PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série tome 48 (62), 1990, 101–109

SIDON TYPE INEQUALITIES

Ferenc Móricz

Abstract. We prove bounds for the L^1-norm over $[0,\pi]$ of a linear combination of consecutive Dirichlet kernels, while generalizing the inequalities due to Sidon, Fomin, Bojanić and Stanojević, etc. In the case of the modified conjugate Dirichlet kernels, we give upper bounds for the L^1-norm over $[\pi/(N+1),\pi]$, where $\bar{D}_N(x)$ is the kernel with the largest subscript occuring in the linear combination. Special cases were proved earlier by Teljakovskii, Bray and Stanojević. We extend all these inequalities to the two-dimensional setting.

1. Introduction. We consider the well-known Dirichlet kernel

$$D_k(x)=\frac{1}{2}+\sum_{j=1}^{k}\cos jx=\frac{\sin(k+1/2)x}{2\sin(x/2)}$$

and the conjugate expression

$$\bar{D}_k(x)=-\frac{\cos(k+1/2)x}{2\sin(x/2)}\qquad (k=0,1,\dots).$$

The latter is connected with the conjugate Dirichlet kernel

$$\tilde{D}_k(x)=\sum_{j=1}^{k}\sin jx=\frac{\cos(x/2)-\cos(k+1/2)x}{2\sin(x/2)}$$

by the identity

$$\tilde{D}_k(x)=\bar{D}_k(x)-\bar{D}_0(x)\qquad (\bar{D}_0(x)=0).$$

In the sequel, $\{a_k : k=0,1,\dots\}$ denotes an arbitrary sequence of real numbers. Sidon [5] proved (apart from the value of the constant) that for all $N=0,1,\dots$

$$I_N=\int_0^{\pi}\left|\sum_{k=0}^{N}a_kD_k(x)\right|dx\le 2(N+1)\max_{0\le k\le N}|a_k|. \tag{1.1}$$

AMS Subject Classification (1985): Primary 42 A 32, Secondary 42 B 99

Later on, Teljakovskii [6] gave an elegant proof of (1.1).

Bojanić and Stanojević [1] proved the following more general inequality: For any p, $1 < p \le 2$, there exists a constant C_p depending only on p such that for all $n = 1, 2, \ldots$

$$\int_0^\pi \left| \sum_{k=n}^{2n-1} a_k D_k(x) \right| dx \le C_p n^{1/q} \left(\sum_{k=n}^{2n-1} |a_k|^p \right)^{1/p}, \tag{1.2}$$

where q is the conjugate exponent to p, i.e. $1/p + 1/q = 1$.

We note that inequality (1.2) is essentially contained in [3] by Fomin. In this paper, C_p will denote a positive constant, depending only on p, and usually not the same at different occurrences. Likewise, C will denote a positive absolute constant, whose value is not necessarily the same at each occurrence.

We also note that the more general inequalities

$$I_N \le C_p (N+1)^{1/q} \left(\sum_{k=0}^{N} |a_k|^p \right)^{1/p} \qquad (N = 0, 1, \ldots) \tag{1.3}$$

and

$$I_N \le C_p \left\{ |a_0| + \sum_{n=1}^{n(N)} 2^{n/q} \left(\sum_{k=2^{n-1}}^{2^n - 1} |a_k|^p \right)^{1/p} \right\} \tag{1.4}$$

can be deduced from (1.2) in standard ways, where the integer $n(N)$ in (1.4) is defined by

$$n(0) = 0, \qquad n(N) = 1 + [\log_2 N] \qquad (N = 1, 2, \ldots), \tag{1.5}$$

$[u]$ being the greatest integer $\le u$; i.e., $2^{n(N)-1} \le N < 2^{n(N)}$.

2. New inequalities. Our main goal is to generalize (1.1) and (1.2) as follows.

THEOREM 1. *There exists an absolute constant C such that for all $0 \le n \le N$, $1 < p \le 2$ and $1/p + 1/q = 1$ we have*

$$\begin{aligned} I_n^N &= \int_0^\pi \left| \sum_{k=n}^{N} a_k D_k(x) \right| dx \\ &\le \frac{C}{(p-1)^{1/p}} (N-n+1)^{1/q} \left(1 + \ln \frac{N+1}{N-n+1} \right) \left(\sum_{k=n}^{N} |a_k|^p \right)^{1/p}. \end{aligned} \tag{2.1}$$

Clearly, inequalities (1.1) and (1.2) are particular cases of (2.1). Besides, for $n = N$ (2.1) gives back the well-known estimate of the Lebesgue constant:

$$L_n = \int_0^\pi |D_n(x)|\, dx \le C\big(1 + \ln(n+1)\big) \qquad (n = 0, 1, \ldots).$$

Actually the proof of Theorem 1 yields a somewhat sharper estimate than (2.1): Under the conditions of Theorem 1, we have

(2.2)
$$I_n^N \le C\left(1+\ln\frac{N+1}{N-n+1}\right)\sum_{k=n}^{N}|a_k| + \frac{C}{(p-1)^{1/p}}(N-n+1)^{1/q}\left(\sum_{k=n}^{N}|a_k|^p\right)^{1/p}.$$

This inequality is obvious by virtue of formulas (3.3)–(3.5) below.

The same sort of reasoning delivers the counterpart of Theorem 1.

THEOREM 2. *Under the conditions of Theorem 1, we have*

$$\begin{aligned}(2.3)\quad \bar I_n^N &= \int_{\pi/(N+1)}^{\pi}\left|\sum_{k=n}^{N} a_k\bar D_k(x)\right|dx\\ &\le C\ln\frac{N+1}{N-n+1}\sum_{k=n}^{N}|a_k| + \frac{C(N-n+1)^{1/q}}{(p-1)^{1/p}}\left(\sum_{k=n}^{N}|a_k|^p\right)^{1/p}.\end{aligned}$$

We note that this right-hand side is also majorized by the right-hand side in (2.1).

A special case of (2.3) says that

$$\begin{aligned}(2.4)\qquad \bar I_N &= \int_{\pi/(N+1)}^{\pi}\left|\sum_{k=0}^{N} a_k\bar D_k(x)\right|dx\\ &\le \frac{C}{(p-1)^{1/p}}(N+1)^{1/q}\left(\sum_{k=0}^{N}|a_k|^p\right)^{1/p}\qquad (N=0,1,\dots),\end{aligned}$$

which is the counterpart of (1.3). The even more special case

$$\bar I_N \le C(N+1)\max_{0\le k\le N}|a_k|\qquad (N=0,1,\dots)$$

was proved by Teljakovskii [6].

In certain cases, the following variants of (2.1) and (2.4) may be useful.

THEOREM 3. *There exists an absolute constant C such that for all $0\le n\le N$, $0<\gamma<\pi$, $1<p\le 2$ we have*

$$(2.5)\qquad \int_{\gamma}^{\pi}\left|\sum_{k=n}^{N} a_k D_k(x)\right|dx \le \frac{C}{(p-1)^{1/p}}\gamma^{-1/q}\left(\sum_{k=n}^{N}|a_k|^p\right)^{1/p},$$

and a similar inequality holds for $\int_{\gamma}^{\pi}\left|\sum_{k=n}^{N} a_k\bar D_k(x)\right|dx$, *too.*

These inequalities are plain by the same argument that leads to (3.5) in the proof of Theorem 1.

We note that the special case $\gamma = \pi/(N-n)$ with $0 \le n < N$ was proved by Bray and Stanojević [2].

Finally, we mention a complex variant of (2.5). In this case, $\{a_k : k = \dots, -1, 0, 1, \dots\}$ is an arbitrary sequence of complex numbers.

THEOREM 4. *There exists an absolute constant C such that for all $n \le N$ and $0 < \gamma < \pi$, $1 < p \le 2$ and $1/p + 1/q = 1$ we have*

$$\int_{\gamma \le |x| \le \pi} \left| \sum_{k=n}^{N} \frac{a_k e^{ikx}}{1 - e^{ix}} \right| dx \le \frac{C\gamma^{-1/q}}{(p-1)^{1/p}} \left(\sum_{k=n}^{N} |a_k|^p \right)^{1/p}.$$

3. Proof of Theorem 1. We split the integral in (2.1) into three parts

(3.1)
$$I_n^N = \left\{ \int_0^{\pi/(N+1)} + \int_{\pi/(N+1)}^{\pi/(N-n+1)} + \int_{\pi/(N-n+1)}^{\pi} \right\} \left| \sum_{k=n}^{N} a_k D_k(x) \right| dx$$
$$= J_1 + J_2 + J_3, \quad \text{say}.$$

Some of these integrals may be trivial (i.e. when the upper and lower limits coincide).

We will use the familiar estimates

(3.2) $$|D_k(x)| \le \min\{k + 1/2, \pi/(2x)\} \qquad \text{for } 0 < x \le \pi.$$

and Hölder's inequality, respectively. Accordingly,

(3.3) $$J_1 \le \frac{\pi}{N+1} \sum_{k=n}^{N} \left(k + \frac{1}{2} \right) |a_k| \le \pi \sum_{k=n}^{N} |a_k|$$
$$\le \pi (N - n + 1)^{1/q} \left(\sum_{k=n}^{N} |a_k|^p \right)^{1/p},$$

(3.4) $$J_2 \le \sum_{k=n}^{N} |a_k| \int_{\pi/(N+1)}^{\pi/(N-n+1)} \frac{\pi}{2x}\, dx = \frac{\pi}{2} \ln \frac{N+1}{N-n+1} \sum_{k=n}^{N} |a_k|$$
$$\le \frac{\pi}{2} (N - n + 1)^{1/q} \ln \frac{N+1}{N-n+1} \left(\sum_{k=n}^{N} |a_k|^p \right)^{1/p}.$$

Now we apply Hölder's inequality and the Hausdorff-Young inequality (see, e.g. [8, Vol. 2, p. 101]) to the system $\{\sin(k + 1/2)x : k = 0, 1, \dots\}$, which is clearly orthogonal on $[0, \pi]$. As a result, we get

(3.5)
$$J_3 \le \left(\int_{\pi/(N-n+1)}^{\pi} \frac{dx}{(2\sin(x/2))^p} \right)^{1/p} \left(\int_{\pi/(N-n+1)}^{\pi} \left| \sum_{k=n}^{N} a_k \sin(k + 1/2)x \right|^q dx \right)^{1/q}$$

$$\leq \frac{\pi}{2}\left(\int_{\pi/(N-n+1)}^{\pi} \frac{dx}{x^p}\right)^{1/p} \pi^{1/2}\left(\sum_{k=n}^{N} |a_k|^p\right)^{1/p}$$
$$\leq \frac{\pi^{1/p+1/2}}{2(p-1)^{1/p}}(N-n+1)^{1/q}\left(\sum_{k=n}^{N} |a_k|^p\right)^{1/p}.$$

Combining (3.1), (3.3)–(3.5) gives (2.1) which was to be proved.

4. Extension to the two-dimensional case. Let $\{a_{jk} : j,k = 0,1,\dots\}$ be a double sequence of real numbers. Teljakovskii [7] extended (1.1) as follows

$$I_{MN} = \int_0^{\pi}\int_0^{\pi}\left|\sum_{j=0}^{M}\sum_{k=0}^{N} a_{jk}D_j(x)D_k(y)\right| dx\, dy$$
$$\leq 4(M+1)(N+1) \max_{0\leq j\leq M,\, 0\leq k\leq N} |a_{jk}| \qquad (M,N = 0,1,\dots).$$

We generalized this inequality (see [4]): There exists an absolute constant C such that for all $M,N = 0,1,\dots$, $1 < p \leq 2$ and $1/p + 1/q = 1$ we have

$$I_{MN} \leq \frac{C}{(p-1)^{2/p}}(M+1)^{1/q}(N+1)^{1/q}\left(\sum_{j=0}^{M}\sum_{k=0}^{N} |a_{jk}|^p\right)^{1/p}.$$

It is not hard to deduce from this inequality that

$$I_{MN} \leq \frac{C}{(p-1)^{2/p}}\left\{|a_{00}| + \sum_{m=1}^{m(M)} 2^{m/q}\left(\sum_{j=2^{m-1}}^{2^m-1} |a_{j0}|^p\right)^{1/p}\right.$$
$$+ \sum_{n=1}^{n(N)} 2^{n/q}\left(\sum_{k=2^{n-1}}^{2^n-1} |a_{0k}|^p\right)^{1/p}$$
$$\left. + \sum_{m=1}^{m(M)}\sum_{n=1}^{n(N)} 2^{(m+n)/q}\left(\sum_{j=2^{m-1}}^{2^m-1}\sum_{k=2^{n-1}}^{2^n-1} |a_{jk}|^p\right)^{1/p}\right\},$$

where $m(M)$ and $n(N)$ are defined by: $m(M) = 1 + [\log_2 M]$, $n(N) = 1 + [\log_2 N]$ for $M,N = 1,2,\dots$ and $m(0) = n(0) = 0$ (cf. (1.5)).

Our claim is to prove the following more general inequalities.

THEOREM 5. *There exists an absolute constant C such that for all $0 \leq m \leq M$, $0 \leq n \leq N$, $1 < p \leq 2$ and $1/p + 1/q = 1$ we have*

$$(4.1) \qquad I_{mn}^{MN} = \int_0^{\pi}\int_0^{\pi}\left|\sum_{j=m}^{M}\sum_{k=n}^{N} a_{jk}D_j(x)D_k(y)\right| dx\, dy$$
$$\leq C\left(1 + \ln\frac{M+1}{M-m+1}\right)\left(1 + \ln\frac{N+1}{N-n+1}\right)\sum_{j=m}^{M}\sum_{k=n}^{N} |a_{jk}|$$

$$+\frac{C}{(p-1)^{1/p}}(M-m+1)^{1/q}\Big(1+\ln\frac{N+1}{N-n+1}\Big)\sum_{k=n}^{N}\Big(\sum_{j=m}^{M}|a_{jk}|^p\Big)^{1/p}$$

$$+\frac{C}{(p-1)^{1/p}}\Big(1+\ln\frac{M+1}{M-m+1}\Big)(N-n+1)^{1/q}\sum_{j=m}^{M}\Big(\sum_{k=n}^{N}|a_{jk}|^p\Big)^{1/p}$$

$$+\frac{C}{(p-1)^{2/p}}(M-m+1)^{1/q}(N-n+1)^{1/q}\Big(\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|^p\Big)^{1/p}.$$

THEOREM 6. *Under the conditions of Theorem 5, we have*

$$\tag{4.2} \bar{I}_{mn}^{MN}=\int_{\pi/(M+1)}^{\pi}\int_{\pi/(N+1)}^{\pi}\Big|\sum_{j=m}^{M}\sum_{k=n}^{N}a_{jk}\bar{D}_j(x)\bar{D}_k(y)\Big|\,dx\,dy$$

$$\le C\ln\frac{M+1}{M-m+1}\ln\frac{N+1}{N-n+1}\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|$$

$$+\frac{C}{(p-1)^{1/p}}(M-m+1)^{1/q}\ln\frac{N+1}{N-n+1}\sum_{k=n}^{N}\Big(\sum_{j=m}^{M}|a_{jk}|^p\Big)^{1/p}$$

$$+\frac{C}{(p-1)^{1/p}}\ln\frac{M+1}{M-m+1}(N-n+1)^{1/q}\sum_{j=m}^{M}\Big(\sum_{k=n}^{N}|a_{jk}|^p\Big)^{1/p}$$

$$+\frac{C}{(p-1)^{2/p}}(M-m+1)^{1/q}(N-n+1)^{1/q}\Big(\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|^p\Big)^{1/p}.$$

THEOREM 7. *Under the conditions of Theorem 5, we have*

(4.3)

$$\bar{\bar{I}}_{mn}^{MN}=\int_{0}^{\pi}\int_{\pi/(N+1)}^{\pi}\Big|\sum_{j=m}^{M}\sum_{k=n}^{N}a_{jk}D_j(x)\bar{D}_k(y)\Big|\,dx\,dy$$

$$\le C\Big(1+\ln\frac{M+1}{M-m+1}\Big)\ln\frac{N+1}{N-n+1}\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|$$

$$+\frac{C}{(p-1)^{1/p}}(M-m+1)^{1/q}\ln\frac{N+1}{N-n+1}\sum_{k=n}^{N}\Big(\sum_{j=m}^{M}|a_{jk}|^p\Big)^{1/p}$$

$$+\frac{C}{(p-1)^{1/p}}\Big(1+\ln\frac{M+1}{M-m+1}\Big)(N-n+1)^{1/q}\sum_{j=m}^{M}\Big(\sum_{k=n}^{N}|a_{jk}|^p\Big)^{1/p}$$

$$+\frac{C}{(p-1)^{2/p}}(M-m+1)^{1/q}(N-n+1)^{1/q}\Big(\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|^p\Big)^{1/p}.$$

The extensions of Theorems 3 and 4 are obvious.

We note that by applying Hölder's inequality to the right-hand sides of (4.1)–(4.3), we obtain the more transparent inequalities

$$I_{mn}^{MN},\ \bar{I}_{mn}^{MN},\ \bar{\bar{I}}_{mn}^{MN} \le \frac{C}{(p-1)^{2/p}}(M-m+1)^{1/q}(N-n+1)^{1/q}\times$$
$$\times\left(1+\ln\frac{M+1}{M-m+1}\right)\left(1+\ln\frac{N+1}{N-n+1}\right)\left(\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|^p\right)^{1/p}.$$

On closing, we emphasize that the extension of these results to d-dimensional trigonometric sums is straightforward, where $d \ge 3$ is a fixed integer. The only reason that we present here the case $d = 2$ is to keep the notation manageable.

5. Proof of Theorem 5. Since the proof follows in great lines the pattern of the proof of Theorem 1, we only sketch it. We begin with splitting the double integral in I_{mn}^{MN} into nine parts while integrating over the intervals

$$\left[0,\frac{\pi}{M+1}\right],\quad \left[\frac{\pi}{M+1},\frac{\pi}{M-m+1}\right],\quad \left[\frac{\pi}{M-m+1},\pi\right]$$

with respect to x, and over the intervals

$$\left[0,\frac{\pi}{N+1}\right],\quad \left[\frac{\pi}{N+1},\frac{\pi}{N-n+1}\right],\quad \left[\frac{\pi}{N-n+1},\pi\right]$$

with respect to y, respectively. We denote by J_{11}, J_{21}, J_{31}, J_{12}, J_{22}, J_{32}, J_{13}, J_{23}, J_{33} the corresponding integrals.

By (3.2),

$$J_{11} \le \int_0^{\pi/(M+1)}\int_0^{\pi/(N+1)}\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|\left(j+\frac{1}{2}\right)\left(k+\frac{1}{2}\right)dx\,dy$$
$$\le \pi^2\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|,$$

$$J_{21} \le \int_{\pi/(M+1)}^{\pi/(M-m+1)}\int_0^{\pi/(N+1)}\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|\frac{\pi}{2x}\left(k+\frac{1}{2}\right)dx\,dy$$
$$\le \frac{\pi^2}{2}\ln\frac{M+1}{M-m+1}\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|,$$

an analogous estimate for J_{12}, and

$$J_{22} \le \int_{\pi/(M+1)}^{\pi/(M-m+1)}\int_{\pi/(N+1)}^{\pi/(N-n+1)}\sum_{j=m}^{M}\sum_{k=n}^{N}|a_{jk}|\frac{\pi^2}{4xy}\,dx\,dy$$

$$= \frac{\pi^2}{4} \ln \frac{M+1}{M-m+1} \ln \frac{N+1}{N-n+1} \sum_{j=m}^{M} \sum_{k=n}^{N} |a_{jk}|.$$

Applying again (3.2) and a similar argument that provides (3.5) gives

$$J_{31} \le \int_{\pi/(M-m+1)}^{\pi} \int_{0}^{\pi/(N+1)} \sum_{k=n}^{N} \left(k + \frac{1}{2}\right) \left| \sum_{j=m}^{M} a_{jk} D_j(x) \right| dx\, dy$$
$$\le \sum_{k=n}^{N} \int_{\pi/(M-m+1)}^{\pi} \left| \sum_{j=m}^{M} a_{jk} D_j(x) \right| dx$$
$$\le \frac{C}{(p-1)^{1/p}} (M-m+1)^{1/q} \sum_{k=n}^{N} \left(\sum_{j=m}^{M} |a_{jk}|^p \right)^{1/p},$$

and an analogous estimate for J_{13},

$$J_{32} \le \int_{\pi/(M-m+1)}^{\pi} \int_{\pi/(N+1)}^{\pi/(N-n+1)} \sum_{k=n}^{N} \frac{\pi}{2y} \left| \sum_{j=m}^{M} a_{jk} D_j(x) \right| dx\, dy$$
$$= \sum_{k=n}^{N} \frac{\pi}{2} \ln \frac{N+1}{N-n+1} \int_{\pi/(M-m+1)}^{\pi} \left| \sum_{j=m}^{M} a_{jk} D_j(x) \right| dx$$
$$\le \frac{C}{(p-1)^{1/p}} (M-m+1)^{1/q} \ln \frac{N+1}{N-n+1} \sum_{k=n}^{N} \left(\sum_{j=m}^{M} |a_{jk}|^p \right)^{1/p},$$

and an analogous estimate for J_{23}.

Finally, by virtue of Hölder's inequality and the Hausdorff-Young inequality extended to two-dimensional Fourier series, we get

$$J_{33} = \int_{\pi/(M-m+1)}^{\pi} \int_{\pi/(N-n+1)}^{\pi} \left| \sum_{j=m}^{M} \sum_{k=n}^{N} a_{jk} \frac{\sin(j+1/2)x}{2\sin(x/2)} \frac{\sin(k+1/2)y}{2\sin(y/2)} \right| dx\, dy$$
$$\le \left(\int_{\pi/(M-m+1)}^{\pi} \int_{\pi/(N-n+1)}^{\pi} \frac{dx\, dy}{(4\sin(x/2)\sin(y/2))^p} \right)^{1/p} \times$$
$$\times \left(\int_{\pi/(M-m+1)}^{\pi} \int_{\pi/(N-n+1)}^{\pi} \left| \sum_{j=m}^{M} \sum_{k=n}^{N} a_{jk} \sin\left(j + \frac{1}{2}\right) x \sin\left(k + \frac{1}{2}\right) y \right|^q dx\, dy \right)^{1/q}$$
$$\le \frac{\pi^{2/p+1}}{4(p-1)^{2/p}} (M-m+1)^{1/q} (N-n+1)^{1/q} \left(\sum_{j=m}^{M} \sum_{k=n}^{N} |a_{jk}|^p \right)^{1/p}.$$

Combining the inequalities obtained for J_{11}, J_{21}, …, J_{33} above completes the proof of Theorem 5.

The proofs of Theorems 6 and 7 run along the same lines as that of Theorem 5. We do not enter into detail.

REFERENCES

[1] R. Bojanić and Č. V. Stanojević, *A class of L^1-convergence*, Trans. Amer. Math. Soc. **269** (1982), 677–683.

[2] W. O. Bray and Č. V. Stanojević, *Tauberian L^1-convergence classes of Fourier series. I*, Trans. Amer. Math. Soc. **275** (1983), 59–69.

[3] G. A. Fomin, *On linear methods of summability of Fourier series*, Math. Sb. (N.S) **65** (**107**)(1964), 144–152 (Russian).

[4] F. Móricz, *On the integrability and L^1-convergence of double trigonometric series*, Studia Math. **98** (1990) (to appear).

[5] S. Sidon, *Hinreichende Bedingungen für den Fourier-Charakter einer trigonometrischen Reihe*, J. London Math. Soc. (Ser. 2) **14** (1939), 158–160.

[6] S. A. Teljakovskii, *On a sufficient condition of Sidon for integrability of trigonometric series*, Mat. Zametki **14** (1973), 317–328 (Russian).

[7] S. A. Teljakovskii, *On the conditions of integrability of multiple trigonometric series*, Trudy Mat. Inst. Akad. Nauk SSSR **164** (1983), 180–188 (Russian).

[8] A. Zygmund, *Trigonometric Series*, Cambridge Univ. Press, Cambridge, 1959.

University of Szeged
Bolyai Institute
Aradi Vértanúk tere 1
6720 Szeged, Hungary

(Received 05 10 1989)