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SOME COMMUTATIVITY THEOREMS FOR s-UNITAL RINGS WITH CONSTRAINTS ON COMMUTATORS

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Abstract. Continuing the investigation of [**1**], [**2**], [**3**] and [**10**], we prove here some commutativity theorems for s-unital rings R satisfying the polynomial identity $x^t[x^n,y]y^{t'} = \pm x^{s'}[x,y^m]y^s$, resp. $x^t[x^n,y]y^{t'} = \pm y^s[x,y^m]x^{s'}$, where m,n,s,s',t and t' are given non-negative integers such that $m>0$ or $n>0$ and $t+n\neq s'+1$ or $m+s\neq t'+1$ for $m=n$. The additional assumption in these theorems concern some torsion freeness of commutators in R.

1. Introduction. Throughout this paper R will be an associative ring (may be without identity 1), $Z(R)$ will represent the center of R, $N(R)$ the set of all nilpotent elements of R, and $C(R)$ the commutator ideal of R. By R' we denote the opposite ring of R, i.e. the ring with the same elements and addition as R, but with opposite multiplication $\circ$ defined by $x\circ y = yx$ for all x,y in R'. We will omit the sign $\circ$ of the multiplication in R', as it is usual for the sign $\cdot$ of the multiplication in R.

A ring R is called left, resp. right s-unital if $x\in Rx$, resp. $x\in xR$ for all x in R. If R is both left and right s-unital, then R is said to be s-unital. If R is s-unital (resp. left or right s-unital), then, for every finite subset F of R, there exists an element e in R such that $ex=xe=x$ (resp. $ex=x$ or $xe=x$) for all x in F.

By $[x,y]$ we denote the commutator $xy-yx$ of two elements x,y in a ring R. If n is a positive integer, then we say for R to have the property $Q(n)$ if commutators in R are n-torsion free, i.e. if $n[x,y]=0$ implies $[x,y]=0$ for all x,y in R. Obviously, any n-torsion free ring R has the property $Q(n)$, and if a ring R has the property $Q(n)$, then R has also the property $Q(m)$ for all divisors m of n. It is clear that R is left, resp. right s-unital if and only if R' is right, resp. left s-unital, and that, for any positive integer n, R has the property $Q(n)$ if and only if R' has this property.

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We investigate here the commutativity of a ring R which satisfies the polynomial identity

$$x^t[x^n, y]y^{t'} = \pm x^{s'}[x, y^m]y^s \qquad \text{for all } x, y \text{ in } R, \tag{1}$$

resp.

$$x^t[x^n, y]y^{t'} = \pm y^s[x, y^m]x^{s'} \qquad \text{for all } x, y \text{ in } R. \tag{1'}$$

For $t' = s' = 0$, the identity (1), resp. (1′) becomes

$$x^t[x^n, y] = \pm[x, y^m]y^s \qquad \text{for all } x, y \text{ in } R, \tag{2}$$

resp.

$$x^t[x^n, y] = \pm y^s[x, y^m] \qquad \text{for all } x, y \text{ in } R. \tag{2'}$$

If an identity with the sign $\pm$ occurring in it is denoted by (k), then we denote by (k_+), resp. (k_-) this identity with the sign $+$, resp. $-$ instead of $\pm$.

In [**10**] Psomopoulos proved the following result.

Theorem P [**10**, Theorem 1 and Theorem 2]. *Let R be a ring with identity 1 satisfying the polynomial identity (2_+) for some positive integers m, n and some non-negative integers s, t. If $n > 1$ and R is n-torsion free, then R is commutative. Also, if m, n are relatively prime, then R is commutative.*

For $s = t' = 0$, the identities (1) and (1′) reduce to

$$x^t[x^n, y] = \pm x^{s'}[x, y^m] \qquad \text{for all } x, y \text{ in } R \tag{3}$$

and

$$x^t[x^n, y] = \pm[x, y^m]x^{s'} \qquad \text{for all } x, y \text{ in } R, \tag{3'}$$

respectively. The commutativity of a left or right s-unital ring R satisfying (3) or (3′) has been investigated in [**1**]. Especially was proved

Theorem AP [**1**, Theorem 1]. *Let R be a left or right s-unital ring with polynomial identity (3) or (3′). If $m > 1$, $n \geq 1$, and R has the property $Q(m)$ for $n > 1$, then R is commutative.*

If $s = s' = 0$, the identities (1) and (1′) reduce to the identity

$$y^t[x^n, y]x^{t'} = \pm[x, y^m] \qquad \text{for all } x, y \text{ in } R \tag{4}$$

considered in [**2**]. For $s = t = 0$, (1) and (1′) become

$$[x^n, y]y^{t'} = \pm x^{s'}[x, y^m] \qquad \text{for all } x, y \text{ in } R \tag{5}$$

and

$$[x^n, y]y^{t'} = \pm[x, y^m]x^{s'} \qquad \text{for all } x, y \text{ in } R, \tag{5'}$$

respectively. Passing to the opposite ring R', the identities (5) and (5′) can be rewritten in the form

$$y^{t'}[x^n, y] = \pm[x, y^m]x^{s'} \qquad \text{for all } x, y \text{ in } R' \tag{6}$$

and

$$y^{t'}[x^n, y] = \pm x^{s'}[x, y^m] \qquad \text{for all } x, y \text{ in } R', \tag{6'}$$

respectively. For R instead of R', the last two identities were considered in [**3**].

For $m = n = 0$, any ring R satisfies both (1) and (1′). If

$$[\,[x, y], x] = 0 \qquad \text{for all } x, y \text{ in } R, \tag{7}$$

especially, if all commutators in R are central, then the identities (1) and (1′) can be rewritten in the form

$$nx^{n+t-1}[x, y]y^{t'} = \pm mx^{s'}[x, y]y^{m+s-1} \qquad \text{for all } x, y \text{ in } R. \tag{8}$$

Thus, for $m = n$, $m + s = t' + 1$ and $n + t = s' + 1$, any ring R satisfying (7), especially any ring R with central commutators, satisfies both (1_+) and $(1'_+)$. Therefore, for non-negative numbers in the identities (1) and (1′) we all along assume that $m > 0$ or $n > 0$, and $m \neq n$ if $n + t - 1 = s'$ and $m + s - 1 = t'$.

2. First we observe that under an additional assumption the integers m and n in Theorem P, can be interchanged. In fact, the theorem can be improved as follows:

THEOREM 1. *Let R be a ring satisfying* (2) *or* (2′) *for $m \geq 1$, $n \geq 1$, and having the property $Q(d)$ for $d = (m, n)$. If, moreover, R is left or right s-unital for $m + s > 1$ and $n + t > 1$, then R is commutative.*

Proof. By an argument used in the proof of [**1**, Lemma 4], we can prove that, form $m + s > 1$ and $n + t > 1$, the ring R is s-unital. Hence, for this case, we can assume that R is a ring with identity 1 (see [**7**, Proposition 1]).

If $n = 1$ and $t = 0$, then R is commutative by a special version of [**11**, Hauptsatz 3] stated in [**1**] which will be cited here as Theorem S.

If $n = 1$ and $t > 0$, then we set in (2), resp. (2′), $x + 1$ for x and combine the identity obtained with (2), resp., (2′) to get $((x + 1)^t - x^t)[x, y] = 0$ for all x, y in R. For $t = 1$ this means that $[x, y] = 0$ for all x, y in R, and thus, R is commutative. If $t > 1$, then the last identity yields $[x, y] = f(x)[x, y]$ for all x, y in R, where $f(X) \in Z[X]$ is a polynomial all monomials of which are of degree at least one. Hence, R is commutative by Theorem S.

Similarly, we can prove that R is commutative for $m = 1$.

Now, we suppose that $m > 1$ and $n > 1$. The proof, we give here for the sake of completeness, differs from the proof of Theorem P only in the final phase where we use Theorem S. To prove that $C(R) \subseteq N(R)$, by [**8**, Theorem 1], it suffices to take

$$x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{in} \quad Z^{2\times 2}$$

for the case of the identity (2). In the case of the identity (2′), one should take $\left[\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right]$ instead of $\left[\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right]$.

Next we prove that $N(R) \subseteq Z(R)$. Let a be an arbitrary element in $N(R)$. Then there exists a positive integer p such that

$$a^k \in Z(R) \quad \text{for all integers } k \geq p,\ p \text{ minimal.} \tag{9}$$

If $p = 1$, then $a \in Z(R)$. Suppose that $p > 1$. We set $b = a^{p-1}$ to get a contradiction. Obviously,

$$[b^k, x] = b^k[b, x] = [b, x]b^k = 0 \text{ for all } x \text{ in } R \text{ and all integers } k > 1. \tag{10}$$

In view of (10), the identity (2), resp. (2′) yields

$$x^t[x^n, b] = 0 \qquad \text{for all } x \text{ in } R. \tag{11}$$

Therefore, setting $1 + b$ for y in (2), resp. (2′), one gets, in account of (10) and (11),

$$m[x, b](1 + sb) = 0 \ \text{ for all } x \text{ in } R, \quad \text{resp.} \quad m(1 + sb)[x, b] = 0 \ \text{ for all } x \text{ in } R.$$

Hence, by (10), $m[x, b]b = 0$, resp. $mb[x, b] = 0$ for all x in R, and thus,

$$m[x, b] = 0 \qquad \text{for all } x \text{ in } R. \tag{12}$$

Similarly, from (2), resp. (2′) one gets

$$[b, y^m]y^s = 0 \qquad \text{for all } y \text{ in } R, \tag{13}$$

resp.

$$y^s[b, y^m] = 0 \qquad \text{for all } y \text{ in } R. \tag{13′}$$

By (13), resp. (13′), from (2), resp. (2′), we easily get

$$n(1 + tb)[b, y] = 0 \ \text{ for all } y \text{ in } R, \quad \text{resp.} \quad n[b, y](1 + tb) = 0 \quad \text{for all } y \text{ in } R,$$

hence, by (10), $nb[b, y] = 0$ resp. $n[b, y]b = 0$ for all y in R, and thus

$$n[x, b] = 0 \qquad \text{for all } x \text{ in } R. \tag{14}$$

Since R has the property $Q(d)$ for $d = (m, n)$, then (12) and (14) imply

$$[x, b] = 0 \quad \text{for all } x \text{ in } R, \quad \text{i.e.} \quad a^{p-1} \in Z(R), \tag{15}$$

which is a contradiction. Thus, we proved that

$$C(R) \subseteq N(R) \subseteq Z(R). \tag{16}$$

In view of (16) and [**9**, Lemma 3], the identities (2) and (2′) can be rewritten in the form

$$nx^{n+t-1}[x, y] = \pm m[x, y]y^{m+s-1} \qquad \text{for all } x, y \text{ in } R. \tag{17}$$

Now, setting $x+1$ for x in (17) and combining the identity obtained with (17), one gets

$$n((x+1)^{n+t-1} - x^{n+t-1})[x,y] = 0 \qquad \text{for all } x,y \text{ in } R. \tag{18}$$

Similarly, from (17), interchanging x and y, and taking in account (16), one derives

$$m((x+1)^{m+s-1} - x^{m+s-1})[x,y] = 0 \qquad \text{for all } x,y \text{ in } R. \tag{19}$$

For $m = dm_1$, $n = dn_1$, the integers m_1, n_1 are relatively prime, and by $Q(d)$, (18) and (19) imply

$$n_1[x,y] = f(x)[x,y] \qquad \text{for all } x,y \text{ in } R \tag{20}$$

and

$$m_1[x,y] = g(x)[x,y] \qquad \text{for all } x,y \text{ in } R, \tag{21}$$

where $f(X)$, $g(X)$ are polynomials in $Z[X]$ all monomials of which have degree at least one. Since m_1, n_1 are relatively prime, then from (20) and (21), for some integers m_2, n_2, it follows

$$[x,y] = (n_2 f(x) + m_2 g(y))[x,y] \qquad \text{for all } x,y \text{ in } R.$$

Hence, R is commutative by Theorem S.

In [**6**, Theorem 8] Harmanci showed that "If $n > 1$ and R is a ring with 1 which satisfies the identities $[x^n, y] = [x, y^n]$ and $[x^{n+1}, y] = [x, y^{n+1}]$ for all $x, y \in R$, then R must be commutative". Bell [**4**, Theorem 6] extended this result to any pair of relatively prime integers m and n instead of n and $n+1$. The following result, generalizing Bell's result, was proved in [**1**] as Theorem 8.

THEOREM 2. *Let $m > 1$ and $n > 1$ be fixed relatively prime integers, $m' \geq 1$, $n' \geq 1$, and r, s and t be given non-negative integers. If R is an s-unital* (*resp. left or right s-unital*) *ring satisfying both identities*

$$x^t[x^{m'}, y] = \pm y^r[x, y^m]x^s \quad \textit{and} \quad x^t[x^{n'}, y] = \pm y^r[x, y^n]x^s \quad \textit{for all } x,y \textit{ in } R,$$

or

$$x^t[x^{m'}, y] = \pm x^s[x, y^m]y^r \quad \textit{and} \quad x^t[x^{n'}, y] = \pm x^s[x, y^n]y^r \quad \textit{for all } x,y \textit{ in } R,$$

(*if $r = 0$*), *then R is commutative.*

Now, we prove the following similar result generalizing also Bell's result.

THEOREM 3. *Let m, n, m', n' be fixed positive, and s, s', t fixed non-negative integers. Further, let R be a ring satisfying both identities*

$$x^t[x^{m'}, y] = \pm x^{s'}[x, y^m]y^s \quad \textit{and} \quad x^t[x^{n'}, y] = \pm x^{s'}[x, y^n]y^s \quad \textit{for all } x,y \textit{ in } R \tag{22}$$

or

$$x^t[x^{m'}, y] = \pm y^s[x, y^m]x^{s'} \quad \textit{and} \quad x^t[x^{n'}, y] = \pm y^s[x, y^n]x^{s'} \quad \textit{for all } x,y \textit{ in } R. \tag{22'}$$

If, moreover, R is s-unital (*resp. left or right s-unital for $s' = 0$*), *and has the property $Q(d)$, where $d = (m,n)$* (*resp. $d = (m,n,m',n')$, for $s' = 0$*), *then R is commutative.*

Proof. Actually, R is s-unital, and thus by [**7**, Proposition 1], we can assume that R is a ring with identity 1.

For $m = 1$ or $n = 1$ (resp. $m' = 1$ or $n' = 1$ if $s' = 0$), we can see, as in the proof of Theorem 1 (using [**5**, Lemma] for $s' > 0$), that R is commutative. For $m > 1$ and $n > 1$ (resp. $m' > 1$ and $n' > 1$ for $s' = 0$), instead of (12), (resp. (14)), we get now (12) and (14) (resp.

$$m'[x,b] = 0 \quad \text{and} \quad n'[x,b] = 0 \quad \text{for all } x \text{ in } R). \tag{23}$$

By the property $Q(d)$ this implies (15). Similarly (for $s' = 0$), instead of (21) (resp. (20)), we have (20) and (21) (and also

$$m_1'[x,y] = f'(x)[x,y] \qquad \text{for all } x,y \text{ in } R, \tag{24}$$

$$n_1'[x,y] = g'(x)[x,y] \qquad \text{for all } x,y \text{ in } R, \tag{25}$$

where $f'(X)$, $g'(x)$ are polynomials in $Z[X]$ all monomials of which are of degree at least equal to one, and $m' = dm_1'$, $n' = dn_1'$ for $d = (m,n,m',n')$).

Since m_1 and n_1 (resp. m_1, n_1, m_1' and n_1') are relatively prime, (12) and (14) (resp. (12), (14), (24) and (25) for $s' = 0$) imply commutativity of R by Theorem S.

3. Now we prove a commutativity theorem for s-unital rings satisfying the polynomial identity (1), resp. (1′), where $m = n = 1$ and one of the other exponents is equal to zero.

THEOREM 4. *Let R be a ring satisfying the polynomial identity* (1), *resp.* (1′) *for $m = n = 1$ and $s' = 0$. Then R is commutative in any of the following cases*:

(a) $t \geq 1$, *and for $s > 0$, R is right, resp. left s-unital*:

(b) $t = 0$, *and $t' = 0$ or $s = 0$*;

(c) $t = 0$, $t' > 0$, $s > 0$, *R is an s-unital* (*resp. left or right s-unital*) *ring which satisfies* (1_-) (*resp.* ($1'_-$)), *or, for $s - t'$ odd,* (1_+) (*resp.* ($1'_+$), *and has the property* $Q(2)$;

(d) $t = 0$, $t' > 0$, $s > 0$, *$s - t'$ even, and R is an s-unital* (*resp. left or right s-unital*) *ring which satisfies* (1_+) *and the property* $Q((|s - t'| + 1)!)$ (*resp.* ($1'_+$) *and the property* $Q((\max\{s,t'\})!)$).

Proof. Case (a): For $s = 0$, R is commutative by Theorem S. If $s > 0$, then it is easy to see that R is in fact s-unital, and thus, by [**7**, Proposition 1], we can assure that for $s > 0$, R is a ring with identity 1.

Now, setting $x+1$ for x in (1), resp. (1′), and combining the identity obtained with (1), resp. (1′), one gets $((x+1)^t - x^t)[x,y]y^{t'} = 0$ for all x,y in R; hence, by [**5**, Lemma], we have $((x+1)^t - x^t)[x,y] = 0$ for all x,y in R.

For $t = 1$, the last identity means that R is commutative, and for $t > 1$, this identity implies the commutativity of R by Theorem S.

The cases (b), (c) and (d) follow from [**1**, Theorem 6].

Obviously, for $m = n = 1$ and any one zero exponent in (1), resp. (1′), we have an analogous result. All these results are corollaries of Theorem 4. We state here only the following one

Corollary 1. *Let R be a ring satisfying the polynomial identity* (1), *resp.* ($1'$) *for $m = n = 1$ and $s = 0$. Then R is commutative in any of the following cases*:

(a) $t' \geq 1$, *and for $s' > 0$, R is left, resp. right s-unital*;

(b) $t' = 0$, *and $t = 0$ or $s' = 0$*;

(c) $t' = 0$, $t > 0$, $s' > 0$, *and R is an s-unital* (*resp. left or right s-unital*) *ring which satisfies* (1_-) (*resp.* ($1'_-$)), *or for $s' - t$ odd,* (1_+) (*resp.* ($1'_+$)) *and has the property $Q(2)$*;

(d) $t' = 0$, $t > 0$, $s' > 0$, $s' - t$ *even, and R is s-unital* (*resp. left or right s-unital*) *ring which satisfies* (1_+) *and the property $Q((|s' - t| + 1)!)$* (*resp.* ($1'_+$) *and the property $Q((\max\{s', t\})!)$*).

Proof. From (1), resp. ($1'$) it follows

$$y^{t'}[x,y]x^t = \pm y^s[x,y]x^{s'} \qquad \text{for all } x, y \text{ in } R',$$

resp.

$$y^{t'}[x,y]x^t = \pm x^{s'}[x,y]y^s \qquad \text{for all } x, y \text{ in } R',$$

and thus, R' is commutative by Theorem 4. Hence, R is also commutative.

4. The assumption that in (1), resp. ($1'$), $s' = t' = 0$, makes Theorem 1 symmetrical with respect to m and n. Here we assume that in (1), resp. ($1'$), m, n, s and t are given positive integers, and that one of the given non-negative integers s' and t' is equal to zero. The result we will prove is the following theorem.

Theorem 5. *Let R be a ring with polynomial identity* (1), *resp.* ($1'$), *where m, n, s and t are given positive, and s', t' are given non-negative integers one of them being equal to zero, and the other positive. Then R is commutative in any of the following cases*:

(a) $s' = 0$ *and R is right, resp. left s-unital and has the property $Q(n)$ for* $n > 1$,

(b) $t' = 0$, *and R is left s-unital and has the property $Q(m)$ form $m > 1$*;

Proof. Case (a): It is easy to see that R is in fact s-unital, and thus we can assume that R is a ring with identity 1. For $n = 1$, by the same argument used in the proof of Theorem 1, one can show that R is commutative. If $n > 1$, using the property $Q(n)$, we can prove, as in the proof of Theorem 1, that $C(R) \subseteq N(R) \subseteq Z(R)$. Hence, the identities (1) and ($1'$) can be rewritten in the form $nx^{n+t-1}[x,y]y^{t'} = \pm m[x,y]y^{m+s-1}$ for all x, y in R. Now, setting in the last identity $x+1$ for x and combining the identity obtained with the one above, we get

$$n((x+1)^{n+t-1} - x^{n+t-1})[x,y]y^{t'} = 0 \qquad \text{for all } x, y \text{ in } R,$$

hence, by [**5**, Lemma] and the property $Q(n)$,

$$((x+1)^{n+t-1} - x^{n+t-1})[x,y] = 0 \qquad \text{for all } x, y \text{ in } R.$$

This yields commutativity of R by Theorem S.

Case (b): Since, for $t' = 0$, (1), resp. (1$'$) can be rewritten in the form

$$y^s[y^m, x]x^{s'} = \pm[y, x^n]x^t \qquad \text{for all } x, y \text{ in } R'$$

resp.

$$y^s[y^m, x]x^{s'} = \pm x^t[y, x^n] \qquad \text{for all } x, y \text{ in } R.$$

Hence, R' resp. R is commutative by the case (a), and thus, R is commutative.

5. In this section the commutativity of an s-unital ring R satisfying the polynomial identity (1) or (1$'$) shall be shown for some other special values of non-negative integers m, n, s, s', and t'. Since every of these results is similar to the corresponding result in [**1**], then they will be stated here without proof.

THEOREM 6. *Let R be an s-unital ring satisfying the polynomial identity* (1) *or* (1$'$). *Then R is commutative provided one of the following conditions is fulfilled*:

(a) $m = 0$ *and R has the property $Q(n)$*;

(b) $n = 0$ *and R has the property $Q(m)$*.

THEOREM 7. *Let R be an s-unital ring which satisfies the polynomial identity* (1) *or* (1$'$). *Suppose that at least one of the integers $n+t-s'-1$ and $m+s-t'-1$ is odd and that R has the property $Q(2)$. If, moreover, R has one of the properties $Q(m)$ and $Q(n)$, especially, if $(m, n) = 2^r$ for some non-negative integer r, then R is commutative.*

THEOREM 8. *Let R be an s-unital ring with polynomial identity* (1) *or* (1$'$). *Suppose that $n+t \neq s'+1$ or $m+s \neq t'+1$, and that R has the property $Q(k)$ for $k = |2^{n+t} - 2^{s'+1}|$ or $k = |2^{m+s} - 2^{t'+1}|$. If, moreover, R has one of the properties $Q(m)$ and $Q(n)$, especially, if $(m, n) = 2^r r'$ for some non-negative integer r and some odd divisor r' of k, then R is commutative.*

REFERENCES

[**1**] H.A.S. Abujabal and V. Perić, *Commutativity of s-unital rings through a Streb result*, Radovi Mat. (to appear).

[**2**] H.A.S. Abujabal and V. Perić, *A commutativity theorem for rings with constraints on commutators*.

[**3**] H.A.S. Abujabal and V. Perić, *Commutativity theorems for s-unital rings with constraints on commutators*.

[**4**] H.E. Bell, *On the power map and ring commutativity*, Canad. Math. Bull. **21** (1978), 399–404.

[**5**] H.E. Bell, *The identity $(xy)^n = x^n y^n$: does it buy commutativity?* Math. Magazine **55** (1982), 165–170.

[**6**] A. Harmanci, *Two elementary commutativity theorems for rings*, Acta Math. Acad. Sci. Hungar. **29** (1977), 23–29.

[**7**] Y. Hirano, Y. Kobayashi and H. Tominaga, *Some polynomial identities and commutativity of s-unital rings*, Math. J. Okayama Univ. **24** (1982), 7–13.

[**8**] T.P. Kezlan, *A note on commutativity of semiprime PI-rings*, Math. Japon. **27** (1982), 267–268.

[**9**] W.K. Nicholson and A. Yaqub, *A commutativity theorem for rings and groups*, Canad. Math. Bull. **22** (1979), 419–423.

[**10**] E. Psomopoulos, *A commutativity theorem for rings and groups with constraints on commutators*, Internat. J. Math. Sci. **7** (1984), 513–517.

[**11**] W. Streb, *Über einen Satz von Herstein und Nakayama*, Rend. Sem. Mat. Univ. Padova. **64** (1981), 159–171.

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