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ON THE FATIGUE CRACK PROPAGATION IN DEFORMABLE MEDIA WITH NONLINEAR CHARACTERISTICS

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1. Introduction

It is a well established fact that the fatigue performances are greatly influenced by internal defects. In practice of major concern are the crack–type defects which are prone to propagate under varying loadings. Even when cracks are not present in a solid other types of defects constitue the preferential locus for the fatigue crack initiation. It appears that the understanding of the fatigue crack propagation process in order to predict the crack growth rate is of greatly fundamental and practical importance.

The first attempts to assess quantitativly the fatigue crack propagation process were based on empirical correlations of the crack propagation rates with the loading parameters (for ex [1]).

The several proposed models ([2] till [10]) for fatigue crack propagation evince a relationship of the general form

$$\frac{da}{dn} = Aa^{m+1} \ f(\sigma) \tag{1}$$

where a is the crack length; n the number of cycles; σ the applied cyclic stress; a and m are empirical constants and $f(\sigma)$ a function which analytic form depends on material and the type of loading . According to various proposals the constant m is independent of the material nature and has a values of $m = -1$ after [1]; $m = 0$ after [3], [4], [7]; $m = 1/2$ after [9] and $m = 1$ after [2], [8], but other experimental results show that m is material dependent [10] ranging from zero up to four.

It appears that eq (1) though resulting from some specific models remains quasi–empirical in nature because parameters as m and A have no particular physical meaning, and as a consequence a wide range of variation is expected which of course impose limitations of the outlined methods for establishing the laws of fatigue crack propagation.

In the followings a new proposed model for fatigue crack propagation is derived based on the evaluation of the cumulative damage in the cyclic–plastically deformed region at the tip of an extending crack. The proposed model correlates the fatigue crack propagation rate with the applied cyclic stress, the geometry of the element involved and the cyclic–plastic properties of the material.

2. The proposed model

In front of a propagating fatigue crack, due to the severe strain and stress concentration, the material undergoes a cyclic plastic straining. For the purpose of the present analysis the material will be considered to have a nonlinear cyclic true stress–strain characteristic of a power form:

$$\Delta\sigma = \sigma_o \Delta\epsilon^{\lambda} \tag{2}$$

where $\Delta\sigma$ is the true stress range, $\Delta\epsilon$ true strain range, λ the cyclic strain hardening exponent and σ_o a strength parameter. Eq(2) was proved to described the cyclic plastic behaviour of a wide class of metallic materials [11].

The stress and strain distribution ahead of a crack in a nonlinear material can be estimated, [12], based on the relationship between the effective stress and strain concentration factors K_σ and K_ϵ considered for the nonlinear material governed by eq.(2) and the elastic stress concentration factor K_e:

$$K_\sigma \, K_\epsilon = K_e^2 \tag{3}$$

This relation proposed by Neuber [12] for a special case of nonlinear media, was proved experimentally, [18], to be also valid for solids with nonlinear characteristic of a power form (eq.2). For the range of variation of stress and strain ahead of the fatigue crack tip, from eqs. (2) and (3) results the effective strain range $\Delta\epsilon$:

$$\Delta\epsilon = \Delta\epsilon_N \left[\frac{\Delta\sigma_{el}}{\Delta\sigma_N} \right]^{\frac{2}{1+\lambda}} \tag{4}$$

where: $\Delta\sigma_N$ is the nominal stress range at the crack tip.

The elastic stress range distribution $\Delta\sigma_{el}$ in the direction r of crack propagation can be inferred based on the fracture mechanics approach [13] according to:

$$\Delta\sigma_{el} = \Delta K / \sqrt{2r} \tag{5}$$

where: ΔK is the stress intensity factor range, which general expression has the from:

$$\Delta K = \Delta\sigma_\infty \sqrt{\beta\,(a,L)\,a} \tag{6}$$

with: $\Delta\sigma_\infty$ the applied stress range in a region remote from the crack; a the crack length and β(a,b) a correction factor pertaining to the geometry of the body in which the crack is propagating. Taking into account that the following relationship is fulfilled:

$$\Delta\sigma_N \left(1 - \frac{a}{L}\right) = \Delta\sigma_\infty \tag{7}$$

where : L is the length over which the crack propagates till the complete separation in the cross section of the stressed body.

Putting:

$$h\,(a,L) = \left(1 - \frac{a}{L}\right) \sqrt{\beta(a,L)\,a} \tag{8}$$

from eqs (4) till (8) results the expression of the effective strain range distribution for a given nominal strain range $\Delta\epsilon_N$, as function of the crack tip distance r in front of the propagating fatigue crack:

$$\Delta\epsilon = \Delta\epsilon_N \left[\frac{h\,(a,L)}{\sqrt{2r}}\right]^{\frac{2}{1+\lambda}} \tag{9}$$

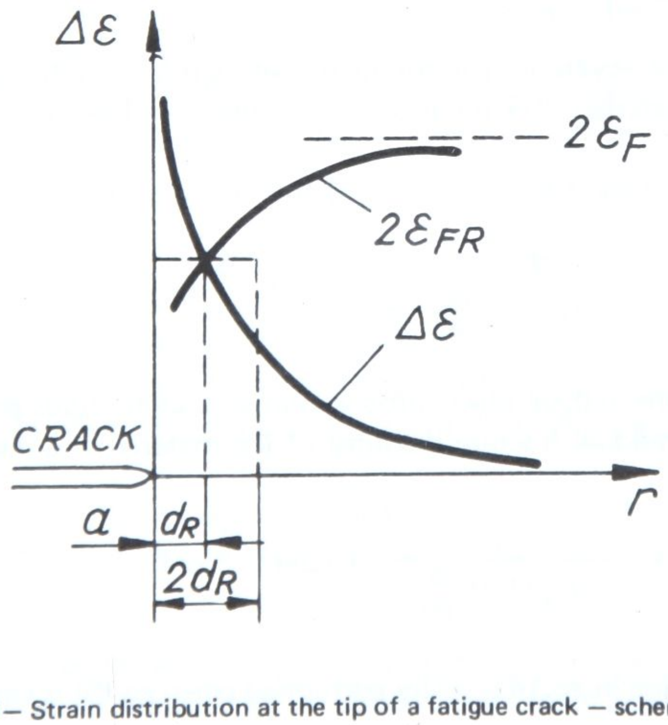

Fig. 1– Strain distribution at the tip of a fatigue crack – schematic

The $\Delta\epsilon - r$ variation is schematically illustrated in figure 1. Now it will be further considered that in a cycle the crack is advancing a distance:

$$\delta_R = \frac{da}{dn} \tag{10}$$

depending on the residual fracture ductility for a singular loading. But the value of is affected by the prior cumulative fatigue damage of the material due to the increasing strain as the fatigue crack tip is approaching.This process can be visualised as a material particle moving towards a stationary crack tip and thus entering a region with a continuously increasing straining in accordance with the effective strain range distribution ahead of the fatigue crack (eq.9). In results that the straining spectrum which determines the residual fracture ductility of the material particle is continuously increasing from cycle to cycle.

Low–cycle fatigue studies [14] reveals that the true residual fracture ductility resnlted after a sequences of n_i cycles at $\Delta\epsilon_i$ strain range levels to which N_i fatigue lives corresponds (i = 1,2. . .k) can be estimated by:

$$\epsilon_{FR} = \epsilon_F \left(1 - \sum_{i=1}^{R} \frac{ni}{Ni}\right)^{\alpha} \tag{11}$$

where: ϵ_F is the fracture ductility corresponding to the singular loading determined in a fracture test or resulted from an adjusting procedure of the low–cycle constant strains range amplitude test data according to a Manson – Coffin type relationship:

$$\Delta\epsilon_i^{1/\alpha} \; N_i = \frac{(2\epsilon_F)^{1/\alpha}}{4} \tag{12}$$

α – is a basic low–cycle fatigue parameter with a representative value of 0.5 for a wide range of materials. Taking into account that n_i = 1 because each strain range

$\Delta\epsilon_i$ appears only once, from eqs.(10),(11) and (12) it results:

$$\epsilon_{FR} = \epsilon_F \left[1 - \frac{4}{(2\epsilon_F)^{1/\alpha} \frac{da}{dn}} \sum_{i=1}^{k} (\Delta\epsilon_i)^{1/\alpha} d_R \right]^{\alpha} \tag{13}$$

Regarding the fatigue crack propagation as a continuous process from (13) results the true residual fracture ductility of the material at a distance r from the crack tip:

$$\epsilon_{FR} = \epsilon_F \left[1 - \frac{1}{(2\epsilon_F)^{1/\alpha} \frac{da}{dn}} \int_r^{\infty} (\Delta\epsilon)^{1/\alpha} \, dr\right]^{\alpha} \tag{14}$$

The integration in eq.14 is easily performed when eq.(9) is considered.

In this stage it will be considered as a basic physical assumption of the present model that the distance over which the material is separated in a cycle corresponds with the distance from the crack tip over which the strain range exceeds the true residual fracture ductility. In the representation illustrated in figure 1, the crack growth for a cycle is considered to be twice the distance d_R determined by the intersection of $2\epsilon_{FR}$ – curve given by eq. (14) and $\Delta\epsilon$ – curve given by eq.(9). The multiplication factor of two for the distance d_R appears from equilibrium condition at the crack tip. Thus:

$$\frac{da}{dn} = \delta_R = 2d_R \qquad (15)$$

where: d_R is the solution in r of the equation:

$$2\epsilon_{FR}(r) = \Delta\epsilon(r) \qquad (16)$$

From eqs.(9),(14),(15) and (16) it results the fatigue crack propagation rate as function of nominal strain range:

$$\frac{da}{dn} = h^2(a,L)\left[1 + \frac{2\alpha(1+\lambda)}{1-\alpha(1+\lambda)}\right]\left(\frac{\Delta\epsilon_N}{2\epsilon_F}\right)^{1+\lambda} \qquad (17)$$

or as function of nominal stress range:

$$\frac{da}{dn} = h^2(a,L)\left[1 + \frac{2\alpha(1+\lambda)}{1-\alpha(1+\lambda)}\right]\left(\frac{\Delta\sigma_N}{2\sigma_F}\right)^{\frac{1+\lambda}{\lambda}} \qquad (18)$$

It is worth to emphasis that the fatigue crack propagation rate as expressed in an alternative forms by eqs.(17)or (18) is determined by geometrical, material and loading parameters explicitly stated, which is not the case with the crack propagation rate relationship of the form (1) which can be applied only after an ,,a priori'' empirical correlation establishment of the pertinent parameters as A or m for the material involved.

Thus, according to the proposed model the fatigue crack propagation rate depends on:

a) the geometrical function $h^2(a,L)$ which concrete analitical form derives from the analitical expression of the stress intensitafactor K,[13], corresponding to the shape and size of the metalic element involved and to the particular loading type imposed.

b) the cyclic–plastic properties of the material, as determined in a low–cycle fatigue test i.e. the cyclic strain hardening exponent λ and the Manson–Coffin coefficient α.

c) the fracture properties as expressed by the true fracture ductility ϵ_F or the true fracture stress σ_F determined in a monotonic static fracture test or by a correlation procedure from the low–cycle fatigue test data. It appears that due to the fact that ϵ_F and σ_F, refers to a situation of triaxial state of stress and strain at the crack tip, the pertinent low–cycle fatigue test or static test data should be determined in conditions of severe notch concentration.

d) the loading intensity expressed by the nominal strain range $\Delta\epsilon_N$ or stress range $\Delta\sigma_N$.

3. The comparison with experimental results

In order to check the validity of the fatigue crack propagation rates as derived from the proposed model, tests were performed with carbon steel of 52 type in as received conditions loaded in plane alternate bending. The test specimens were 30 mm wide and 2 mm thick with a 2 mm long nad 0,2 mm wide central slit. The crack propagation was measured under microscope with a micrometric device.

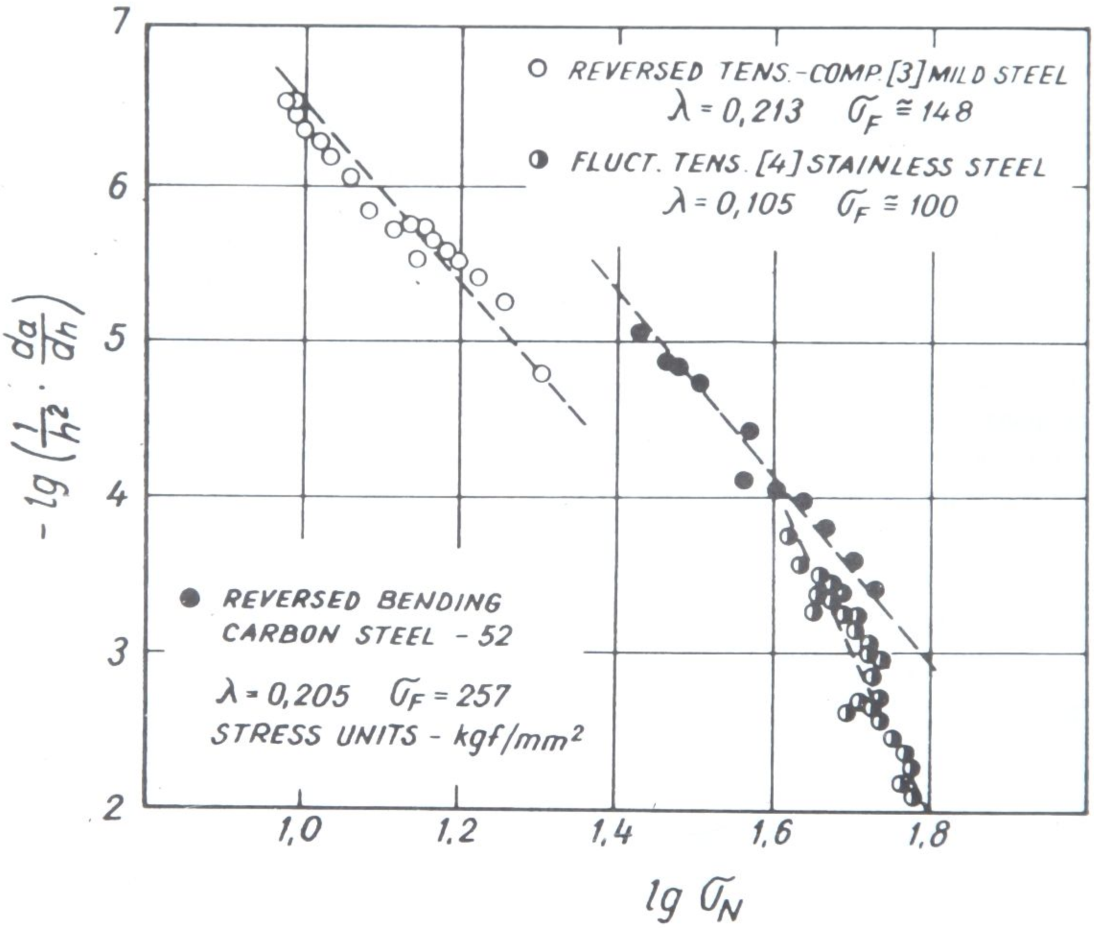

Fig. 2– Comparison of the experimental data with the theoretical prediction – eq.18

The obtained experimental results are plotted in figure 2 in appropriate coordinates which enables a linear correlation to be made based on eq.18 i.e. $y = \lg \left(\frac{1}{h^2} \frac{da}{dn}\right)$ and $\chi = \lg \sigma_N$. It can be seen from figure 2 that the experimental results follow reasonably a stright line with a slope $A = \frac{1+\lambda}{\lambda}$ and an ordinate at origin:

$$C = \lg \left\{ 2\sigma_F^{-\frac{1+\lambda}{\lambda}} \left[1 + \frac{2\alpha(1+\lambda)}{1-\alpha(1+\lambda)} \right]^{\alpha} \right\} \qquad (19)$$

From the slope A, the cyclic strain hardening exponent λ can be determined, while from C (eq.19) the true fracture stress due to the fact that α has a representative constant value as stated previously.

This comparative analysis was extended to some known experimental results as those obtained by Frost [15] with a carbon steel with 0,22 % C, subjected to cyclic alternate axial loading and those obtained by Radhakrishnan [16] with a 18–8 type stainless steel heated to 1050°C and oil quenched, subjected to fluctuating tensile loading. The analysis of these experimental results illustrated in figure 2 evince the same linear trend predicted by the theory.

It can be seen that the resulted values of the cyclic strain hardening exponent and true fracture stress of the stainless steel are smaller than of the normal carbon steels which is in line with the known experimental results. The true fracture stress of the carbon steel was found to be 148 daN/mm² approaching the value obtained with the same type of steel by Safta [17] in a static tensile stress with notched specimens with the theoretical stress concentration facotr K_e = 4,4 who found a true fracture stress of 137 daN/mm². The difference can be accounted on the greater degree of stress concentration in case of a propagating fatigue crack.

4. Conclusions

A model for fatigue crack propagation in solids was derived based on the evaluation of the cumulative damage in the cyclic–plastically deformed region at the tip of an extending crack. The proposed model correlates the fatigue crack propagation rate with the nominal stress range, geometric factors as expressed by a function of instantaneous crack length, which concrete analytical form is given by the crack dependent part of the stress intensity factor, the nonlinear cyclic plastic and fracture properties of the solid. It may, be remarked that the parameters involved in the present fatigue crack propagation model have a clear physical meaning in terms of the basic plastic and fracture properties.

The results of experimental invesigations concerning the fatigue crack propagation in steels support well the proposed model.

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Zusammenfassung

Aufgrund der Ermittlung der Schädigungshäufung in der zyklisch–plastisch verformter Zone an der Spitze eines sich ausbreitenden Risses wurde ein Modell für die Fortpflanzung von Ermüdngsrissen in verformbaren Mitteln abgeleitet. Das Modell stellt eine Beziehung zwischen der Fortpflanzungsgeschwindigkeit von Ermüdungsrissen und den für die Belastung Versuchsprobenform bestimmenden Parameter, Risslänge und die zyklischplastischen sowie die Brucheigenschaften des Materials her. Es wird eine gute Übereinstimmung der Versuchsergebnisse mit dem vorgeschlagenen Modell für die Fortpflanzung von Ermüdungsrissen in Stählen festgestellt.

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