int_v \rho\,\dot{w}\,dv = \int_v (\rho\, l^{ij}\nu_{ij} + \varepsilon^i\dot{p}_i)\,dv +$$
$$+ \int_v \left(t^{ij} v_{i,j} + m^{ijk}_{\ ,k}\nu_{ij} + m^{ijk}\nu_{ij,k} + r^{ij}_{\ ,j}\dot{p}_i + r^{ik}\dot{p}_{i,k}\right) dv,$$

which must be form-invariant with respect to the superposed velocity of rigid rotation.

If the velocity of rigid rotation is superposed on the real velocities in Eq. (3.14), all of its quantities remain unchanged except $v_{i,j}$, ν_{ij}, $\dot{p}_i$ and $\dot{p}_{i,k}$, which should be replaced by $v_{i,j} + \Omega_{ij}$, $\nu_{ij} + \Omega_{ij}$, $\dot{p}_i + \Omega_{ij} p^j$ and $\dot{p}_{i,k} + \Omega_{ij} p^j{}_{k}$, where Ω_{ij} is a skew-symmetric tensor which determines the superposed velocity of rigid rotation. To make Eq. (3.14) invariant to this superposition, we must have

(3.15)
$$t^{[ij]} + m^{ijk}_{\ ,k} + \rho(l^{ij} - \Gamma^{ij}) + r^{[i}p^{j]} + r^{[i|k|}p^{j]}_{\ ,k} = 0,$$

where

(3.16)
$$r^i = r^{ik}_{\ ,k} + \varepsilon^i - \rho\Gamma^i$$

is the local electrical vector of the field.

Equation (3.15) is Cauchy's second law of motion, i.e. a necessary and sufficient condition for the balance of the moment of momentum, and it represents a system of three partial differential equations of motion. Equation (3.16) also represents a system of three partial differential equations for determining the polarization vector field.

From Eq. (3.15) we obtain the equality

$$m^{ijk}_{\quad ,k}+\rho\left(l^{ij}-\Gamma^{ij}\right)=-t^{[ij]}-r^{[i}p^{j]}-r^{[i|k|}p^{j]}_{\ ,k}, \tag{3.17}$$

which, when substituted in Eq. (3.14), reduces this equation to the form

$$\begin{aligned}\int_v \rho\,\dot{w}\,dv=\int_v \Big[t^{ij}v_{i,j}-\left(t^{[ij]}+r^{[i}p^{j]}+r^{[i|k|}p^{j]}_{\ ,k}\right)\nu_{ij}+\\ +m^{ijk}\nu_{ij,k}+r^{i}\dot{p}_i+r^{ij}\dot{p}_{i,j}\Big]\,dv,\end{aligned} \tag{3.18}$$

which yields the local equation of balance for the specific energy of deformation and polarization in the form

$$\begin{aligned}\rho\,\dot{w}=t^{ij}v_{i,j}-\left(t^{[ij]}+r^{[i}p^{j]}+r^{[i|k|}p^{j]}_{\ ,k}\right)\nu_{ij}+\\ +m^{ijk}\nu_{ij,k}+r^{i}\dot{p}_i+r^{ij}\dot{p}_{i,j},\end{aligned} \tag{3.19}$$

invariant with respect to superposed rigid motions.

Using the relations

$$\begin{aligned}v_{i,j}=g_{il}X^{K}_{;j}\dot{x}^{l}_{;K},\quad \nu_{ij}=g_{il}\chi^{K}_{\cdot j}\dot{\chi}^{l}_{\cdot K},\\ \nu_{ij,k}=g_{il}X^{L}_{;k}\chi^{K}_{\cdot j}\dot{\chi}^{l}_{\cdot K;L}+g_{il}\chi^{K}_{\cdot j;k}\dot{\chi}^{l}_{\cdot K},\quad \dot{p}_{i,j}=g_{il}X^{K}_{;j}\dot{p}^{l}_{;K},\end{aligned} \tag{3.20}$$

we can present Eq. (3.19) in the form

$$\begin{aligned}\rho\,\dot{w}=g_{il}t^{ij}X^{K}_{;j}\dot{x}^{l}_{;K}-g_{il}\Big(t^{[ij]}\chi^{K}_{\cdot j}+r^{[i}p^{j]}\chi^{K}_{\cdot j}+\\ +r^{[i|k|}p^{j]}_{;L}X^{L}_{;k}\chi^{K}_{\cdot j}-m^{ijk}\chi^{K}_{\cdot j;k}\Big)\dot{\chi}^{l}_{\cdot K}+\\ +g_{il}m^{ijk}X^{L}_{;k}\chi^{K}_{\cdot j}\dot{\chi}^{l}_{\cdot K;L}+g_{il}r^{i}\dot{p}^{l}+g_{il}r^{ij}X^{K}_{;j}\dot{p}^{l}_{;K},\end{aligned} \tag{3.21}$$

concluding that the specific internal energy of deformation and polarization is a function of the following arguments:

$$w=w\left(x^{k}_{;K},\ \chi^{k}_{\cdot K},\ \chi^{k}_{\cdot L;K},\ p^{k},\ p^{k}_{;K}\right). \tag{3.22}$$

We also conclude from (3.21) that the quantities t^{ij}, m^{ijk}, r^{i} and r^{ij} must be determined by means of constitutive equations. Accordingly, to determine nine unknown functions of v_i, ν_{ij}, and p_i we have at our disposal a system of nine partial differential equations (3.13), (3.15) and (3.16).

4. Constitutive equations

By differentiating (3.22) with regard to time and comparing it with (3.21), that is (3.19), we obtain the following constitutive equations:

$$
\begin{aligned}
t^{ij} &= \rho g^{il} \frac{\partial w}{\partial x^{l}_{;K}} x^{j}_{;K}, \\
t^{[ij]} &= -\rho \frac{\partial w}{\partial \chi^{l}_{;K}} g^{l[i} \chi^{j]}_{\cdot K} - \rho \frac{\partial w}{\partial \chi^{l}_{\cdot L;K}} g^{l[i} \chi^{j]}_{\cdot L;K} - \rho \frac{\partial w}{\partial p^{l}} g^{l[i} p^{j]} - \\
&\quad - \rho \frac{\partial w}{\partial p^{l}_{;K}} g^{l[i} p^{j]}_{;K}, \\
m^{ijk} &= \rho \frac{\partial w}{\partial \chi^{l}_{\cdot L;K}} g^{l[i} \chi^{j]}_{\cdot L} x^{k}_{;K}, \\
r^{i} &= \rho g^{il} \frac{\partial w}{\partial p^{l}}, \\
r^{ij} &= \rho g^{il} \frac{\partial w}{\partial p^{l}_{;K}} x^{j}_{;K}.
\end{aligned}
\tag{4.1}
$$

The first two of above equations must be compatible, i.e.

$$
\left(g^{il} \frac{\partial w}{\partial x^{l}_{;K}} x^{j}_{;K} + g^{il} \frac{\partial w}{\partial \chi^{l}_{\cdot K}} \chi^{j}_{\cdot K} + g^{il} \frac{\partial w}{\partial \chi^{l}_{\cdot L;K}} \chi^{j}_{\cdot L;K} + g^{il} \frac{\partial w}{\partial p^{l}} p^{j} + g^{il} \frac{\partial w}{\partial p^{l}_{;K}} p^{j}_{;K} \right)_{[ij]} = 0. \tag{4.2}
$$

This condition, in the form of three partial differential equations that must be satisfied by the specific internal energy of deformation and polarization, represents a *condition of objectivity* for the specific internal energy of deformation and polarization (3.22) and for constitutive equations (4.1).

If condition (4.2) is satisfied, then the equation $(4.1)_2$ is superfluous, because it is included in $(4.1)_1$ as its skew-symmetric part. In this case, therefore, the complete system of constitutive equations takes the form:

$$
\begin{aligned}
t^{ij} &= \rho g^{il} \frac{\partial w}{\partial x^{l}_{;K}} x^{j}_{;K}, \\
m^{ijk} &= \rho \frac{\partial w}{\partial \chi^{l}_{\cdot L;K}} g^{l[i} \chi^{j]}_{\cdot L} x^{k}_{;K}, \\
r^{i} &= \rho g^{il} \frac{\partial w}{\partial p^{l}}, \\
r^{ij} &= \rho g^{il} \frac{\partial w}{\partial p^{l}_{;K}} x^{j}_{;K}.
\end{aligned}
\tag{4.3}
$$

The specific internal energy of deformation and polarization (3.22) is a function of 33 independent variables $x^k_{;K}$, $\chi^k_{\cdot K}$, $\chi^k_{\cdot L;K}$, p^k and $p^k_{;K}$. Since (4.2) represents the system of three partial differential equations, then it admits 33 —3=30 independent integrals. These integrals we shall take in the form[1]

(4.4)
$$\begin{aligned}
\varepsilon_{KL} &= \chi_{kK}\, x^k_{;L} - G_{KL},\\
K_{KLM} &= \chi_{kK}\, \chi^k_{\cdot L;M} = -\chi_{kL}\, \chi^k_{\cdot K;M} = -K_{LKM},\\
\Pi_K &= g_{kl}\, \chi^k_{\cdot K}\, p^l,\\
\Pi_{KL} &= g_{kl}\, \chi^k_{\cdot K}\, p^l_{;L},
\end{aligned}$$

so that system (4.2) has the following solution

(4.5)
$$w = w(\varepsilon_{KL},\ K_{KLM},\ \Pi_K,\ \Pi_{KL}).$$

Substituting (4.5) in (4.3) and using (4.4), we obtain nonlinear constitutive equations for an anisotropic micropolar elastic dielectric which are form-invariant with respect to superposed rigid motions, in the form

(4.6)
$$\begin{aligned}
t^{ij} &= \rho \frac{\partial w}{\partial \varepsilon_{KL}}\, \chi^i_{\cdot K}\, x^j_{;L},\\
m^{ijk} &= \rho \frac{\partial w}{\partial K_{KLM}}\, \chi^i_{\cdot K}\, \chi^j_{\cdot L}\, x^k_{;M},\\
r^i &= \rho \frac{\partial w}{\partial \Pi_K}\, \chi^i_{\cdot K},\\
r^{ij} &= \rho \frac{\partial w}{\partial \Pi_{KL}}\, \chi^i_{\cdot K}\, x^j_{;L}.
\end{aligned}$$

Further reduction of the constitutive equations depends on the material symmetries of the dielectrics.

The deformation gradients $x^k_{;K}$ are correlated with the gradients of displacement vector $u^k_{;K}$ by the relation

(4.7)
$$x^k_{;K} = g^k_K + u^k_{;K},$$

where g^k_K are the coordinates of Euclidean shifters. If the gradients of microdisplacement $\varphi^k_{\cdot K}$ are introduced in a similar manner,

(4.8)
$$\chi^k_{\cdot K} = g^k_K + \varphi^k_{\cdot K},$$

then, on the ground of (2.6), it can be concluded that the gradients of microdisplacement vector satisfy the relation

(4.9)
$$\varphi_{KL} + \varphi_{LK} + \varphi_{MK}\, \varphi^M_{\cdot L} = 0,$$

---

[1] The reason for choosing these integrals is given in Appendix.

which, in the linear theory, has the form

(4.10) $$\varphi_{KL}+\varphi_{LK}=0.$$

Making use of (4.7), (4.8) and (4.9), we can express material tensors (4.4) in the form

(4.11)
$$\begin{gathered}
\varepsilon_{KL}=u_{K,L}+\varphi_{LK}+\varphi_{MK}\overset{M}{u}_{,L}=u_{K,L}-\varphi_{KL}+(u_{M,L}-\varphi_{ML})\,\varphi^{M}_{\cdot K},\\
K_{KLM}=\varphi_{KL,M}+\varphi_{SK}\varphi^{S}_{\cdot L,M}=-\varphi_{LK,M}-\varphi_{SL}\varphi^{S}_{\cdot K,M},\\
\Pi_K=g_{lK}p^l+\varphi_{lK}p^l=p_K+\varphi_{SK}p^S,\\
\Pi_{KL}=g_{lK}\overset{l}{p}_{;L}+\varphi_{lK}\overset{l}{p}_{;L}=p_{K,L}+\varphi_{SK}\overset{S}{p}_{,L}.
\end{gathered}$$

In the linear theory these tensors have the form

(4.12)
$$\begin{gathered}
\varepsilon_{KL}=u_{K,L}+\varphi_{LK}=u_{K,L}-\varphi_{KL}=u_{K,L}-\varepsilon_{KLS}\varphi^S,\\
K_{KLM}=\varphi_{KL,M}=-\varphi_{LK,M}=\varepsilon_{KLS}\overset{S}{\varphi}_{,M},\\
\Pi_K=g_{lK}p^l=p_K,\\
\Pi_{KL}=g_{lK}\overset{l}{p}_{;L}=p_{K,L},
\end{gathered}$$

where $\vec{\varphi}=\{\varphi^S\}$ is the vectorial representation of the angle of independent rotation of the dielectric's particles.

Appendix

The balance equation for the specific internal energy of deformation and polarization (3.19) obviously may be written in the form

(1) $$\rho\dot{w}=t^{ij}(v_{i,j}-\nu_{ij})+m^{ijk}\nu_{ij,k}+r^i(\dot{p}_i-p^j\nu_{ij})+r^{ik}(\dot{p}_{i,k}-\overset{j}{p}_{,k}\nu_{ij}).$$

The tensors $v_{i\,j}-\nu_{ij}$, $\nu_{ij,k}$, $\dot{p}_i-p^j\nu_{ij}$ and $\dot{p}_{i,k}-\overset{j}{p}_{,k}\nu_{ij}$, all of them being objective, may be expressed as follows:

(2)
$$\begin{gathered}
v_{i,j}-\nu_{ij}=\dot{\varepsilon}_{KL}\chi^{K}_{\cdot i}X^{L}_{;j},\qquad \nu_{ij,k}=\dot{K}_{KLM}\chi^{K}_{\cdot j}\chi^{L}_{\cdot j}X^{M}_{;k},\\
\dot{p}_i-p^j\nu_{ij}=\dot{\Pi}_K\chi^{K}_{\cdot i},\quad \dot{p}_{i,k}-\overset{j}{p}_{,k}\nu_{ij}=\dot{\Pi}_{KL}\chi^{K}_{\cdot i}X^{L}_{;k},
\end{gathered}$$

where

(3)
$$\begin{gathered}
\varepsilon_{KL}=\chi_{kK}\overset{k}{x}_{;L}-G_{KL},\quad K_{KLM}=\chi_{kK}\chi^{k}_{\cdot L;M},\\
\Pi_K=\chi_{kK}p^k,\quad \Pi_{KL}=\chi_{kK}\overset{k}{p}_{;L}.
\end{gathered}$$

By means of expressions (2) , Eq. (1) can be written in the form

(4) $$\rho\dot{w}=t^{ij}\chi^{K}_{\cdot i}X^{L}_{;j}\dot{\varepsilon}_{KL}+m^{ijk}\chi^{K}_{\cdot i}\chi^{L}_{\cdot j}X^{M}_{;k}\dot{K}_{KLM}+r^i\chi^{K}_{\cdot i}\dot{\Pi}_K+r^{ik}\chi^{K}_{\cdot i}X^{L}_{;k}\dot{\Pi}_{KL},$$

from which we conclude that the specific internal energy of deformation and polarization is a function of the form

(5) $$w=w(\varepsilon_{KL},\ K_{KLM},\ \Pi_K,\ \Pi_{KL}).$$

By differentiating expression (5) with respect to time, we obtain

$$\dot{w}=\frac{\partial w}{\partial \varepsilon_{KL}}\dot{\varepsilon}_{KL}+\frac{\partial w}{\partial K_{KLM}}\dot{K}_{KLM}+\frac{\partial w}{\partial \Pi_K}\dot{\Pi}_K+\frac{\partial w}{\partial \Pi_{KL}}\Pi_{KL},$$

and then, comparing this with (4), we obtain the nonlinear constitutive equations, (4.6), which are form-invariant with respect to superposed rigid motions.

REFERENCES

[1] Toupin R. A., *The elastic Dielectrics*, J. Rat. Mech. Anal., **5**, pp 849—915, 1956.

[2] Toupin R. A., *A Dynamical Theory of Elastic Dielectrics*, Int. J. Engng. Sci., **1**, pp 101—126, 1963.

[3] Mindlin R. D., *Polarization Gradient in Elastic Dielectrics*, Int. J. Solids and Structures, **4**, pp 637—642, 1968.

[4] Mindlin R. D., *Polarization Gradient in Elastic Dielectrics*, CISM, Udine, 1970.

[5] Suhubi E. S., *Elastic Dielectrics with Polarization Gradient*, Int. J. Engng., Sci. **7**, pp 993—997, 1969.

[6] Dixon R. C. and A. C. Eringen, *A Dynamical Theory of Polar Elastic Dielectrics* — I, Int. J. Engng. Sci., **3**, pp 359—377, 1965.

[7] Dixon R. C. and A. C. Eringen, *A Dynamical Theory of Polar Elastic Dielectrics*, — II, Int. J. Engng. Sci., **3**, pp 379—398, 1965.

[8] Bauchert J., *Das elastische Dielektrum als orientiertes elastisches Kontinuum*, Acta Mechanica, **16**, 65—76, 1973.

[9] Green A. E. and R. S. Rivlin, *Simple force and Stress Multipoles*, Arch. Rat. Mech. Anal., **16**, pp 325—353, 1964.

[10] Plavšić M. and M. Gligorić, *Elastic Dielectric with Microstructure*, Teorijska i primenjena mehanika (Theoretical and Applied Mechanics), Yugoslav Society of Mechanics, **1**, pp 49—59, 1975.

[11] Gligorić M., *Teorija elastičnog dielektričnog kontinuuma* (in Serbian), Tehnika, **7—8**, str. 4—9, 1975.

[12] Suhubi E. S. and A. C. Eringen, *Nonlinear Theory of Microelastic Solids* — II, Int. J. Engng. Sci., **2**, pp 389—404, 1964.

[13] Eringen A. C., *Theory of Micropolar Elasticity*, J. Math. Mech., **15**, pp 909—924, 1966.

[14] Eringen A. C., *Theory of Micropolar Continua*, Proc. of 9th Midw. Mech. Conf., Madison, Wisconsin, 1965.

[15] Eringen A. C., *Linear Theory of Micropolar Viscoelasticity*, Int. J. Engng. Sci., **5**, pp 191—204, 1967.

[16] Kafadar C. B. and A. C. Eringen, *Micropolar media*, Int. J. Engng. Sci., **9**, pp 271—305, 1971.

[17] Plavšić M. and J. Jarić, *The Principle of virtual work and the constitutive equations in generalized theories of elasticity*, Rozprawy inzynierskie (Engineering Transactions), Polska Akademia Nauk, **22**, 2, pp 181—211, 1974.

Adress of the authors:

Institute of Mechanics
Faculty of Sciences
University of Belgrade
11000 Belgrade
Studentski Trg 16, P.O. Box 550
Yugoslavia

MIKROPOLARE ELASTISCHE DIELEKTRISCHE MATERIALEN

M. Plavšić, S. Đurić and *M. Gligorić*

Zusammenfassung

Das Verhalten des elastischen Dielektrik unter dem Einfluss des elektromagnetischen Feldes wird untersucht und zwar für das mechanisches Modell des mikropolaren Materials. Nach der Einführung der ensprechender mechanischer und elektromagnetischer Kräfte, die globale Form des Energiebilanzes wird festgestellt. Auf Grund der Forderung, dass der Energiesatz invariant sei unter den überlagerten starren Bewegungen, werden die Bewegungsgleichungen sowie der Ausdruck für die innere Energie ausgeführt. Schliesslich, unter Berücksichtigung des Ausdrucks für die spezifische innere Verformungs — und Polarisationsenergie werden die nichtlineare Materialgleichungen hergestellt.

МИКРОПОЛАРНИ ЕЛАСТИЧНИ ДИЕЛЕКТРИК

М. Плавшић, С. Ђурић, М. Глиіорић

Резиме

У раду се посматра модел микрополарног еластичног материјала као диелектрик у електромагнетном пољу. Користећи одговарајуће механичке и електромагнетске силе, постулиран је облик глобалне једначине баланса енергије деформације и поларизације. Из захтева инваријантности ове једначине на суперпонирана крута кретања, изведене су диференцијалне једначине кретања и једначине баланса специфичне унутрашње енергије деформације и поларизације, из које су добијене нелинеарне конститутивне једначине.