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L^1-CONVERGENCE OF FOURIER SERIES WITH COEFFICIENTS MONOTONIC WITH RESPECT TO REGULARLY VARYING SEQUENCES

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Abstract. Let $\{R(n)\}$ be a non-decreasing sequence of positive numbers such that the sequence

$$\left\{\frac{1}{n}\sum_{k=1}^{n} k\left(\frac{R(k)}{R(k-1)}-1\right)\right\}, \qquad R(0)=1,$$

of bounded variation. If $f \in {}'L^1(0,\pi)$ is even and the sequence $\{\hat{f}(n)/R(n)\}$ is monotonic then $\{S_n(f)\}$ and $\{\sigma_n(f)\}$ are equiconvergent if and only if $\hat{f}(n)\lg n = o(1)$. Consequently a necessary and sufficient condition for L^1-convergence follows.

1. Introduction. Wide L^1-convergence classes have been obtained in [**1**–**8**] through Tauberian L^p-methods. However, none of these L^1-convergence classes contain sequences of Fourier coefficients with purely regular properties such as monotonicity and its generalizations. A typical result regarding generalized monotonicity is the well-known theorem of Telyakovskii and Fomin [**9**].

THEOREM A. *Let the Fourier coefficients of $f \in L^1(0,\pi)$ be positive. If for some $\alpha \geq 0$ the sequence $\{\hat{f}(n)/n^\alpha\}$ is monotonic, then $\|S_n(f)-f\| = o(1)$, $n \to \infty$, is equivalent to $\hat{f}(n)\lg n = o(1)$, $n\to\infty$, where $S_n(f,x) = S_n(f) = \frac{1}{2}\hat{f}(0)+\sum_{k=1}^{n}\hat{f}(k)\cos kx$, $x\in(0,\pi)$, and where $\|\cdot\|$ denotes the $L^1(0,\pi)$-norm.*

The fact that $\{\hat{f}(n)/n^\alpha\}$ is monotonic is called the quasimonotonicity of $\{\hat{f}(n)\}$, a term introduced by O. Szasz [**10**]. W. O. Bray and Č. V. Stanojević [**11**] attempted to generalize Theorem A in the following way. Let $\{R(n)\}$ be a sequence of positive numbers. If $\lim_n R([\lambda n])/R(n) = \lambda^\rho$ where $\lambda > 0$ and $\rho > 0$, it is said, following J. Karamata [**12**], that $\{R(n)\}$ is regularly varying. In [**11**] quasimonotonicity of $\{\hat{f}(n)\}$ is defined for a non-decreasing regularly varying sequence

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$\{R(n)\}$ by requiring that $\{\hat{f}(n)/R(n)\}$ be monotonic. However, due to various errors, their proof is not complete as V. B. Stanojević pointed out in [**13**]. As a matter of fact, she has further generalized the concept of quasimonotonicity with respect to regularly varying sequences.

A non-decreasing sequence $\{R(n)\}$ of positive numbers is O-regularly varying if $\overline{\lim}_n R([\lambda n])/R(n)$ is finite, for $\lambda > 1$. A null sequence of positive numbers is O-regularly varying quasimonotonic if, for some O-regularly varying sequence $\{R(n)\}$, the sequence $\{a_n/R(n)\}$ is monotonic. In [**13**] V. B. Stanojević proved the following theorem.

THEOREM B. *Let $\{a_n\}$ be a O-regularly varying quasimonotonic sequence, and for some even $f \in L^1(0,\pi)$ let $\hat{f}(n) = a_n$, $n = 0,1,2,\ldots$. Then the necessary and sufficient condition for $\|S_n(f) - f\| = o(1)$, $n \to \infty$ is $a_n \lg n = o(1)$, $n \to \infty$.*

However not all sequences $\{R(n)\}$ mentioned in the definition of O-regularly varying quasimonotonic sequences in [**13**] will generalize the theorem of Telyakovskii-Fomin. Only those $\{R(n)\}$ for which $\left\{n\left(\frac{R(n+1)}{R(n)} - 1\right)\right\}$ is unbounded will effectively generalize the quasimonotonic sequences in the sense of Szasz [**10**]. To see this, note that from the monotonicity of $\{a_n/R(n)\}$ we have $a_{n+1} \le \left[\left(\frac{R(n+1)}{R(n)} - 1\right) + 1\right] a_n$. If $\frac{R(n+1)}{R(n)} = 1 + O(1/n)$ then $\frac{R(n+1)}{R(n)} - 1 < \frac{C}{n}$ and $a_{n+1} \le \left(1 + \frac{C}{n}\right) a_n$. Hence, O-regularly varying quasimonotonicity is reduced to the Telyakovskii-Fomin case.

In this paper we shall prove a refinement of Theorem B using the concept of strongly O-regularly varying sequences suggested by the above observation. A non-decreasing sequence $\{R(n)\}$ of positive numbers is strongly O-regularly varying if $\left\{\frac{1}{n}\sum_{k=1}^{n} k\left(\frac{R(k+1)}{R(k)} - 1\right)\right\}$ is of bounded variation. In view of the previous remarks we should restrict ourselves to those sequences $\{R(n)\}$ for which $\left\{n\left(\frac{R(n+1)}{R(n)} - 1\right)\right\}$ is unbounded. Let $r_n = n\left(\frac{R(n+1)}{R(n)} - 1\right)$. Since $\left\{\frac{1}{n}\sum_{k=1}^{n} r_k\right\}$ is assumed to be of bounded variation, consider a particular case. Let $\alpha_n > 0$ and $\sum_{n=1}^{\infty} \alpha_n < \infty$ but $\{n\alpha_n\}$ is not bounded. Define $\{R(n)\}$ by

$$R(n) = R(1)\prod_{k=1}^{n}\left(1 + \alpha_k + \frac{\beta_k}{k}\right), \tag{1.1}$$

where $\left\{\frac{1}{n}\sum_{k=1}^{n} \beta_k\right\}$ is a sequence of bounded variation and $\{\beta_n\}$ is bounded away from zero. Clearly $R(n) \uparrow \infty$ since $\sum_{k=1}^{n} \frac{\beta_k}{k}$ diverges.

A useful observation is that if the sequence $\{R(n)\}$ is strongly O-regularly varying then it is O-regularly varying. This follows easily from the inequality

$$\overline{\lim_n} \frac{R([\lambda n])}{R(n)} \leq \exp\left\{\overline{\lim_n} \frac{1}{[\lambda n]-1} \sum_{k=2}^{[\lambda n]} k\left(\frac{R(k)}{R(k-1)} - 1\right)\right\}.$$

From this last remark it follows that our main result, in the next section, is indeed a refinement of Theorem B in [**13**].

Before we present our results we need the following definition and notation. A null sequence $\{a_n\}$ of positive numbers is strongly O-regularly varying quasimonotonic if for some strongly O-regularly varying sequence $\{R(n)\}$, the sequence $\{a_n/R(n)\}$ is monotonic. Throughout this paper $\sigma_n(f,x) = \sigma_n(f)$ denotes the $(C,1)$ means of the Fourier cosine series of f. We are now ready to state and prove our results.

2. Results. The next theorem is a slightly more general statement than L^1-convergence of Fourier series.

THEOREM. *Let $\{a_n\}$ be a strongly O-regularly varying quasimonotonic sequence and let $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx$ be the Fourier series of some even $f \in L^1(0,\pi)$. Then $\|\sigma_n(f) - S_n(f)\| = o(1)$, $n \to \infty$, is equivalent to $a_n \lg n = o(1)$, $n \to \infty$.*

Proof of sufficiency. Let $\sigma_n(f,x) = \sigma_n(f) = \frac{1}{n+1}\sum_{k=0}^{n} S_k(f,x)$. Using summation by parts we obtain

$$S_n(f,x)-\sigma_n(f,x) = \frac{1}{n+1}\sum_{k=1}^{n-1} k\Delta a_k D_k(x) - \frac{1}{n+1}\sum_{k=0}^{n-1} a_{k+1}D_k(x) + \frac{n}{n+1}a_n D_n(x),$$

where $D_n(x) = \frac{1}{2} + \sum_{k=1}^{n} \cos kx$ is the Dirichlet kernel for which $\|D_n\| = 4\pi^{-2}\lg n + O(1)$, $n \to \infty$. Taking norms and majoring the right-hand side we obtain

$$\begin{aligned} \|\sigma_n(f) - S_n(f)\| &\leq \frac{4}{\pi^2}\frac{1}{n+1}\sum_{k=1}^{n-1} k|\Delta a_k| \lg k \\ &+ \frac{4}{\pi^2}\frac{1}{n+1}\sum_{k=1}^{n-1} a_{k+1}\lg k + \frac{n}{n+1}a_n \lg n + O(a_n). \end{aligned} \tag{2.1}$$

The hypothesis $a_n \lg n = o(1)$, $n \to \infty$, implies that the second and the third terms of the right-hand side of (2.1) are $o(1)$, $n \to \infty$. Hence, it remains to show that

$$\frac{1}{n+1}\sum_{k=1}^{n} k|\Delta a_k| \lg k = o(1), \qquad n \to \infty. \tag{2.2}$$

From the monotonicity of $\{a_n/R(n)\}$ we obtain

$$|\Delta a_n| \leq 2\Delta a_n + 3\left(\frac{R(n+1)}{R(n)} - 1\right)a_n.$$

Hence (2.2) becomes

(2.3)
$$\frac{1}{n+1}\sum_{k=1}^{n} k|\Delta a_k|\lg k \leq \frac{2}{n+1}\sum_{k=1}^{n} k\Delta a_k \lg k + \frac{3}{n+1}\sum_{k=1}^{n} k\left(\frac{R(k+1)}{R(k)} - 1\right)a_k \lg k.$$

For the first term of (2.3) we apply summation by parts

$$\begin{aligned}\frac{1}{n+1}\sum_{k=1}^{n} k\Delta a_k \lg k &= \frac{1}{n+1}\sum_{k=1}^{n-1} k \lg\left(1+\frac{1}{k}\right)a_{k+1} \\ &+ \frac{1}{n+1}\sum_{k=1}^{n-1} a_{k+1}\lg(k+1) - \frac{n}{n+1}a_{n+1}\lg n.\end{aligned} \tag{2.4}$$

The first term of the right-hand side of (2.4) is $o(1)$, $n \to \infty$, because $\{a_n\}$ is a null sequence. The second and the third terms are $o(1)$, $n \to \infty$, because $a_n \lg n = o(1)$, $n \to \infty$.

It remains to estimate the second term of the right-hand side of (2.3). If

$$\rho_n = \frac{1}{n}\sum_{k=1}^{n} r_k \qquad \text{where} \quad r_k = k\left(\frac{R(k+1)}{R(k)} - 1\right),$$

then $n(\rho_n - \rho_{n-1}) + \rho_{n-1} = r_n$. Thus, the last term in (2.3) becomes

$$\begin{aligned}\frac{1}{n+1}\sum_{k=1}^{n} k\left(\frac{R(k+1)}{R(k)} - 1\right)a_k \lg k &= \frac{1}{n+1}\sum_{k=1}^{n} r_k a_k \lg k \\ &= \frac{1}{n+1}\sum_{k=1}^{n} k(\rho_k - \rho_{k-1})a_k \lg k + \frac{1}{n+1}\sum_{k=1}^{n} \rho_{k-1} a_k \lg k \\ &\leq \frac{n}{n+1}\left(\frac{1}{n}\sum_{k=1}^{n} k|\Delta\rho_k| a_k \lg k\right) + \frac{n}{n+1}\left(\frac{C}{n}\sum_{k=1}^{n} a_k \lg k\right).\end{aligned}$$

Since $\{\rho_k\}$ is of bounded variation and $a_n \lg n = o(1)$, $n \to \infty$, both terms are $o(1)$, $n \to \infty$. Therefore $\|\sigma_n(f) - S_n(f)\| = o(1)$, $n \to \infty$.

Proof of necessity. Since $f \in L^1(0,\pi)$ and $\|\sigma_n(f) - S_n(f)\| = o(1)$, $n \to \infty$, then $\|f - S_n(f)\| = o(1)$, $n \to \infty$. We now use the well-known inequality

$$\|f - S_n(f)\| \geq A\sum_{k=1}^{n} \frac{a_{n+k}}{k},$$

where A is an absolute constant. Hence,

$$\|f - S_n(f)\| \geq A\sum_{k=1}^{n} \frac{a_{n+k}}{k}\frac{R(n+k)}{R(n+k)} \geq \frac{Aa_{2n}}{R(2n)}\sum_{k=1}^{n}\frac{R(n+k)}{k} \geq \frac{a_{2n}}{R(2n)}R(n)\cdot C \cdot \lg n$$

where $C \neq 0$ is an absolute constant. Thus

$$(2.5) \qquad a_{2n} \lg n \leq \frac{1}{C} \frac{R(2n)}{R(n)} \|f - S_n(f)\|.$$

Since $\{R(n)\}$ is O-regularly varying, $\overline{\lim}_n R(2n)/R(n)$ is finite. Thus the right-hand side of (2.5) is $o(1)$, $n \to \infty$, which implies $a_n \lg n = o(1)$, $n \to \infty$. This concludes the proof of the Theorem.

The next corollary gives the necessary and sufficient condition for L^1-convergence of cosine Fourier series with strongly O-regularly varying quasimonotonic coefficients. In [13] it is mentioned that the estimate $\|S_n(f) - \sigma_n(f)\| = o(1)$, $n \to \infty$, is too weak for the conclusion of Theorem B. However, our result shows that the estimate $\|S_n(f) - \sigma_n(f)\| = o(1)$, $n \to \infty$, suffices to obtain the desired result.

COROLLARY 1. *Let the conditions of Theorem hold. Then* $\|S_n(f) - f\| = o(1)$, $n \to \infty$, *is equivalent to* $a_n \lg n = o(1)$, $n \to \infty$.

Our next result concerns Theorem B in [**13**]. As we shall shortly show we can prove a version of Theorem B if we require that

$$(2.6) \qquad \overline{\lim_n} \frac{R(2n)}{R(n)} \quad \text{is finite}$$

because (2.6) is equivalent to the fact that $\{R(n)\}$ is O-regularly varying sequence; and by estimating $\|S_n(f) - \sigma_n(f)\|$ we have a slight generalization of Theorem B.

PROPOSITION. *Let* $\{a_n\}$ *be a null sequence of positive numbers and let* $\{a_n/R(n)\}$ *be monotonic, where* $\{R(n)\}$ *is a non-decreasing sequence of positive numbers such that* $\overline{\lim}_n R(2n)/R(n)$ *is finite. If* $\hat{f}(n) = a_n$, $n = 0, 1, 2, \ldots$, *then the necessary and sufficient condition for* $\|S_n(f) - \sigma_n(f)\| = o(1)$, $n \to \infty$, *is* $a_n \lg n = o(1)$, $n \to \infty$.

Proof of sufficiency. Consider the identity

$$(2.6) \qquad f(x) - S_n(f,x) = \frac{1}{n} \sum_{k=n+1}^{2n} \sum_{j=n+1}^{k} \Delta a_j D_j(x) - \frac{2n+1}{n}(\sigma_{2n}(f) - f)$$
$$+ \frac{n+1}{n}(\sigma_n(f) - f) + \frac{1}{n} \sum_{k=n+1}^{2n} a_{k+1} D_k(x) - a_{n+1} D_n(x).$$

This implies

$$(2.7) \qquad \sigma_n(f,x) - S_n(f,x) = \frac{1}{n} \sum_{k=n+1}^{2n} \sum_{j=n+1}^{k} \Delta a_j D_j(x)$$
$$- \frac{2n+1}{n}(\sigma_{2n}(f) - \sigma_n(f)) + \frac{1}{n} \sum_{k=n+1}^{2n} a_{k+1} D_k(x) - a_{n+1} D_n(x).$$

Taking norms of both sides of (2.7) and majoring the right-hand side we obtain

(2.8)

$$\|\sigma_n(f) - S_n(f)\| \le 2\sum_{k=n+1}^{2n} |\Delta a_k| \lg k + \frac{2n+1}{n}\|\sigma_{2n}(f) - \sigma_n(f)\| + \frac{1}{n}\sum_{k=n+1}^{2n} a_{k+1}\lg(k+1) + a_{n+1}\lg(n+1) + o(1).$$

Taking the lim sup of both side of (2.8) we get

$$\begin{aligned}(2.9)\qquad &\overline{\lim_n}\|\sigma_n(f) - S_n(f)\| \\ &\le 2\overline{\lim_n}\sum_{k=n+1}^{2n} |\Delta a_k|\lg k + \overline{\lim_n}\frac{2n+1}{n}\cdot\overline{\lim_n}\|\sigma_{2n}(f) - \sigma_n(f)\| \\ &+ \overline{\lim_n}\left(\frac{1}{n}\sum_{k=n+1}^{2n} a_{k+1}\lg(k+1)\right) + \overline{\lim_n} a_{n+1}\lg(n+1).\end{aligned}$$

The second term of the right-hand side of (2.9) is $o(1)$, $n \to \infty$, since $f \in L^1(0,\pi)$. The last two terms are $o(1)$, $n \to \infty$, since $a_n \lg n = o(1)$, $n \to \infty$. Therefore it remains to show

$$(2.10)\qquad \sum_{k=n+1}^{2n} |\Delta a_k|\lg k = o(1), \qquad n \to \infty.$$

From the monotonicity of $\{a_n/R(n)\}$ we have

$$|\Delta a_k|\lg k \le \Delta\left(\frac{a_k}{R(k)}\right)R(k)\lg k + \left(1 - \frac{R(k)}{R(k+1)}\right)a_{k+1}\lg(k+1).$$

Hence,

$$\begin{aligned}&\sum_{k=n+1}^{2n} |\Delta a_k|\lg k \\ &\le \sum_{k=n+1}^{2n}\Delta\left(\frac{a_k}{R(k)}\right)R(k)\lg k + \sum_{k=n+1}^{2n}\left(1 - \frac{R(k)}{R(k+1)}\right)a_{k+1}\lg(k+1) \\ &\le R(2n)\lg(2n)\sum_{k=n+1}^{2n}\Delta\left(\frac{a_k}{R(k)}\right) \\ &+ \max_{n+1\le k\le 2n}(a_{k+1}\lg(k+1))\sum_{k=n+1}^{2n}\left(1 - \frac{R(k)}{R(k+1)}\right) \\ &\le R(2n)\lg(2n)\frac{a_{n+1}}{R(n)}\frac{\lg(n+1)}{\lg n}\end{aligned}$$

$$+\max_{n+1\le k\le 2n}(a_{k+1}\lg(k+1))\sum_{k=n+1}^{2n}\left(1-\frac{R(k)}{R(k+1)}\right).$$

Taking lim sup of both sides of the inequality we get

$$(2.11)\quad \overline{\lim_n}\sum_{k=n+1}^{2n}|\Delta a_k|\lg k\le \overline{\lim_n}\frac{R(2n)}{R(n)}\cdot\overline{\lim_n}\frac{\lg(2n)}{\lg n}\cdot\overline{\lim_n}(a_{n+1}\lg(n+1))$$
$$+\overline{\lim_n}\max_{n+1\le k\le 2n}(a_{k+1}\lg(k+1))\cdot\overline{\lim_n}\sum_{k=n+1}^{2n}\left(1-\frac{R(k)}{R(k+1)}\right).$$

The first term of the right-hand side of (2.11) is $o(1)$, $n\to\infty$, since $\overline{\lim}_n R(2n)/R(n)$ is finite and $a_n\lg n=o(1)$, $n\to\infty$. It remains to show

$$\overline{\lim_n}\sum_{k=n+1}^{2n}\left(1-\frac{R(k)}{R(k+1)}\right)$$

is finite. We have

$$\sum_{k=n+1}^{2n}\left(1-\frac{R(k)}{R(k+1)}\right)\le\frac{R(2n)}{R(n)}\sum_{k=n+1}^{2n}\left(\frac{R(k+1)}{R(k)}-1\right)$$
$$\le\frac{R(2n)}{R(n)}\prod_{k=n+1}^{2n}\left[1+\left(\frac{R(k+1)}{R(k)}-1\right)\right]\le\frac{R(2(n+1))}{R(n+1)}.$$

Thus $\overline{\lim_n}\sum_{k=n+1}^{2n}\left(1-\frac{R(k)}{R(k+1)}\right)$ is finite. Therefore $\|S_n(f)-\sigma_n(f)\|=o(1)$, $n\to\infty$.

Proof of necessity. Since $f\in L^1(0,\pi)$ and $\|\sigma_n(f)-S_n(f)\|=o(1)$, $n\to\infty$, then $\|S_n(f)-f\|=o(1)$, $n\to\infty$. Using the well-known inequality $\|S_n(f)-f\|\ge A\sum_{k=1}^{n}a_{n+k}/k$, where A is an absolute constant, and the fact that $\{a_n/R(n)\}$ is monotonic we obtain

$$\|S_n(f)-f\|\ge A\sum_{k=1}^{n}\frac{a_{n+k}}{k}\frac{R(n+k)}{R(n+k)}\ge\frac{Aa_{2n}}{R(2n)}\sum_{k=1}^{n}\frac{R(n+k)}{k}\ge\frac{a_{2n}}{R(2n)}R(n)\cdot C\cdot\lg n$$

where $C\ne 0$ is an absolute constant.

Thus

$$a_{2n}\lg n\le\frac{1}{C}\frac{R(2n)}{R(n)}\|S_n(f)-f\|.$$

Taking lim sup of both sides we get $a_n\lg n=o(1)$, $n\to\infty$. This concludes the proof of the Proposition.

Clearly Theorem B is the following corollary to Proposition.

COROLLARY 2. *Let the conditions of Proposition hold. then* $\|S_n(f)-f\|=o(1)$, $n\to\infty$, *is equivalent to* $a_n\lg n=o(1)$, $n\to\infty$.

Here again, we have a generalization of Szasz quasimonotonicity [10] only if $\left\{n\left(\frac{R(n+1)}{R(n)}-1\right)\right\}$ is unbounded. We consider sequences $\{R(n)\}$ such that $R(n)=R(1)\prod_{k=1}^{n}(1+\alpha_k+\beta_k/k)$, where $\{\alpha_n\}$ and $\{\beta_k\}$ are as in (1.1). Notice that for such sequences, $\overline{\lim}_n R(2n)/R(n)$ is finite. This follows from the inequality

$$\overline{\lim_n}\frac{R(2n)}{R(n)}\le 2^{\sup_n \beta_n}.$$

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