PREFACE

This special issue of the journal Publications de l'Institut Mathématique is devoted to the topic Quasiconformal and harmonic mappings (Geometric function theory). Quasiconformal (qc) and harmonic mappings play an important role, among other places, in the theory of Riemann surfaces, theory of univalent functions, Teichmüller theory, complex dynamics (qc surgery), complex manifolds, geometric theory and differential geometry, hyperbolic 3-manifolds and string theory.

We shall discuss the connections between some of these topics below.

The natural question of how to make a concrete surface in R^3 into a Riemann surface leads to qc. Gauss considered a piece of a smooth oriented surface in Euclidean space and embedded it conformally into the complex plane. In studying Riemann surfaces, one is naturally led to a study of their moduli including Teichmüller's moduli space of surfaces.

1. *Theory of univalent functions.* The interplay between the theory of univalent functions and the theory of Teichmüller space is the main theme of Lehto's book [L]. Bers approach to Teichmüller theory, initiated in the early sixties, leads to quadratic differentials; this method can be applied to noncompact surfaces. The quadratic differentials are now Schwarzian derivatives of conformal extensions of qc mappings, obtained by the use of solutions of Beltrami differential equations. With this approach there arise several interesting problems belonging to the classical theory.

2. *Teichmüller theory and extremal problems.* The theory of Teichmüller spaces studies the different conformal structures on a Riemann surface. A classical approach deals with classes consisting of qc mappings of a Riemann surface which are homotopic modulo conformal mappings. There is a remarkable connection with quadratic differentials. On a compact Riemann surface of genus greater than one, every holomorphic quadratic differential determines a qc mapping which is a unique extremal in its homotopy class (in the sense that it has the smallest deviation from conformal mappings) and all extremals can be obtained in this manner.

For more details and further developments we refer to the bibliography below and the references therein.

3. *Geometric theory and differential geometry.* We refer to Tromba's book (see the review [W]) for the links between Teichmüller theory and differential geometry (Dirichlet problem, harmonic maps and Teihmüller theory) in which one is naturally, via the Plateau–Douglas problem, led to the study of the Dirichlet energy

integral $D(u)$ and its critical points, the harmonic maps, in their dependence upon complex structure and the points it represents in moduli space.

There is an idea to devote a number of special issues to the above topics. In this issue we cover mainly classical topics.

Reich has written in a recent survey [R] that during the last several years, important progress has been done in characterizing the conditions under which unique extremality occurs. This development is partially related to the Seminar at the University of Belgrade and it perhaps justifies publication of special issue in Belgrade. In order to popularize this subject further, we have invited experts in this field to contribute papers to this special issue.

We would like to thank the editors of Publications de l'Institut Mathématique for accepting this idea and Edgar Reich for support. We are also grateful to all the colleagues who contributed papers for this special issue.

The volume is divided into three parts. Part I contains 11 papers; 9 are related to qc mappings, including Zorich's survey which also points the way to the theory of qc mappings in several variables and Vasil'ev's paper on the parametric method for conformal map with qc extensions. The contribution of Brakalova and Jenkins includes their well-known previous result. Papers of Earle and Krushkal are related to Teichmüller theory.

Part II contains 3 papers related to harmonic mappings. Part III contains 6 papers on closely related topics including the Bers space that play an important role in qc theory (the paper of Fletcher and Marković on zeros of functions in the Bers space).

References

[K] Irwin Kra, Review of the book: F. P. Gardiner and N. Lakic, *Quasiconformal Teichmüller Theory*, Bull. Am. Math. Soc. 38(2) (2001), 255–265.

[L] Olli Lehto, *Univalent Functions and Teichmüller Spaces*, Springer-Verlag, 1987.

[R] Edgar Reich, *Extremal quasiconformal mappings of the disk*, in: R. Kühnau (ed.), *Handbook of Complex Analysis: Geometric Function Theory, Volume 1*, Elsevier, 2002.

[W] Michael Wolf, Review of the book: A. J. Tromba, *Teichmüller Theory in Riemannian Geometry*, Bull. Am. Math. Soc. 29(2) (1993), 285–290.

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