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THERMODIFFUSION IN ELASTIC, MAGNETICALLY SATURATED, CURRENT CONDUCTING MEDIA. I. CONSTITUTIVE EQUATIONS

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Ferromagnetic bodies are strongly magnetized even in weak magnetic fields. Until the field of temperature of the regarded medium stays below the temperature assigned as the Curie point, the intensity of magnetization may be treated as being constant. Under such assumption we attach to each point of the medium a rigid director, which is free to perform rotation independently of the rotation of the neighbouring material volume element and which represents the magnetization in that point. The high tendency to orientation, i.e. to align parallel the neighbouring electron spins, may be explained taking account of exchange forces acting across the surfaces of separation between neighbouring volume elements of the continuum.

The mechanics of ferromagnetic materials was treated in a certain number of works [1], [2], [3]. G. A. Maugin and A. C. Eringen [4] proposed a variational approach to the study of nonlinear theormoelastic deformation of magnetically saturated solids. H. F. Tiersten and C. F. Tsai [5] studied the behaviour of finitely deformable, polarizable and magnetizable continuum introducing the model of an electric charge and spin continuum coupled to a lattice continuum which in itself consists of two interpenetrating ionic continua. H. Parkus [6], [7] extends the magneto-thermoelastic theory to the case of current conducting materials.

In the present article the process of heat conduction and electric current conduction are analysed on the ground of thermdynamics of irreversible processes. Starting from the first and second laws of thermodynamics, the dissipation function and the free energy are determined, as well as the corresponding constitutive equations.

It is assumed throughout the paper, that macroscopic material velocities are considerable less than the speed of light.

1. External electromagnetic field.

The external field is characterized by the electric field strength – $\vec{E}$, electrical displacement – $\vec{D}$, magnetic field strength – $\vec{H}$ and magnetic induction – $\vec{B}$

The field quantities must satisfy the Maxwell equations, which in Gaussian units take the form:

$$C \operatorname{rot} \vec{H} = \frac{\partial \vec{D}}{\partial t} + 4\pi C \vec{J}_{(e)}; \quad \operatorname{div} \vec{D} = 4\pi \rho_{(e)}$$

(1.1) $$C \operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \qquad \operatorname{div} \vec{B} = 0,$$

where C is the speed of light, $\rho_{(e)}$ — charge per unit volume, $\vec{J}_{(e)}$ — electric current density. The low velocity relations between external field quantities are:

(1.2) $$\vec{D} = \vec{E}; \quad \vec{H} = \vec{B} - 4\pi \vec{M}$$

where $\vec{M}$ is magnetization vector per unit volume. The free-space electromagnetic energy per unit volume is given by:

(1.3) $$u_{(e)} = \frac{1}{8\pi}(E_i E^i + B_i B^i).$$

The quantities appearing in Eqs (1.1) are those, as observed from a frame fixed in space.

2. The spin angular momentum.

We define the magnetization vector density:

(2.1) $$\vec{\mu} = \frac{1}{\rho} \vec{M}$$

and since the medium is considered to be saturated, it follows:

(2.2) $$\vec{\mu} \cdot \vec{\mu} = \mu_S^2 = \text{const} \rightarrow \vec{\mu}\, \dot{\vec{\mu}} = 0 \rightarrow \dot{\vec{\mu}} = \vec{\omega} \times \vec{\mu},$$

where $\vec{\omega}$ is the angular velocity of magnetization vector.

We assign to particles such as electrons an intrinsic angular momentum $\vec{s}$ called spin angular momentum, related to magnetizaton vector by:

(2.3) $$\vec{s} = \Gamma^{-1} \vec{\mu},$$

where the quantity Γ^{-1} is called-the gyromagnetic ratio.

Introducing the spin angular momentum per unit volume:

(2.4) $$\vec{G} = \Gamma^{-1} \vec{M},$$

we consider $d\vec{G}/dt$ as d'Alamberian inertia couple. From (2.2) we conclude:

(2.5) $$\frac{d\vec{G}}{dt} \cdot \vec{\omega} = 0,$$

i. e. its rate of work vanishes.

3. Kinematics.

We assume that there exists a referent configuration in which the material is undeformed and unstressed, the temperature field and the magnetization field--uniform, the electric current-zero. We assign to the referent configuration the moment $t=0$. The position of the material points of the body in the referent configuration is related to the material system of coordinates X^K. The point X^K at $t>0$ is carried to the spatial place x^k. Relations:

$$x^k = x^k(X^K, t) \tag{3.1}$$

represent equations of motion. The velocity of the point X^K at t is given by:

$$v^k = \dot{x}^k(X^K, t). \tag{3.2}$$

4. The equations of balance.

We assume that during the thermodynamic process, the law of conservation of mass remains to be valid:

$$\rho_0\, dV = \rho\, dv \rightarrow \overline{\dot{dm}} = 0 \rightarrow \frac{\partial \rho}{\partial t} + (\rho v^k)_{,k} = 0, \tag{4.1}$$

where ρ_0, dV relate to the reference configuration and ρ, dv are density and volume element in an arbitrary configuration.

The law of conservation of charge may be written in the form:

$$\frac{1}{C}\dot{\rho}_{(e)} + \operatorname{div}\vec{J}_{(e)} = 0, \tag{4.2}$$

where $\rho_{(e)}$ is the density of charge.

The volume concentration of diffused mass at a point of the body is denoted by $c(x^k, t)$. We assume that there is no body sources of mass production and obtain the local form of balance equation:

$$\rho\dot{c} = \operatorname{div}\vec{J}_{(m)} \tag{4.3}$$

with notation $\vec{J}_{(m)}$ for the flux vector of diffused mass.

The balance of spin angular momentum will be written in accordance to Eqs. (2.2)—(2.5), motivated by the classical formula of precession. The couples acting at the electronic spin continuum are connected not only to the external magnetic induction, but also to the local material magnetization field $\vec{B}_{(L)}$, as well as to the action of magnetic exchange field $\vec{F}^{(\mu)}$. The couples acting on the material are of the form $\vec{M}\times\vec{B}$ [5]. Starting from the global form of balance, we obtain:

$$\frac{d}{dt}\int_v \rho/\Gamma\,\vec{\mu}\,dv = \int_v \rho\vec{\mu}\times(\vec{B}+\vec{B}_{(L)})\,dv + \oint_s \rho\vec{\mu}\times\vec{F}^{(\mu)}\,ds. \tag{4.4}$$

Application of (4.4) to an elementary tetrahedron yields in usual manner the definition of the magnetic exchange tensor A_{kl}:

$$F^{(\mu)k} = A^{kl} n_l \tag{4.5}$$

where $\vec{n}$ is the unit normal at a point of the boundary s enclosing the arbitrary volume v. By virtue of Eq. (4.5), without any loss of generality, it may be taken [5]:

$$A_{\dot{k}}{}^{l} \mu^k = 0; \qquad B_{(L)k} \mu^k = 0. \tag{4.6}$$

Substituting from Eq. (4.6) into Eq. (4.4) and utilizing the divergence theorem, we obtain the local form of balance of spin angular momentum:

$$\Gamma^{-1} \dot{\mu}^i = e^{ijk} \mu_j [B_k + B_{(L)k} + \rho^{-1} (\rho A_{\dot{k}}{}^{l}),_l], \tag{4.7}$$

with $(A_{\dot{k}}{}^{l} \mu_{j,l})_{[j,k]} = 0$. The last condition is connected with Eq. (2.2).

5. Balance of energy.

The global form of energy balance yields:

$$\dot{U} + \dot{K} + \dot{U}_{(e)} = P + Q_{(T)} + Q_{(m)} + Q_{(e)} + Q_{(\mu)}, \tag{5.1}$$

where on the left-hand side there are the rates of internal, kinetic and electromagnetic energy and the right-hand side provides the mechanical power-P, the heating-$Q_{(T)}$, rate of working due to thermodiffusion $-Q_{(m)}$, due to the electromagnetic field $-Q_{(e)}$ and due to the exchange forces $-Q_{(\mu)}$. Introducing the density u of the internal energy, we represent Eq. (5.1) in the form:

$$\begin{aligned} &\int_v \rho (\dot{u} + \dot{v}^k v_k)\, dv + \frac{d}{dt} \int_v u_{(e)}\, dv = \int_v \rho f^i v_i\, dv + \oint_s T^i v_i\, ds + \int_v \rho h\, dv + \\ &+ \oint_s q\, ds + \int_v \rho M \dot{c}\, dv + \oint_s \left[u_{(e)} v^k - \frac{C}{4\pi} (\vec{E} \times \vec{H})^k \right] n_k\, ds + \oint_s \rho (\vec{\mu} \times \vec{F}^{(\mu)}) \vec{\omega}\, ds . \end{aligned} \tag{5.2}$$

In Eq. (5.2) f^i is density of body forces, T^i – surface tractions, h – density of body supply of heat, q – influx of heat per unit area in unit of time, M – chemical potential, and the two last integrals contain: the convective flux of electromagnetic energy, the Pointing vector representing the influx of electromagnetic energy and the rate of working of surface couples due to exchange forces.

We introduce the convection current [8]:

$$\vec{j}_{(e)} = \vec{J}_{(e)} - \frac{1}{C} \rho_{(e)} \vec{v} \tag{5.3}$$

and the electromotive intensity at a point moving with the particles of the body:

$$\vec{\varepsilon} = \vec{E} + \frac{1}{C} \vec{v} \times \vec{B}. \tag{5.4}$$

Bearing in mind that:

$$\frac{d}{dt}\int_v u_{(e)}\,dv=\int_v \frac{\partial u_{(e)}}{\partial t}\,dv+\oint_s u_{(e)}\,v^k n_k\,ds \tag{5.5}$$

and after performing an identical transformation, we obtain from Eq. (5.2), the following expression:

$$\begin{aligned}\int_v \rho\,(\dot u+\dot v^k v_k)\,dv= &\oint_s (T^i-t^{ij}n_j)\,v_i\,ds+\oint_s (q-q^i n_i)\,ds+\\ &+\oint_s \rho\,\dot\mu^k (F_k^{(\mu)}-A_k^{\cdot l} n_l)\,ds+\int_v \rho\,(f^k v_k+h+M\dot c)\,dv+\\ &+\int_v\Big[t^{ij}{}_{,j}v_i+t^{ij}v_{i,j}+q^i{}_{,i}+\varepsilon_i j^i_{(e)}+\frac{1}{C}e^{ijk}J_{(e)i}B_j v_k+\\ &+\frac{1}{C}\rho_{(e)}\varepsilon^k v_k-\rho\mu_k\dot B^k+\rho\mu_k v^l B^k{}_{,l}+(\rho\dot\mu^k A_k^{\cdot l})_{,l}\Big]dv.\end{aligned} \tag{5.6}$$

Eq. (5.6) holds for an arbitrary volume v of the body, enclosed by s. The application of Eq. (5.6) to an elementar tetrahedron yields boundary conditions:

$$\begin{aligned}&t^{ij}n_j=T^i \qquad A_k^{\cdot l}n_l=F_k^{(\mu)}.\\ &q^i n_i=q\end{aligned} \tag{5.7}$$

After substituting the conditions (5.7) into Eq. (5.6) and introducing the relation following from Eq. (4.7):

$$\dot\mu^k\,[B_k+\rho^{-1}(\rho A_k^{\cdot l})_{,l}]=-\dot\mu^k B_{(L),k}, \tag{5.8}$$

we may write Eq. (5.6) in the form:

$$\begin{aligned}\int_v \rho\,(u+\dot v^k v_k)\,dv=\int_v\Big(&\rho f^i v_i+\rho h+\rho M\dot c+t^{ij}v_{i,j}+t^{ij}{}_{,j}v_i+q^i{}_{,i}++\varepsilon_i j^i_{(e)}+\\ &+\frac{1}{C}e^{ijk}J_{(e)i}B_j v_k+\frac{1}{C}\rho_{(e)}\varepsilon^i v_i+\rho A^{kl}\dot\mu_{k,l}+\mu^j B^{j,i}v_i-\rho\dot\mu^k B_{(L)k}-\dot{\overline{\mu^k B_k}}\Big)dv.\end{aligned} \tag{5.9}$$

The balance equation (5.9) must be invariant under superposed rigid body motions. We introduce following transformation into Eq. (5.9):

$$v_i\to v_i+a_i \tag{5.9}$$

$$v_{i,j}\to v_{i,j}+\Omega_{ij};\quad \dot\mu_i\to\dot\mu_i+\Omega_{ij}\mu^j;\quad \dot\mu_{i,k}\to\dot\mu_{i,k}+\Omega_{ij}\mu^j{}_{,k} \tag{5.10}$$

and require the invariance of this expression under transformation (5.10). This procedure renders equations of motion in the form:

$$t^{ij}{}_{,j}+\rho f^i+\frac{1}{C}e^{ijk}J_{(e)j}B_k+\frac{1}{C}\rho_{(e)}\varepsilon^i \tag{5.11}$$

and

(5.12) $$t^{[ij]} = \rho\, B^{[i}_{(L)}\, \mu^{j]}.$$

Finally we obtain the invariant form of internal energy equation. Its local form may be written by:

(5.13) $$\rho\, \dot{u} = t^{ij} v_{i,j} + q^i{}_{,i} + \rho\, h + \rho\, M\, \dot{c} + \varepsilon_i\, \overset{i}{j}_{(e)} + \rho\, A^{kl}\, \dot{\mu}_{k,l} - \rho\, \dot{\mu}^k B_{(L)k} - \overline{\mu_k\, B^k}.$$

We define now, analogically to Tiersten and Tsai [5], following function, which takes over the role of the thermodynamic potential $\chi = u + \mu^k B_k$ and find

(5.14) $$\rho\, \dot{\chi} = t^{ij} v_{i,j} + \rho\, h + q^i{}_{,i} + \rho\, M\, \dot{c} + \rho\, A^{kl}\, \mu_{k,l} - \rho\, \dot{\mu}_k\, B^k_{(L)} + \varepsilon_i\, \overset{i}{j}_{(e)}.$$

6. The dissipation function and thermodynamic forces.

In the case of a mechanically reversible model of behaviour, the entropy balance equation may be written in following local form:

(6.1) $$\rho\, T\dot{\eta} = \rho\, h + q^k{}_{,k} + \varepsilon_i\, \overset{i}{j}_{(e)}.$$

The equation (6.1) states that the increase of the entropy during the regarded process is influenced by heat and current conduction. We further express Eq. (6.1) in the following way:

(6.2) $$\rho\dot{\eta} = \frac{\rho h}{T} + \left(\frac{q^k}{T}\right)_{,k} + \frac{q^k T_{,k}}{T^2} + \frac{\varepsilon_i\, \overset{i}{j}_{(e)}}{T}.$$

The first two terms of the right-hand side of Eq. (6.2) represent the reversible part of entropy production:

(6.3) $$\rho\, \dot{\eta}_{(r)} = \frac{\rho h}{T} + \left(\frac{q^k}{T}\right)_{,k}$$

while the remaining part of Eq. (6.2) determines the entropy production due to the existence of irreversible process in the body:

(6.4) $$\rho\sigma = \frac{q^i\, T_{,i}}{T^2} + \frac{\varepsilon_i\, \overset{i}{j}_{(e)}}{T}.$$

In a irreversible process, the entropy production is non-negative, as well as the dissipation function defined by:

(6.5) $$\rho\Phi = \frac{q^i\, T_{,i}}{T} + \varepsilon_i\, \overset{i}{j}_{(e)}$$

Hence:

(6.6) $$\rho\sigma \geqq 0; \quad \rho\Phi \geqq 0.$$

We represent the dissipation function in terms of products of irreversible thermodynamic forces and corresponding fluxes:

(6.7) $$\rho\, \Phi = Q_{(a)}\, \dot{q}^{(a)},$$

In our case we choose for independent variables, following forces:

$$\left\{\frac{T_{,k}}{T};\ \varepsilon_k\right\}. \tag{6.8}$$

As a sequence of expressions (6.8) following fluxes may be identified:

$$\{q^k;\ \dot{j}^k_{(e)}\}. \tag{6.9}$$

The nonlinear relations between irreversible thermodynamic forces and fluxes can be obtained using the Ziegler's [9] principle of least irreversible force. In that way, representing the dissipation function as function of following arguments:

$$\Phi = \Phi\left(\frac{T_{,k}}{T},\ \varepsilon_k\right), \tag{6.10}$$

we write the nonlinear constitutive equations for the heat flux vector and for convection current in the form:

$$q^k = \lambda \frac{\partial \Phi}{\partial\left(\frac{T_{,k}}{T}\right)};\quad \dot{j}^k_{(e)} = \lambda \frac{\partial \Phi}{\partial \varepsilon_k} \tag{6.11}$$

where:

$$\lambda = \rho\Phi\left\{\frac{\partial \Phi}{\partial\left(\frac{T_{,k}}{T}\right)} \frac{T_{,k}}{T} + \frac{\partial \Phi}{\partial \varepsilon_k}\varepsilon_k\right\}^{-1}. \tag{6.12}$$

7. The free energy and the constitutive equations.

Introducing the free energy density:

$$\psi = \chi - T\eta \tag{7.1}$$

we obtain:

$$\rho\dot{\psi} = t^{ij} v_{i,j} + \rho M\dot{c} + \rho A^{kl}\dot{\mu}_{k,l} - \rho\dot{\mu}_k B^k_{(L)} - \rho\eta\dot{T}. \tag{7.2}$$

Equation (7.2) may be written in the following way:

$$\rho\dot{\psi} = g_{il} t^{ij} X^K_{;j}\dot{x}^l_{;K} + \rho g_{il} A^{ij} X^K_{;j}\dot{\mu}^l_{;K} + \rho M\dot{c} - \rho g_{kl} B^k_{(L)}\dot{\mu}^l - \rho\eta\dot{T}. \tag{7.3}$$

From (7.3) we conclude that the free energy depends on following arguments:

$$x^l_{;K},\ \mu^l,\ \mu^l_{;K},\ T,\ c. \tag{7.4}$$

Now we must recall that 23 derivatives appearing on the right-hand side of Eq. (7.3) are not independent, but on account of (2.2) are connected by four relations:

$$\mu_l\dot{\mu}^l = 0;\quad \mu_l\dot{\mu}^l_{;K} + \mu_{l;K}\dot{\mu}^l = 0. \tag{7.5}$$

Consequently, we must introduce four Lagrangian multipliers and on the ground of (7.3) and (7.5) we proceed as follows:

$$
\begin{aligned}
(7.6)\qquad & \left[g_{il}\, t^{ij} X^K_{;j} - \rho \frac{\partial\psi}{\partial x^l_{;K}}\right] \dot{x}^l_{;K} + \rho\left(g_{il}\, A^{ij} X^K_{;j} - \frac{\partial\psi}{\partial \mu^l_{;K}} - N^K \mu_l\right) \dot{\mu}^l_{;K} - \\
& - \rho\left(g_{kl}\, B^k_{(L)} + \frac{\partial\psi}{\partial \mu^l} + \nu\mu_l + N^K \mu_{l;K}\right) \dot{\mu}^l + \rho\left(M - \frac{\partial\psi}{\partial c}\right) \dot{c} - \rho\left(\eta + \frac{\partial\psi}{\partial T}\right) \dot{T} = 0;
\end{aligned}
$$

where ν and $N^K (K=1,\ 2,\ 3)$ are the undetermined Lagrangian multipliers. Since we have introduced the required number of multipliers, we treat now all 23 velocities, as if they were independent and so we find:

$$
\begin{aligned}
& t^{ij} = \rho\, g^{il} \frac{\partial\psi}{\partial x^l_{;K}}\, x^j_{;K}; \\
& A^{ij} = g^{il} \frac{\partial\psi}{\partial \mu^l_{;K}}\, x^j_{;K} + N^K X^j_{;K}\, \mu^i; \\
(7.7)\qquad & B^k_{(L)} = -\left(g^{kl} \frac{\partial\psi}{\partial \mu^l} + \nu\mu^k + N^K \mu^k_{;K}\right); \\
& M = \frac{\partial\psi}{\partial c} \\
& \eta = -\frac{\partial\psi}{\partial T}.
\end{aligned}
$$

The Lagrangian multipliers may be determined, reminding the introduced relations (4.6). Substituting from (7.6/2,3) respectively into Eqs. (4.6), we obtain:

$$
(7.8)\qquad \left(g^{il} \frac{\partial\psi}{\partial \mu^l_{;K}}\, \mu^j_{;K} + N^K X^j_{;K}\, \mu^i\right) \mu_i = 0; \qquad \left(g^{kl} \frac{\partial\psi}{\partial \mu^l} + \nu\mu^k + N^K \mu^k_{;K}\right) \mu_k = 0\,.
$$

From Eqs. (7.8) result following expressions for N^K and ν:

$$
(7.9)\qquad N^K = -\frac{\partial\psi}{\partial \mu^l_{;K}} \frac{\mu^l}{\mu_s^2}; \quad \nu = -\frac{\partial\psi}{\partial \mu^l} \frac{\mu^l}{\mu_s^2}.
$$

In that way we finely obtain the constitutive expressions which are in accordance to the saturation condition (2.2):

$$
\begin{aligned}
& t^{ij} = \rho\, g^{il} \frac{\partial\psi}{\partial x^l_{;K}}\, x^j_{;K}; \quad A^{ij} = \frac{\partial\psi}{\partial \mu^l_{;K}}\, x^j_{;K} \left(g^{il} - \frac{\mu^i \mu^l}{\mu_s^2}\right); \\
(7.10)\qquad & B^k_{(L)} = \frac{\partial\psi}{\partial \mu^l}\left(\frac{\mu^k \mu^l}{\mu_s^2} - g^{kl}\right) + \frac{\partial\psi}{\partial \mu^l_{;K}} \frac{\mu^k_{;K}\mu^l}{\mu_s^2}; \quad M = \frac{\partial\psi}{\partial c}; \quad \eta = -\frac{\partial\psi}{\partial T}.
\end{aligned}
$$

Expressions for A^{ij} and $B^k_{(L)}$ coincide with corresponding results in the theory of Tiersten and Tsai [5].

The conditions of objectivity (5.12) represent now a system of three linear partial differential equations:

$$\left(g^{il}\frac{\partial\psi}{\partial x^{l}_{;K}}x^{j}_{;K}-\mu^{[i}g^{j]l}\frac{\partial\psi}{\partial\mu^{l}}+\mu^{i}\mu^{j]}_{;K}\frac{\partial\psi}{\partial\mu^{l}_{;K}}\frac{\mu^{l}}{\mu_s^2}\right)=0. \tag{7.11}$$

From Eqs. (7.11) we need to obtain $23-4-3=16$ independent integrals.

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ТЕРМОДИФФУЗИЯ В УПРУГИХ, МАГНИТНО ЗАСЫЩЕННЫХ СРЕДАХ, ПРОВОДЯЩИХ ТОК ЭЛЕКТРИЧЕСТВА И ТЕПЛА. I. КОНСТИТУТИВНЫЕ УРАВНЕНИЯ

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Резюме

На основании первого и второго принципов термодинамики как и принципа найменьшего термодинамического усилия, предложенного Циглером, выведенны нелинейные уравнения описивающие поведение назначеного материяла в внешнем электромагнитном поле. Полученные уравнения представляют основ для построики сопряженных полевых уравнений по параметрам состояния.

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