

## ONE — AND DOUBLEPERIODIC PLANE ELASTOSTATIC BOUNDARY VALUE PROBLEM

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### I.

The purpose of the present article is to study the first and the second boundary value problem in plane linear elastostatics together with a detailed analysis regarding both similarities and differences. The first periodic boundary value problem has been treated by Savin [1], and by a number of other authors. The first detailed study of the double periodic (second) boundary value problem, however, reaches only as far back as 1972 [2]. In this article, we shall rely exclusively upon the widely known method of D. J. Sherman [3], [4], [5] which enables us to provide an elegant proof of the existence of the problem solution; besides, compared with [2], the procedure will be both simplified and completed.

Let the entire plane  $z$  be divided into congruent — periodic infinite parallelograms. (Figure 1). In each infinite parallelogram, there is a finite number of holes limited by curves of finite length. As regards the smoothness of the curves, the usual limitations are valid [5].

The period of the infinite parallelograms is designated by  $\Omega$ . Let us introduce the designations

$$G(\alpha) = -\alpha \varphi(z) + z \overline{\varphi'(z)} + \overline{\psi(z)}$$

$D$  is the interior region of a chosen parallelogram. Hole boundary curves are designated by  $C_k$ ,  $k = 1, 2, \dots, N$ , interior region of  $C_k$  by  $D_k$ .

Let us examine the functions

$$\varphi_1(z) = \sum_{k=1}^{k=N} A_k \ln \sin \frac{\pi}{\Omega} (z - z_k)$$

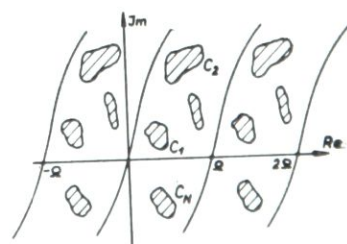


Fig. 1

$$(1-1) \quad \psi_1(z) = -\alpha \sum_{k=1}^{k=N} A_k \ln \sin \frac{\pi}{\Omega} (z - z_k) - \frac{\pi}{\Omega} z \sum_{k=1}^{k=N} A_k \operatorname{ctg} \frac{\pi}{\Omega} (z - z_k)$$

where  $z_k \in D_k$  stands for an arbitrary chosen point in  $D_k$ , besides, the following should be valid

$$(1-2) \quad \sum_{k=1}^{k=N} A_k = 0$$

In  $D$ , functions  $\varphi_1(z)$  and  $\varphi_1(z)$  are holomorphic, we can write

$$(1-3) \quad \begin{aligned} [\varphi_1(z)]_z^{z+\Omega} &= 0 & \varphi_1'(z) &= o(|y|^{-1}) \\ [G_1(x)]_z^{z+\Omega} &= 0 & G_1(x) &= o(y), \quad \lim |y| = \infty \end{aligned}$$

Here, the constants  $A_k$  retain the usual meaning as in non-periodic problems.

Let us write

$$(1-4) \quad \begin{aligned} \varphi(z) &= \varphi_1(z) + Az + \varphi_0(z) \\ \psi(z) &= \psi_1(z) + A'z + \psi_0(z) \end{aligned}$$

where  $A$  and  $A'$  are arbitrary given constants.

The first boundary value problem (the problem of boundary value displacements) requires the determination of holomorphic functions  $\varphi_0(z)$ ,  $\psi_0(z)$  and constants  $A_k$  in agreement with (1-2) so that

$$(1-5) \quad G(x) = f(z), \quad z \in C_k, \quad k = 1, 2, \dots, N$$

where  $f(z)$  is a given function with the usual properties [5]. In the case of the second boundary value problem (the problem of boundary value stresses) functions  $\varphi_0(z)$ ,  $\psi_0(z)$ , holomorphic in  $D$ , and constants  $\beta_1, \dots, \beta_N$  have to be determined, so that

$$(1-5') \quad \begin{aligned} G(-1) &= f(z) + \beta_k \\ z &\in C_k, \quad k = 1, 2, \dots, N \end{aligned}$$

where  $f(z)$  is a given function on  $C$ .

For  $\varphi_0(z)$  and  $\psi_0(z)$ , the following is required:

$$(1-6) \quad \begin{aligned} [\varphi_0'(z)]_z^{z+\Omega} &= 0 \\ [G_0(x)]_z^{z+\Omega} &= 0 \\ \varphi_0'(z) &= o(|y|^{-1}) \\ G_0(x) &= o(|y|), \quad \lim |y| = \infty \end{aligned}$$

From (1-3), (1-4) and (1-6) follows

$$(1-6') \quad [G(x)]_z^{z+\Omega} = [-x A + \bar{A} + \bar{A}'] \Omega$$

According to the aforementioned procedure [5] for the first boundary value problem, the following is valid for  $f(z) = 0$  and  $A = A' = 0$ :

$$(1-7) \quad \begin{aligned} \varphi(z) &= \varphi_0(z) = c_1 \\ \psi(z) &= \psi_0(z) = x \bar{c}_1 \\ A_1 &= A_2 = \dots = A_N = 0 \end{aligned}$$

while in the second boundary value problem the constants  $A_k$  are known; therefore

$$(1-7') \quad \begin{aligned} \varphi(z) &= \varphi_0(z) = c_1 \\ \psi(z) &= \psi_0(z) = c_2 \\ \beta_1 &= \beta_2 = \dots = \beta_N \end{aligned}$$

If we choose e.g.  $\beta_1 = 0$ , then

$$(1-8) \quad \begin{aligned} c_2 &= -\overline{c_1} \\ \beta_1 &= \dots = \beta_N = 0 \end{aligned}$$

In our article,  $c$  designates arbitrary complex constants.

## II.

The function

$$(2-1) \quad v(z) = \pi^{-1} \Omega \sum_{n=-\infty}^{n=\infty} (n=0) [n \Omega (z - n \Omega)^{-2} - (n \Omega)^{-1}]$$

has the following properties

$$(2-2) \quad \begin{aligned} v(-z) &= -v(z) \\ v(0) &= 0 \\ [v(z)]_z^{z+\Omega} &= \pi \sin^{-2}(\Omega^{-1} \pi z) \end{aligned}$$

It has singular points (poles) only at  $z = n \Omega$ , where  $n$  means an arbitrary integer different from 0.

By means of this function, the description of  $\varphi_0(z)$  and  $\psi_0(z)$  will be made possible. Let us write

$$(2-3) \quad \begin{aligned} \varphi_0(z) &= \frac{1}{2 \pi i} \int_C \text{ctg} \frac{\pi}{\Omega} (t - z) \omega(t) dt + K \\ \psi_0(z) &= \frac{1}{2 \pi i} \int_C \text{ctg} \frac{\pi}{\Omega} (t - z) [-x \overline{\omega(t)} - \overline{t} \omega'(t)] dt + \frac{1}{2 \pi i} \int_C v(t - z) \omega(t) dt \\ K &= \int_{C_N} \omega(t) ds, \quad A_k = \int_{C_k} \omega(t) ds, \quad k = 1, 2, \dots, N - 1 \end{aligned}$$

All of the properties required in (1-6) are valid for functions  $\varphi_0(z)$  and  $\psi_0(z)$ . Considering that under the usual conditions Cauchy's integral

$$F(z) = \frac{1}{2 \pi i} \int_C \text{ctg} \frac{\pi}{\Omega} (t - z) \omega(t) dt$$

behaves as

$$\frac{1}{2 \pi i} \int_C \frac{\omega(t) dt}{t - z}$$

which means that Plemelj's equations [6] are applicable in the former case

$$\begin{aligned}
 F^+(z_0) &= \Omega\pi^{-1} \cdot \frac{1}{2} \omega(z_0) + F(z_0) \\
 (2-4) \quad F^-(z_0) &= -\Omega\pi^{-1} \cdot \frac{1}{2} \omega(z_0) + F(z_0) \\
 &z_0 \in C
 \end{aligned}$$

since

$$(2-5) \quad \operatorname{ctg} \pi\Omega^{-1}(t-z) = \Omega\pi^{-1}(t-z)^{-1} + \operatorname{reg} \cdot f.$$

For  $t$  in the appropriate surroundings of point  $z$ , in the case of the first boundary value problem we have to obtain Sherman's integral equation; with respect to (2-5) this equation differs from the classical one only as regards the additive element representing a bounded operator on function  $\omega$ . Therefore, the expression itself is not of essential weight, consequently, it will not be introduced. It is important, however, that the homogeneous equation corresponding to the aforementioned equation, which is of the Fredholm type of the second order, has only a trivial solution. Let the solution of the corresponding homogeneous one be  $\omega_0(t)$ . Let us form functions  $\varphi_0(z)$  and  $\psi_0(z)$  by this function according to Eqns (2-3). With regard to (1-7)

$$\begin{aligned}
 (2-6) \quad c_1 &= \frac{1}{2\pi i} \int_C \operatorname{ctg} \frac{\pi}{\Omega} (t-z) \omega_0(t) dt + K \\
 {}_x \bar{c}_1 &= \frac{1}{2\pi i} \int_C \operatorname{ctg} \frac{\pi}{\Omega} (t-z) [-{}_x \overline{\omega_0(t)} - \bar{t} \omega_0'(t)] dt + \\
 (2-7) \quad &+ \frac{1}{2\pi i} \int_C \nu(t-z) \omega_0(t) dt, \quad z \in D
 \end{aligned}$$

From the first equation it follows that the function  $\varphi^*(z)$  where

$$(2-8) \quad \varphi^*(t) = i \omega_0(t), \quad t \in C$$

is a holomorphic function in  $D_k$ . Hence we obtain

$$\int_C \nu(t-z) \omega_0(t) dt = -i \int_C \nu(t-z) \varphi^*(t) dt = 0$$

and function  $\psi^*(z)$ , for which

$$(2-9) \quad \psi^*(t) = -i [{}_x \overline{\omega_0(t)} + \bar{t} \omega_0'(t)], \quad t \in C$$

is again a holomorphic function in  $D_k$ ; here

$$(2-10) \quad G^*(x) = 0 \quad t \in C$$

Thus we have

$$\begin{aligned} \varphi^*(z) &= c_{1k} \\ (2-11) \quad \psi^*(z) &= \kappa \bar{c}_{1k}, \quad z \in D_k \end{aligned}$$

Hence it follows that  $\omega_0(t)$  is a constant, generally different on any curve  $C_k$ . Following from (2-7) is  $c_1=0$ , from (2-6)  $K=0$ , and from the last two equations in (2-3)

$$(2-12) \quad \omega_0(t) = 0, \quad t \in C$$

which had to be proved. This means that the problem treated can be solved by means of expressions (2-3).

A similar approach is used in the case of the second boundary value problem. We write

$$\begin{aligned} \varphi_0(z) &= \frac{1}{2\pi i} \int_C \operatorname{ctg} \frac{\pi}{\Omega} (t-z) \omega(t) dt + \sum_{k=1}^{k=N} b_k \operatorname{ctg} \frac{\pi}{\Omega} (z-z_k) \\ \psi_0(z) &= \frac{1}{2\pi i} \int_C \operatorname{ctg} \frac{\pi}{\Omega} (t-z) [\overline{\omega(t)} - \bar{t} \omega'(t)] dt + \\ &+ \sum_{k=1}^{k=N} b_k \operatorname{ctg} \frac{\pi}{\Omega} (z-z_k) + \frac{1}{2\pi i} \int_C \nu(t-z) \omega(t) dt + \sum_{k=1}^{k=N} b_k \nu(z-z_k) \\ b_k &= \frac{1}{2\pi i} \int_{C_k} [\overline{\omega(t)} dt - \omega(t) \bar{d}t], \quad 1 \leq k \leq N \end{aligned}$$

$$(2-13) \quad \beta_k = \int_{C_k} \omega(t) ds, \quad 1 \leq k \leq N$$

Reasoning in a way analogous to that in the previous case we obtain again the result (2-12). The variants of this problem have often been dealt with, and since there occur no essential differences in the process — with regard to the first boundary value problem — the proof is omitted.

### III.

Let us examine also the study of twoperiodic boundary value problem with respect to the possibility of dealing with it in oneperiodic conditions.

Let the basic periods be  $\Omega_1$  and  $\Omega_2$ , with  $\Omega_1$  being real. We shall see that dealing with the twoperiodic boundary value problem with regard to the oneperiodic boundary value problem we are unable to achieve a complete analogy between the two, yet they are similar to one another. This is ascertained already by the fact that there is no twoperiodic function having only a single pole.

Let each periodic parallelogram have  $O$  holes limited by  $C_k, k = 1, 2, \dots, N$ . The interior region of an arbitrarily chosen parallelogram is designated by  $D$ , the hole interior by  $D_k, 1 \leq k \leq N$ .

Let us write

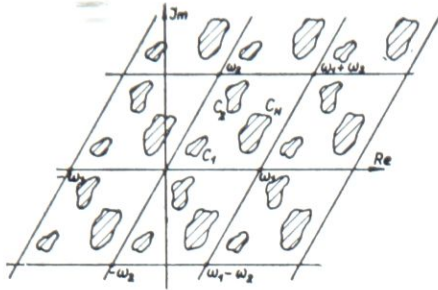


Fig. 2

$$\varphi_1(z) = \sum_{k=1}^{k=N} A_k [\ln \sigma(z - z_k) + z_k \zeta(z - z_k)]$$

$$\psi_1(z) = -\chi \sum_{k=1}^{k=N} \bar{A}_k [\ln \sigma(z - z_k) + z_k \zeta(z - z_k)] +$$

$$+ \sum_{k=1}^{k=N} A_k z_k v_1(z - z_k) p(z - z_k) +$$

$$+ \sum_{k=1}^{k=N} A_k v_2(z - z_k) \zeta(z - z_k)$$

$$(3-1) \quad \sum_{k=1}^{k=N} A_k = 0$$

where  $\sigma(z)$ ,  $\zeta(z)$  and  $p(z)$  are Weierstrass sigma, zeta and  $p$  functions. Further

$$(3-2) \quad v_1(z) = A_{11} z + A_{12} \zeta(z),$$

$$(3-2') \quad v_2(z) = A_{21} z + A_{22} \zeta(z) + A_{23}$$

$$(3-3) \quad [v_1(z)]_z^{z+\Omega_m} = A_{11} \Omega_m + A_{12} \eta_m, \quad m = 1, 2$$

$$(3-4) \quad [v_2(z)]_z^{z+\Omega_m} = A_{21} \Omega_m + A_{22} \eta_m, \quad m = 1, 2$$

$$(3-5) \quad A_{11} \Omega_m + A_{12} \eta_m = \bar{\Omega}_m, \quad m = 1, 2$$

$$(3-6) \quad A_{21} \Omega_m + 2 A_{22} \eta_m = -\bar{\Omega}_m, \quad m = 1, 2$$

$$(3-7) \quad \Omega_2 \eta_1 - \Omega_1 \eta_2 = 2 \pi i$$

$$(3-8) \quad v_2(z - z_k) = A_{21} (z - z_k) + A_{22} \zeta(z - z_k) + A_{23, k}$$

$$(3-9) \quad A_{23, k} = z_k A_{21}, \quad k = 1, 2, \dots, N$$

From these equations follows

$$(3-10) \quad [\varphi_1(z)]_z^{z+\Omega_m} = 0, \quad m = 1, 2$$

$$(3-11) \quad [G_1(\chi)]_z^{z+\Omega_m} = 0, \quad m = 1, 2$$

$$(3-12) \quad [G_1(-1)]_z^{z+\Omega_m} = 0, \quad m = 1, 2$$

Consider now the following equations

$$(3-13) \quad \varphi(z) = \varphi_1(z) + \left[ A z + B \sum_{k=1}^{k=N} \zeta(z - z_k) \right] + \varphi_0(z)$$

$$(3-14) \quad \psi(z) = \psi_1(z) + \left[ A' z + B' \sum_{k=1}^{k=N} \zeta(z - z_k) + \sum_{k=1}^{k=N} v_2(z - z_k) p(z - z_k) \right] + \psi_0(z)$$

where  $A, A', B, B'$  are arbitrary given constants,  $\varphi_0(z)$  and  $\psi_0(z)$  two functions holomorphic in  $D$ , such that

$$(3-15) \quad [\varphi_0(z)]_z^{z+\Omega m} = 0, \quad m = 1, 2$$

$$(3-16) \quad [G_0(x)]_z^{z+\Omega m} = 0, \quad m = 1, 2$$

$$(3-17) \quad [G_0(-1)]_z^{z+\Omega m} = 0, \quad m = 1, 2$$

and

$$(3-18) \quad v_3(z) = Cz + D\zeta(z)$$

$$(3-19) \quad C\Omega_m + D\gamma_{lm} = \Omega_m B, \quad m = 1, 2$$

From these definitions follows

$$(3-20) \quad [\varphi'(z)]_z^{z+\Omega m} = 0, \quad m = 1, 2$$

$$(3-21) \quad [G(x)]_z^{z+\Omega m} = -x(A\Omega_m + BN\gamma_{lm}) + \bar{A}\Omega_m + \bar{A}'\bar{\Omega}_m + \bar{B}'N\bar{\gamma}_{lm}$$

$$(3-22) \quad [G(-1)]_z^{z+\Omega m} = (A\Omega_m + BN\gamma_{lm}) + \bar{A}\Omega_m + \bar{A}'\bar{\Omega}_m + \bar{B}'N\bar{\gamma}_{lm}$$

$$m = 1, 2$$

If the numbers  $[G(x)]_z^{z+\Omega m}$  and  $[G(-1)]_z^{z+\Omega m}$ ,  $m = 1, 2$  are arbitrary given, say equal  $P_m$  and  $Q_m$ , then the system of equations

$$(3-23) \quad [G(x)]_z^{z+\Omega m} = P_m, \quad m = 1, 2$$

$$[G(-1)]_z^{z+\Omega m} = Q_m, \quad m = 1, 2$$

has a unique solution  $A, A', B, B'$ :

$$(3-24) \quad (x+1)[A\Omega_m + B\gamma_{lm}N] = Q_m - P_m, \quad m = 1, 2$$

$$(3-25) \quad (x+1)[A'\Omega_m + B'N\gamma_{lm}] = \bar{P}_m + x\bar{Q}_m - (x+1)A\bar{\Omega}_m, \quad m = 1, 2$$

At  $f(z) = 0$  and  $A = B = A' = B' = 0$  we have [5] in the first boundary value problem

$$\varphi(z) = \varphi_0(z) = c_1$$

$$(3-26) \quad \psi(z) = \psi_0(z) = \bar{x}c_1$$

$$A_1 = A_2 = \dots = A_N = 0$$

and in the second

$$\varphi(z) = \varphi_0(z) = c_1$$

$$(3-27) \quad \psi(z) = \psi_0(z) = c_2$$

$$c_1 + \bar{c}_1 = \beta_1 = \beta_2 = \dots = \beta_N$$

In these expressions,  $c_1$  and  $c_2$  are arbitrary complex constants.

## IV.

In order to form functions  $\varphi_0(z)$  and  $\psi_0(z)$ , we need a two-periodic function, holomorphic in  $D$  and having only one pole at  $z=t \in C$ . This function can assume the form

$$K(t, z) = \zeta(t-z) + \frac{1}{N} \sum_{k=1}^{k=N} \zeta(z-z_k), \quad z \in D, \quad z_k \in D \cup C$$

$$(4-1) \quad [K(t, z)]_z^{z+\Omega m} = 0, \quad m = 1, 2$$

By appropriate reduction, we can obtain the following expressions for  $\varphi_0(z)$  and  $\psi_0(z)$  in the first boundary value problem

$$\varphi_0(z) = \frac{1}{2\pi i} \int_C K(t, z) \omega(t) dt + C_0$$

$$(4-2) \quad \begin{aligned} \psi_0(z) = & \frac{1}{2\pi i} \int_C K(t, z) [-\kappa \overline{\omega(t)} - \bar{t} \omega'(t)] dt + \\ & + \frac{1}{2\pi i} \int_C \nu_4(t-z) \omega(t) dt - \frac{1}{N} \sum_{k=1}^{k=N} \frac{1}{2\pi i} \int_C \nu_4(z-z_k) \omega(t) dt \end{aligned}$$

where the function  $\nu_4(z)$  is defined as

$$(4-3) \quad \nu_4(z) = \sum_{m, n=-\infty}^{m, n=\infty} \binom{m=0}{n=0} [\overline{\Omega} (z-\Omega)^{-2} - 2z \overline{\Omega} \Omega^{-3} - \overline{\Omega} \Omega^{-2}]$$

$$\Omega = m \Omega_1 + n \Omega_2$$

for which we can write

$$\nu_4(-z) = -\nu_4(z)$$

$$(4-4) \quad \nu_4(0) = 0$$

$$[\nu_4(z)]_z^{z+\Omega m} = \overline{\Omega}_m p(z) + \gamma_m, \quad m = 1, 2$$

where  $\gamma_1$  and  $\gamma_2$  are two constants for which the following is valid [2]:

$$(4-5) \quad \gamma_2 \Omega_1 - \gamma_1 \Omega_2 = \eta_1 \overline{\Omega}_2 - \eta_2 \overline{\Omega}_1$$

It follows from (4-2) — (4-5), that the requirements (3-15) — (3-17) are satisfied.

After defining

$$(4-6) \quad A_k = \int_{C_k} \omega(t) ds, \quad 1 \leq k \leq N-1$$

$$(4-7) \quad C_0 = \int_{C_N} \omega(t) ds$$



we obtain Sherman's integral equation for function  $\omega(t)$ , since the integral

$$F(t) = \frac{1}{2\pi i} \int_C K(t, z) \omega(t) dt$$

behaves analogously to

$$\frac{1}{2\pi i} \int_C \frac{\omega(t) dt}{t-z}$$

In this case, Plemelj's equations are valid in unchanged conditions for  $\omega(t)$ : Sherman's equation, again of Fredholm type of the second order, differs from the classical one as regards the additive element representing a bounded operator on  $\omega(t)$ , since we can write

$$K(t, z) = \frac{1}{t-z} + \text{reg. } f.$$

for  $t$  in the appropriate surroundings of point  $z$ .

Let us prove that the homogeneous equation corresponding to that of Sherman has only a trivial solution. Let the solution of the homogeneous equation be  $\omega_0(t)$ . Let us form — by this function — functions  $\varphi_0(z)$  and  $\psi_0(z)$  according to formulas (4—2), (4—6) and (4—7). From (3—26) we obtain

$$(4-8) \quad c_1 = \frac{1}{2\pi i} \int_C K(t, z) \omega_0(t) dt + C_0$$

$$(4-8') \quad \begin{aligned} \alpha \bar{c}_1 = & \frac{1}{2\pi i} \int_C K(t, z) [-\alpha \overline{\omega_0(t)} - \bar{t} \omega_0'(t)] dt + \\ & + \frac{1}{2\pi i} \int_C \nu_4(t-z) \omega_0(t) dt - \frac{1}{N} \sum_{k=1}^{k=N} \frac{1}{2\pi i} \int_C \nu_4(z-z_k) \omega_0(t) dt \end{aligned}$$

It follows, that function  $\varphi^*(z)$ , for which

$$(4-9) \quad \varphi^*(t) = i \omega_0(t), \quad t \in C$$

is holomorphic in  $D_k$ .

Following from the equation (4—8') is

$$(4-10) \quad \alpha \bar{c}_1 = \frac{1}{2\pi i} \int_C K(t, z) [-\alpha \overline{\omega_0(t)} - \bar{t} \omega_0'(t)] dt$$

and, besides, that  $\psi^*(z)$  where

$$(4-11) \quad \psi^*(t) = -i [\alpha \overline{\omega_0(t)} + \bar{t} \omega_0'(t)], \quad t \in C$$

is holomorphic in  $D_k$ . With regard to (4—9) and (4—11)

$$(4-12) \quad G^*(\alpha) = 0, \quad t \in C$$

and further

$$\begin{aligned} \varphi^*(z) &= c_k^* \\ (4-13) \quad \psi^*(z) &= \kappa c_k^*, \quad z \in D_k, \quad k = 1, 2, \dots, N \end{aligned}$$

Following from (4—9) is that  $\omega_0(t)$  on  $C$  is a constant, different on each  $C_k$  in general. From (4—10) we have immediately

$$(4-14) \quad c_1 = 0$$

and from (4—8)

$$(4-15) \quad C_0 = 0$$

With regard to the last equation of (3—26), (4—6), and (4—15) we have finally

$$(4-16) \quad \omega_0(t) = 0, \quad t \in C$$

which had to be proved.

The details of deduction of the second boundary value problem will not be treated. It follows from (3—27), that there occur no essential differences in the process. In comparison with [2], where the momentum condition is necessary to be satisfied, the present treatment does not require that.

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#### EIN — UND ZWEIPERIODISCHES RANDWERTPROBLEM IN DER EBENEN ELASTOSTATIK

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#### Zusammenfassung

In diesem Aufsatz werden Ein- und Zweiperiodische Randwertprobleme in der ebenen Elastostatik behandelt. Die Deduktion ist in Details nur für Randverschiebungsprobleme ausgeführt, indem sie für das Randspannungsproblem nur angedeutet ist. Es hat sich nämlich erwiesen, dass die Momentbedingung bei zitierten Definition des Randwertproblem — ähnlich wie bei allen unendlichen Bereichen — nicht nötig ist auszufüllen, zum Unterschied von Ausführung von [2], wo die Erfüllung der Momentbedingung obligat ist.