ON THE DISTRIBUTION OF THE EXCENTRICITIES AND INCLINATIONS OF ASTEROIDS

By

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INTRODUCTION

In volume I of these Publications I gave a short summary [1] of some conclusions obtained from researches on the distribution of the longitudes of the ascending nodes and perihelia of the orbits of the minor planets known at the present time. This paper is intended to summarise some remarks on the distribution of the excentricities of the orbits and the inclinations of the orbital planes of these bodies. The first investigations into these elements go back nearly a hundred years. Actually, no more than fifteen such bodies were known at that time, but already endeavours were being made [2] to see whether there existed any law in the distribution of these elements, or connection between the elements of different bodies. But these researches did not yield any significant or precise conclusion. Neither attempts with individual orbits and suitable groupings of them, nor with orbits related to the invariable plane of the solar system or to the solar equatorial plane, led to definite results of any importance.

Nevertheless, some astronomers at that time believed it possible to infer from their studies a relation between the mean inclinations (related to the solar equator) and those of the excentricities of groups of minor planets; whereas others, surprised by the discrepancies between the computed an observed (mean) values, refused to admit its existence.

But, modest as they may seem, the results of these first investigations indicated that it would be necessary first to obtain knowledge of a far greater number of these bodies and then to proceed to tackle the problem of the structure of the asteroid ring. In other words, they contributed to convince astronomers, in spite of Gauss' opinion [3]: „Man möge unter den kleinen Planeten die interessanteren und helleren auswählen, sich mit ihnen anhaltend beschäftigen und die übrigen ihrem Schicksal überlassen", — not only that one should not abandon a single minor planet to its fate, but rather that should try to discover as many new ones as possible. And, indeed, this latter alternative was chosen by the astronomers of that time. It is true that it was incomparably more exacting and onerous than the former, both for observers and computers, but for astronomy it was undoubtedly more fruitful. Thus, inasmuch as the number of the known asteroids increased, the prospects for the investigation of the structure of the system in general, as well as of the forms and positions of their orbits in particular, became more promising. This is also confirmed by the long list of works [4 - 17] consecrated to these problems.

Now, if we consider the results and conclusions obtained from these studies, we have the impression that our present knowledge, as far as is known, of the constitution of the asteroid ring, is still rather modest. For instance, only one common property of the ring can be inferred from the researches on excentricities, i. e. that the mean excentricities of the orbits of minor planets increase when their longitudes of perihelia approach that of Jupiter, and decrease when their longitudes of perihelia approach the aphelion of Jupiter.

The relationship between the mean excentricities and inclinations, pointed out in the first researches and reiterated several times since [6, 7], has not been affirmed, at least not in a sufficiently clear and general manner, as has the above mentioned property.

As for the distribution of inclinations, this needs, in our opinion, more detailed and profound consideration.

In this paper are given some results of a study of the distribution of the excentricities and inclinations of the orbits of minor planets, made during the period 1942–1944. It was based on the values of the orbital elements of 1500 minor planets, borrowed from the pamphlet „Kleine Planeten" for the year 1942.

The distribution of these 1500 bodies, according to both elements, was studied by dividing the whole material into six separate sections, $S_A - S_F$, each comprising 250 minor planets, following the chronological order (or nearly so) of their discovery. In proceeding thus, two objects were aimed at. On the one hand, it was hoped to establish whether there is anything permanent and to what degree, in the character studied, or not. On the other hand, it was hoped to show whether in any degree the dimensions of these bodies had any influence upon the character studied.

Further, in order to obtain an idea of the behaviour of the character of the entire ring according to the increasing number of known planets, numerical results have been given also for the groups of consecutive sections. In other words, the whole material was studied in the following manner:

Sections	*N⁰. of m. pl.*	*Groups*	*of Sections*	*N⁰. of m. pl.*
S_A . . .	1 – 250	G_A	S_A	1 – 250
S_B . . .	251 – 500	G_B	S_A to S_B	1 – 500
S_C . . .	501 – 750	G_C	S_A to S_C	1 – 750
S_D . . .	751 – 1000	G_D	S_A to S_D	1 – 1000
S_E . . .	1001 – 1250	G_E	S_A to S_E	1 – 1250
S_F . . .	1251 – 1500	G_F	S_A to S_F	1 – 1500

I. DISTRIBUTION OF EXCENTRICITIES

Table I shows the numbers which served to characterize numerically the distribution of the excentricities. The first column of the table gives the excentricities divided into intervals of 0·05; the next six columns contain the numbers of minor planets according to the excentricity in each of the six sections, $S_A - S_F$; the last column gives the mean numbers of the minor planets, expressed in percentage for each class-interval, of the six sections.

Table I

Excentricity e	Numbers of minor planets						Means in $^0/_0$
	S_A	S_B	S_C	S_D	S_E	S_F	
0·000 – 0·049	20	21	21	13	21	21	8·0
·050 – ·099	45	63	47	45	45	57	20·0
·100 – ·149	63	59	66	60	68	54	24·8
·150 – ·199	59	51	59	69	57	65	24·0
·200 – ·249	44	38	34	43	28	35	14·8
·250 – ·299	12	8	15	15	22	12	5·6
·300 – ·349	6	9	4	2	4	4	2·0
·350 – ·399	1	1	2	1	1	1	0·4
·400 – 0·449	0	0	1	0	1	0	0·0
0·450 – <	0	0	1	2	3	1	0·4

Since the distributions in each of the six sections rather deviate from the normal frequency curve, we have chosen for their approximation the frequency function called by Charlier type B. We recall that this function has the form

$$F(x) = B_0\psi(x) + B_1\Delta\psi(x) + B_2\Delta^2\psi(x) + \; . \; . \; . \; . \; . \; ,$$

B_0, B_1, B_2, ... being coefficients independent of x and the successive differences $\Delta\psi(x)$, $\Delta^2\psi(x)$, ... of the function of probability $\psi(x)$ whose form is

$$\psi(x) = e^{-\lambda}\frac{\lambda^x}{x\,!}.$$

The coefficients of the frequency function for each of the six sections obtained by means of Table I are shown in Table II

Table II

Coeff.	S_A	S_B	S_C	S_D	S_E	S_F
e_m	0·150	0·142	0·150	0·157	0·153	0·145
φ_m	8°6	8°2	8°6	9°0	8°8	8°3
σ_e	0·072	0·076	0·078	0·075	0·084	0·074
σ_φ	4°1	4°1	4°5	4°3	4°8	4°2

In this table are designated: by e_m the mean value of the excentricity; by φ_m the mean value of the angle of excentricity; by σ the value of the standard deviation.

By examining attentively the above figures it can be noticed that both the mean values of the excentricities and the coefficients of the corresponding standard deviations differ very slightly from section to section. That is to say, if the values obtained are taken into account it becomes manifest that the mean excentricities, from section to section of the 250 minor planets, remain steady, nearly equal to 0·150 or the excentricity angles nearly equal to 8°·6.

For the six groups of minor planets, $G_A - G_F$, we obtain the following values for the coefficients of the different frequency curves:

Table III

Coeff.	G_A	G_B	G_C	G_D	G_E	G_F
e_m	0·150	0·146	0·147	0·150	0·151	0·150
φ_m	8°6	8°4	8°5	8°6	8°7	8°6
σ_e	0·072	0·074	0·078	0·076	0·085	0·077
σ_φ	4°1	4°2	4°5	4°4	4°9	4°4

The results of this table indicate that, although, as we know, the excentricities of a large number of minor planets discovered in the course of the last hundred and fifty years varied within the extensive limits (from 0·01 · · · to 0 650 or the angles of excentricity between 0° . . . and 40°), the mean value of the entire system of the known bodies does not show any tendency to depart from the value $e_m = 0{\cdot}150$. At the same time we might remark also that the mean values of the excentricity do not change with the decreasing dimensions of the planets.

However, one pecularity in the distribution of the excentricity of minor planets has been established. It was, to the best of our knowledge, A. von Brunn [5] who pointed it out first. He has, indeed, shown and verified that, under the action of Jupiter's secular perturbations, the mean value of the excentricity of minor planets having their perihelia situated from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$ on both sides of the Jovian perihelion is superior to that of the minor planets having their perihelia within $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$ of the Jovian aphelion. He reached the following results for the 414 minor planets then known:

Semi-circumf.	*Number of pl.*.	*Mean Excentr.*
around the perihelion of ♃	267	0·1544
around the aphelion of ♃	147	0·1275

In order to see how this pecularity in the distribution of excentricities manifests itself at present, we have proceeded in the following manner. Four sectors of 10° each on both sides of two rectangular diameters were taken: one very near to the line of apsides of Jupiter's orbit, the other perpendicular to this latter. These four sectors correspond respectively to the longitudes of perihelia 351° – 11°, 171° – 191°, 81° – 101° and 261° – 281°. Having then counted the minor planets whose perihelia were situated in each of these sectors and classified them as above into intervals of (0·05), we obtained the values given in table IV.

Table IV

Excentricity e	Numbers of minor planets			
	351°–11°	81°–101°	171°–191°	261°–281°
0 000 – 0·049	6	4	4	6
050 – ·099	14	15	12	14
·100 – ·149	24	15	9	12
·150 – ·199	23	21	7	16
·200 – ·249	21	17	6	12
·250 ·299	26	4	1	8
·300 – ·349	12	7	0	3
·350 – ·399	5	0	0	1
·400 - 0·449	6	1	0	0
0·450 – <	0	1	0	0
Σ	137	85	39	72

Starting from these data we find for the coefficients of frequency functions, representing the distribution of the excentricities in each of the four sectors, the following values:

Table V

Sector	351°–11°	81°–101°	171°–191°	261°–281°
Number of m. p.	137	85	39	72
e_m	0·166	0·140	0·102	0·130
φ_m	9°6	8°0	5°9	7°5

It follows that the sector adjacent to Jupiter's perihelion, containing 3·5 times more bodies than the sector diametrically opposed, is in reality characterized by a mean excentricity more than 10% greater than the general mean excentricity and more than 60% greater than the mean excentricity of planets having their perihelia in the sector comprising Jupiter's aphelion. Moreover, this table shows that the mean excentricity of minor planets indeed decreases as their perihelia move away and increases as their perihelia approach that of Jupiter.

II. DISTRIBUTION OF INCLINATIONS

The same procedure was applied to the study of the distribution of the inclinations, but by taking, instead of the angles, the sine of the inclination of the orbital plane in relation to the ecliptic. In order to express numerically the distribution of the minor planets they have been arranged in class-intervals increasing by 0·05 of the sine of inclination.

The results of counts are given in the Table VI

Tables VI

Inclinations (sin i)	Numbers of minor planets						Means in %
	S_A	S_B	S_C	S_D	S_E	S_F	
0·000 – 0·049	41	25	24	24	28	34	11·6
·050 – ·099	64	49	37	47	60	53	20·8
·100 – ·149	50	55	45	33	39	48	18·0
·150 – ·199	35	44	52	47	49	40	17·8
·200 – ·249	29	34	33	37	27	29	12·6
·250 – ·299	12	16	29	33	23	15	8·5
·300 – ·349	4	15	14	13	10	9	4·4
·350 – ·399	8	6	6	9	5	12	3·1
·400 – ·449	6	5	7	4	6	5	2·2
·450 – ·499	0	1	0	1	2	2	0·4
·500 – 0·549	0	0	2	1	0	0	0·1
0·550 – <	1	0	1	1	1	3	0·5

In order to obtain the approximation of the distributions in the different sections, the frequency function of type B has been used as before. Thus the following values have been obtained for the coefficients of the frequency functions:

Table VII

Coeff.	S_A	S_B	S_C	S_D	S_E	S_F
$(\sin i)_m$	0·142	0·163	0·180	0·179	0·161	0·164
i_m	8°2	9°4	10°4	10°3	9°3	9°4
σ_s	0·101	0·097	0·106	0·104	0·111	0·113
σ_i	5°8	5°6	6°1	6°0	6°4	6°5

It will be noticed here, first of all, that the mean values of the different sections do not remain, as in the case of the excentricities, (nearly) constant, but the mean inclinations vary from one section to the other. It appears that the section of the most massive minor planets is characterized by the smallest mean value $(\sin i)_m = 0{\cdot}142$; then the mean value slowly increases till it reaches the mean value $(\sin i)_m = 0{\cdot}179$, in sections $S_C - S_D$ corresponding to 500 minor planets numbered from 501 – 1000, to redescend then, in the two next sections, which include the 500 bodies recently discovered, to the mean value $(\sin i)_m = 0{\cdot}164$.

The standard deviations, although slightly more marked than those found for the excentricities, show here a vague tendency to increase.

Starting from these results, we also easily arrive at the coefficients (analogous to those for the sections), which characterise the distributions of the minor planets according to the second

Table VIII

Coeff.	G_A	G_B	G_C	G_D	G_E	G_F
$(\sin i)_m$	0·142	0·152	0·162	0·166	0·165	0·165
i_m	8°2	8°8	9°3	9°5	9°5	9°5
σ_s	0·101	0·100	0·101	0·107	0·107	0·106
σ_i	5°8	5°8	5°8	6°0	6°2	6°1

classification, i.e. divided into six groups, $G_A - G_F$, as explained in the Introduction. These numbers will show us if and how the mean inclinations of the whole system of minor planets have varied in the course of the last hundred and fifty years.

For the sake of comparison the mean values of the excentricities and inclinations are given in the following graph.

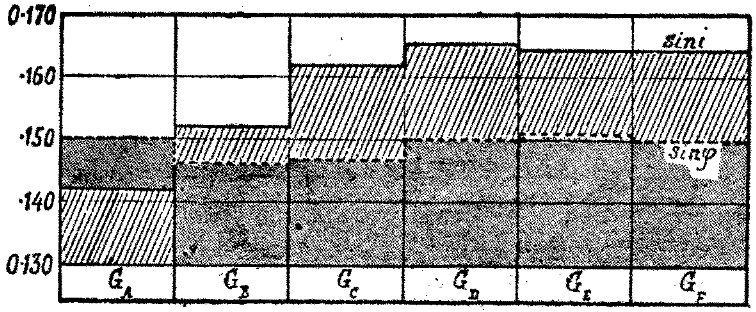

Fig. 1.

By examining the sequence of the mean values of the inclinations corresponding to the six groups, the question arises whether these numbers reproduce the studied character of the system of known planets, or if they could or should not be interpreted, at least partly, as a result of the conditions and circumstances that have governed the methodes of search and discovery of these bodies. Especially, whether the stop in the upward slope of the curve of the mean inclinations in sections S_A - S_C and the slight downward slope in the last two sections is real or only the consequence of the methods of observation and search. For, as is well known, the search for new minor planets was from the beginning rather limited to one relatively narrow band (zodiacal) situated symmetrically on both sides of the ecliptic. So, then, we may say that observers do not so much look for minor planets, as, so to speak, wait for them to pass through these (zodiacal) regions.

For it is clear that, given equal conditions, the chances of the discovery of planets differ according to whether their entire trajectories or only part of them are situated in the zone under observation. In other words, in so far as the greater part of the trajectory is in this zone, that is, in so far the inclination is under a certain value, the planet spends more time in the field under observation and we shall therefore discover it the more easily. The applied method of observation and of search for new minor planets is obviously more favourable to the discovery of objects with a small or moderate inclination, and less favourable to those with greater inclinations. There are, therefore, grounds for the opinion that the mean of the deduced inclination from the known orbital elements of the minor planets may be less than its real mean value, i.e. the mean value of the whole asteroid ring.

It is, nevertheless, a hypothesis which requires verification. In any case, taking into account confirmed facts only, we can state: while the excentricity of the mean orbit of the planetary ring has remained constant for these 150 years, the mean inclination of its orbital plane has gradually increased with the number of planets within it.

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О РАСПОРЕДУ ЕКСЦЕНТРИЧНОСТИ И НАГИБА ПЛАНЕТОИДСКИХ ПУТАЊА

Од

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У овом раду изложени су резултати проучавања распореда ексцентричности путања и нагиба путањских равни скупа од 1500 планетоида, познатих елемената, објављених у „Kleine Planeten“ за 1942.

У циљу да би се могле истаћи како сталне карактеристике распореда путањских елемената тако и евентуална зависност тих карактеристика од димензија планетоида, — проучен је распоред елемената, с једне стране, по „секцијама“ од по 250 планетоида према њиховим редним бројевима (дакле, приближно, према хронолошком реду њихових проналазака), а, с друге стране, по групама узастопних секција.

Из добивених резултата види се:

1) да средња ексцентричност планетоидских путања остаје са повећавањем броја планетоида тако рећи непромењена: $e = 0{\cdot}150$ или $\varphi = 8^0{\cdot}6$;

2) да су средње ексцентричности путања планетоида чији се перихели налазе у околини ($\pm 10^0$) Јупитерова перихела за око $10^0/_0$ већи ($e = 0{\cdot}166$) од опште средње ексцентричности, а за преко $60^0/_0$ већи од средње ексцентричности путања планетоида ($e = 0{\cdot}102$) чији се перихели налазе у околини ($\pm 10^0/_0$) Јупитерова афхела;

3) да су средњи нагиби путањских равни са повећавањем броја планетоида постепено расли: полазећи од $i = 8^0{\cdot}2$, код најмасивнијих, преко $i = 10^0{\cdot}4$, код средњих, до $i = 9^0{\cdot}4$, код најмање масивних познатих планетоида.

Објашњење тока последње карактеристике могло би се, можда, наћи у самој методи трагања за новим планетоидима, која су се досад ограничавала, углавном, на релативно уску зону око еклиптике.

Са практичног гледишта, прва карактеристика указује да би, при одређивању првих путањских елемената, природније било у првој апроксимацији полазити, место од кружних, од елиптичких елемената, са ексцентричношћу $e = 0 \cdot 150$.

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