

ALL GENERAL SOLUTIONS OF FINITE EQUATIONS

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Abstract. Recently Prešić determined in [4] all reproductive solutions of the finite equation, where a finite equation is an equation the solution set of which is a subset of a given finite set. In this paper we determine all general solutions of such equations. Especially we also get all reproductive solutions.

Let E be a given non-empty set and $q(x)$ be any x -equation (x is an unknown element of E and q is a given unary relation of E) supposing that $q(x)$ has at least one solution.

Definition 1. Let $f : E \rightarrow E$ be a given function. The formula $x = f(t)$ represents a general solution of x -equation $q(x)$ if and only if $(\forall t \in E)q(f(t)) \wedge (\forall x \in E)(q(x) \implies (\exists t \in E)x = f(t))$.

Definition 2. Let $g : E \rightarrow E$ be a given function. The formula $x = g(t)$ represents a reproductive solution of x -equation $q(x)$ if and only if $(\forall t \in E)q(g(t)) \wedge (\forall t \in E)(q(t) \implies x = g(t))$.

Let $B = \{b_0, b_1, \dots, b_m\}$ be a given set of $m + 1$ elements and $S = \{0, 1\}$. Define the operation x^y by

$$x^y = \begin{cases} 1, & \text{if } x = y \\ 0, & \text{otherwise} \end{cases} \quad (x, y \in B \cup S).$$

The standard Boolean operations $+$ and \cdot (“or” and “and”) are described by the following tables:

$+$	0	1	\cdot	0	1
0	0	1	0	0	0
1	1	1	1	0	1

Extend these operations to the partial operations of the set $B \cup S$ by

$$x + 0 = x, \quad 0 + x = x, \quad x \cdot 0 = 0, \quad x \cdot 1 = x, \quad 0 \cdot x = 0, \quad 1 \cdot x = x, \quad (x \in B \cup S).$$

We consider the x -equation

$$s_0 x^{b_0} + s_1 x^{b_1} + \cdots + s_m x^{b_m} = 0 \quad (1)$$

where $s_i \in \{0, 1\}$ are given elements and $x \in B$ is unknown.

Obviously the equation (1) is consistent if and only if

$$s_0 s_1 \cdots s_m = 0. \quad (2)$$

Definition 3. Let $(a_0, a_1, \dots, a_m) \in S^{m+1}$. Then the set $Z(a_0, \dots, a_m)$ (“the zero-set of (a_0, \dots, a_m) ”) is defined by

$$b_i \in Z(a_0, \dots, a_m) \iff a_i = 0 \quad (i = 0, 1, \dots, m). \quad (3)$$

For instance, if $m = 3$ we have

$$Z(1, 1, 1, 1) = \emptyset, \quad Z(1, 0, 1, 0) = \{b_1, b_3\}, \quad Z(0, 0, 0, 0) = \{b_0, b_1, b_2, b_3\}.$$

Let $M = \{0, 1, 2, \dots, m\}$.

Definition 4. Let $s_0 \cdots s_m = 0$. A function $A : B \rightarrow B$ of the form

$$A(x) = A_0(s_0, \dots, s_m) x^{b_0} + \cdots + A_m(s_0, \dots, s_m) x^{b_m}$$

is a repro-function if and only if each coefficient $A_k(s_0, \dots, s_m)$ is determined by some equality of the form

$$A_k(s_0, \dots, s_m) = b_k s_k^0 + \sum_{a_k \neq 0, a_0 \cdots a_m = 0} F_k(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m},$$

where

$$\begin{aligned} & (\forall k \in M) (\forall a_0, \dots, a_m \in S) (a_k \neq 0 \wedge a_0 \cdots a_m = 0 \\ & \implies F_k(a_0, \dots, a_m) \in Z(a_0, \dots, a_m)). \end{aligned}$$

Definition 5. Let $s_0 \cdots s_m = 0$. A function $A : B \rightarrow B$ of the form

$$A(x) = A_0(s_0, \dots, s_m) x^{b_0} + \cdots + A_m(s_0, \dots, s_m) x^{b_m}$$

is a gener-function if and only if there is a function $\psi : M \xrightarrow{1-1} M$ such that each coefficient $A_k(s_0, \dots, s_m)$ is determined by some equality of the form

$$A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_{a_{\psi(k)} \neq 0, a_0 \cdots a_m = 0} F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m},$$

where

$$\begin{aligned} & (\forall k \in M) (\forall a_0, \dots, a_m \in S) (a_{\psi(k)} \neq 0 \wedge a_0 \cdots a_m = 0 \\ & \implies F_{\psi(k)}(a_0, \dots, a_m) \in Z(a_0, \dots, a_m)). \end{aligned}$$

Let (1) be denoted by $g(x) = 0$.

LEMMA 1. *Let $A(x) = A_0(s_0, \dots, s_m)x^{b_0} + \dots + A_m(s_0, \dots, s_m)x^{b_m}$ assuming that $A_0(s_0, \dots, s_m), \dots, A_m(s_0, \dots, s_m) \in B$. Then the formula*

$$(\forall x \in B)(g(x) = 0 \implies (\exists t \in B)x = A(t))$$

holds if and only if there is a function $\psi : M \xrightarrow{1-1} M$ such that each coefficient $A_k(s_0, \dots, s_m)$ is determined by the equality

$$A_k(s_0, \dots, s_m) = b_{\psi(k)}s_{\psi(k)}^0 + \sum_{a_{\psi(k)} \neq 0, a_0, \dots, a_m = 0} F_{\psi(k)}(a_0, \dots, a_m)s_0^{a_0} \cdots s_m^{a_m},$$

where

$$\begin{aligned} & (\forall k \in M)(\forall a_0, \dots, a_m \in S)(a_{\psi(k)} \neq 0 \wedge a_0 \cdots a_m = 0 \\ & \implies F_{\psi(k)}(a_0, \dots, a_m) \in B). \end{aligned}$$

The proof follows from the following equivalences.

$$\begin{aligned} & (\forall x \in B)(g(x) = 0 \implies (\exists t \in B)x = A(t)) \\ \iff & (\forall x \in B)(x \in Z(s_0, \dots, s_m) \implies (\exists t \in B)x = A(t)) \\ \iff & (\forall x \in Z(s_0, \dots, s_m))(\exists t \in B)x = A(t) \\ \iff & (\exists \bar{f} : Z(s_0, \dots, s_m) \rightarrow B)(\forall x \in Z(s_0, \dots, s_m))x = A(\bar{f}(x)) \end{aligned}$$

(by the axiom of choice)

$$\iff (\exists \bar{f} : Z(s_0, \dots, s_m) \xrightarrow{1-1} B)(\forall x \in Z(s_0, \dots, s_m))x = A(\bar{f}(x)).$$

(Assuming $b_p \in Z(s_0, \dots, s_m)$, $b_r \in Z(s_0, \dots, s_m)$, $b_p \neq b_r$ and $\bar{f}(b_p) = \bar{f}(b_r) = b_u$, we get from $x = A_0(f(x))^{b_0} + \dots + A_m(f(x))^{b_m}$ the following implications:

$$x = b_p \implies b_p = A_u, \quad x = b_r \implies b_r = A_u \quad \text{i.e.} \quad b_p = b_r.$$

Thus \bar{f} is $\xrightarrow{1-1}$).

$$\iff (\exists f : B \xrightarrow{1-1} B)(\forall x \in Z(s_0, \dots, s_m))x = A(f(x))$$

(f is an extension of \bar{f})

$$\begin{aligned} \iff & (\exists f : B \xrightarrow{1-1} B)(\forall x \in B)(x \in Z(s_0, \dots, s_m) \implies x = A(f(x))) \\ \iff & (\exists f : B \xrightarrow{1-1} B)(\forall x \in B)(s_0x^{b_0} + \dots + s_mx^{b_m} \\ & \implies x = A_0(s_0, \dots, s_m)(f(x))^{b_0} + \dots \\ & + A_m(s_0, \dots, s_m)(f(x))^{b_m} \wedge (\forall k \in M)A_k(s_0, \dots, s_m) \in B) \end{aligned}$$

$$\begin{aligned}
&\iff (\exists f : B \xrightarrow{1-1} B)(\forall k \in M)(s_0 b_k^{b_0} + \dots + s_m b_k^{b_m} = 0 \\
&\quad \implies b_k = A_0(s_0, \dots, s_m)(f(b_k))^{b_0} + \dots \\
&\quad + A_m(s_0, \dots, s_m)(f(b_k))^{b_m} \wedge A_k(s_0, \dots, s_m) \in B) \\
&\iff (\exists \varphi : M \xrightarrow{1-1} M)(\forall k \in M)(s_k = 0 \\
&\quad \implies b_k = A(\varphi(k))(s_0, \dots, s_m) \wedge A_k(s_0, \dots, s_m) \in B)
\end{aligned}$$

(φ is defined by $(\forall c, d \in M)(\varphi(c) = d \iff f(b_c) = b_d)$)

$$\begin{aligned}
&\iff (\exists \varphi : M \xrightarrow{1-1} M)(\forall k \in M) \left(A_{\varphi(k)}(s_0, \dots, s_m) \right. \\
&\quad = b_k s_k^0 + \sum_{a_k \neq 0, a_0 \dots a_m = 0} F_k(a_0, \dots, a_m) s_0^{a_0} \dots s_m^{a_m} \\
&\quad \left. \wedge \forall a_0, \dots, a_m \in S(a_k \neq 0 \wedge a_0 \dots a_m = 0 \implies F_k(a_0, \dots, a_m) \in B) \right) \\
&\iff (\exists \psi : M \xrightarrow{1-1} M)(\forall k \in M) \left(A_k(s_0, \dots, s_m) \right. \\
&\quad = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \dots s_m^{a_m} \\
&\quad \left. \wedge (\forall a_0, \dots, a_m \in S)(C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \right)
\end{aligned}$$

(ψ is φ^{-1} , C is the conjunction $a_{\psi(k)} \neq 0 \wedge a_0 \dots a_m = 0$ and \sum_C means the sum over all $(a_0, \dots, a_m) \in S^{m+1}$ such that C is satisfied).

THEOREM 1. *Let $A(x) = A_0(s_0, \dots, s_m)x^{b_0} + \dots + A_m(s_0, \dots, s_m)x^{b_m}$ and $A_0(s_0, \dots, s_m), \dots, A_m(s_0, \dots, s_m) \in B$. If*

$$s_0 x^{b_0} + \dots + s_m x^{b_m} = 0 \tag{1}$$

is a consistent equation then the formula $x = A(t)$ (t is any element of B) represents a general solution of (1) and only if the function A is a gener-function.

Proof. It we denote (1) by $g(x) = 0$ we have

$$\begin{aligned}
&(\forall x \in B)g(A(x)) = 0 \wedge (\forall x \in B)(g(x) = 0 \implies (\exists t \in B)x = A(t)) \\
&\iff (\forall x \in B)s_0(A_0(s_0, \dots, s_m)x^{b_0} + \dots + A_m(s_0, \dots, s_m)x^{b_m})^{b_0} + \dots \\
&\quad + s_m(A_0(s_0, \dots, s_m)x^{b_0} + \dots + A_m(s_0, \dots, s_m)x^{b_m})^{b_m} = 0 \\
&\quad \wedge (\exists \psi : M \xrightarrow{1-1} M)(\forall k \in M) \left(A_k(s_0, \dots, s_m) \right. \\
&\quad = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \dots s_m^{a_m}
\end{aligned}$$

$$\wedge (\forall a_0, \dots, a_m \in S) (C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B)$$

(by Lemma 1)

$$\begin{aligned} &\iff (\forall i, k \in M) s_i A_k^{b_i}(s_0, \dots, s_m) = 0 \\ &\quad \wedge (\exists \psi : M \xrightarrow{1-1} M) (\forall k \in M) \left(A_k(s_0, \dots, s_m) \right. \\ &\quad = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \\ &\quad \left. \wedge (\forall a_0, \dots, a_m \in S) (C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \right) \end{aligned}$$

(This part of the proof is based on the following general facts:

If a_0, \dots, a_n, b , are any elements of B then

- 1° $(a_0 x^{b_0} + \dots + a_n x^{b_n})^b = a_0^b x^{b_0} + \dots + a_n^b x^{b_n}$ (for all $x \in B$)
- 2° $(\forall x \in B) a_0 x^{b_0} + \dots + a_n x^{b_n} = 0 \implies (\forall i \in M) a_i = 0$.

$$\begin{aligned} &\iff (\exists \psi : M \xrightarrow{1-1} M) (\forall i, k \in M) \left(s_i A_k^{b_i}(s_0, \dots, s_m) = 0 \right. \\ &\quad \wedge (\forall a_0, \dots, a_m \in S) (C \implies F_{\psi(k)}(a_0, \dots, a_m)) \\ &\quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \right) \\ &\iff (\exists \psi : M \xrightarrow{1-1} M) (\forall i, k \in M) \left(s_i \left(b_{\psi(k)} s_{\psi(k)}^0 \right. \right. \\ &\quad \left. \left. + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \right)^{b_i} = 0 \right. \\ &\quad \wedge (\forall a_0, \dots, a_m \in S) (C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\ &\quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(s_0, \dots, s_m) s_0^{a_0} \cdots s_m^{a_m} \right) \\ &\iff (\exists \psi : M \xrightarrow{1-1} M) (\forall i, k \in M) \left(s_i \left(\sum_C F_{\psi(k)}^{b_i}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \right) = 0 \right. \\ &\quad \wedge (\forall a_0, \dots, a_m \in S) (C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\ &\quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \right) \end{aligned}$$

(we have used the identity $s_p s_r^0 b_r^{b_p} = 0$)

$$\iff (\exists \psi : M \xrightarrow{1-1} M) (\forall i, k \in M) \left(\sum_{(a_0, \dots, a_m) \in S^{m+1}} a_i \cdot s_0^{a_0} \cdots s_m^{a_m} \right)$$

$$\begin{aligned} & \cdot \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} = 0 \\ & \wedge (\forall a_0, \dots, a_m \in S)(C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\ & \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \end{aligned}$$

(because of the identity $s_i = \sum_{(a_0, \dots, a_m) \in S^{m+1}} a_i s_0^{a_0} \cdots s_m^{a_m}$)

$$\begin{aligned} & \iff (\exists \psi : M \xrightarrow{1-1} M)(\forall i, k \in M) \left(\sum_C a_i F_{\psi(k)}^{b_i}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} = 0 \right. \\ & \quad \wedge (\forall a_0, \dots, a_m \in S)(C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\ & \quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \right) \\ & \iff (\exists \psi : M \xrightarrow{1-1} M)(\forall i, k \in M) \left((\forall a_0, \dots, a_m \in S) \right. \\ & \quad \left. (C \implies a_i F_{\psi(k)}(a_0, \dots, a_m) = 0) \right. \\ & \quad \wedge (\forall a_0, \dots, a_m \in S)(C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\ & \quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \right) \\ & \iff (\exists \psi : M \xrightarrow{1-1} M)(\forall i, k \in M) \left((\forall a_0, \dots, a_m \in S)(C \implies a_i b_{h(k)}^{b_i} = 0) \right. \\ & \quad \wedge (\forall a_0, \dots, a_m \in S)(C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\ & \quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \right) \end{aligned}$$

(we have used the following facts:

$$(I) (\forall k \in M) F_{\psi(k)} \in B \iff (\forall k \in M)(\exists j \in M) F_{\psi(k)} = b_j;$$

$$(II) (\forall k \in M)(\exists j \in M) F_{\psi(k)} = b_j \iff (\exists h : M \rightarrow M)(\forall k \in M) F_{\psi(k)} = b_{h(k)},$$

by the axiom of choice)

$$\begin{aligned} & \iff (\exists \psi : M \xrightarrow{1-1} M)(\forall k \in M) \left((\forall i \in M)(\forall a_0, \dots, a_m \in S)(C \implies a_i b_{h(k)}^{b_i} = 0) \right. \\ & \quad \wedge (\forall a_0, \dots, a_m \in S)(C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\ & \quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m} \right) \\ & \iff (\exists \psi : M \xrightarrow{1-1} M)(\forall k \in M) \left((\forall a_0, \dots, a_m \in S)(C \implies a_{h(k)} = 0) \right. \\ & \quad \left. \wedge (\forall a_0, \dots, a_m \in S)(C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \right) \end{aligned}$$

$$\begin{aligned}
& \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \dots s_m^{a_m} \\
& \iff (\exists \psi : M \xrightarrow{1-1} M) (\forall k \in M) \left((\forall a_0, \dots, a_m \in S) \right. \\
& \quad (C \implies b_{h(k)} \in Z(a_0, \dots, a_m)) \\
& \quad \wedge (\forall a_0, \dots, a_m \in S) (C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\
& \quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \dots s_m^{a_m} \right)
\end{aligned}$$

(Definition 3)

$$\begin{aligned}
& \iff (\exists \psi : M \xrightarrow{1-1} M) (\forall k \in M) \left((\forall a_0, \dots, a_m \in S) \right. \\
& \quad (C \implies F_k(a_0, \dots, a_m) \in Z(a_0, \dots, a_m)) \\
& \quad \wedge (\forall a_0, \dots, a_m \in S) (C \implies F_{\psi(k)}(a_0, \dots, a_m) \in B) \\
& \quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \dots s_m^{a_m} \right) \\
& \iff (\exists \psi : M \xrightarrow{1-1} M) (\forall k \in M) \left((\forall a_0, \dots, a_m \in S) (\implies \right. \\
& \quad F_k(a_0, \dots, a_m) \in Z(a_0, \dots, a_m) \wedge F_{\psi(k)}(a_0, \dots, a_m) \in B) \\
& \quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \dots s_m^{a_m} \right)
\end{aligned}$$

(because $(p_1 \implies p_2) \wedge (p_1 \implies p_3) \iff (p_1 \implies p_2 \wedge p_3)$ is a tautology)

$$\begin{aligned}
& \iff (\exists \psi : M \xrightarrow{1-1} M) (\forall k \in M) \left((\forall a_0, \dots, a_m \in S) \right. \\
& \quad (C \implies F_k(a_0, \dots, a_m) \in Z(a_0, \dots, a_m)) \\
& \quad \left. \wedge A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \dots s_m^{a_m} \right) \\
& \iff A \text{ is a gener-function.}
\end{aligned}$$

The following Theorem 2 can be obtained from Theorem 1 if we assume that $\psi : B \rightarrow B$ is the identical mapping.

THEOREM 2. [4] If

$$s_0 x^{b_0} + \dots + s_m x^{b_m} = 0 \quad (1)$$

is a consistent equation, then the formula

$$x = A(p) \quad (p \text{ is any element of } B)$$

represents a reproductive solution of the equation (1) if and only if the function A is a repro-function.

Example 1. Let $s_0x^{b_0} + s_1x^{b_1} + s_2x^{b_2} = 0$ be a consistent equation i.e. $s_0s_1s_2 = 0$. If $\psi = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ then the formula $x = A_0P^{b_0} + A_1P^{b_1} + A_2P^{b_2}$ represents a general solution of $s_0x^{b_0} + s_1x^{b_1} + s_2x^{b_2} = 0$ provided

$$\begin{aligned} A_0 &= b_2s_2^0 + F_2(0, 0, 1)s_0^0s_1^0s_2^1 + F_2(0, 1, 1)s_0^0s_1^1s_2^1 + F_2(1, 0, 1)s_0^1s_1^0s_2^1 \\ A_1 &= b_0s_0^0 + F_0(1, 0, 0)s_0^1s_1^0s_2^0 + F_0(1, 0, 1)s_0^1s_1^0s_2^1 + F_0(1, 1, 0)s_0^1s_1^1s_2^0 \\ A_2 &= b_1s_1^0 + F_1(0, 1, 0)s_0^0s_1^1s_2^0 + F_1(0, 1, 1)s_0^0s_1^1s_2^1 + F_1(1, 1, 0)s_0^1s_1^1s_2^0 \end{aligned}$$

i.e.

$$\begin{aligned} A_0 &= b_2s_2^0 + (b_0 \text{ or } b_1)s_0^0s_1^0s_2^1 + b_0s_0^0s_1^1s_2^1 + b_1s_0^1s_1^0s_2^1 \\ A_1 &= b_0s_0^0 + (b_1 \text{ or } b_2)s_0^1s_1^0s_2^0 + b_1s_0^1s_1^0s_2^1 + b_2s_0^1s_1^1s_2^0 \\ A_2 &= b_1s_1^0 + (b_0 \text{ or } b_2)s_0^0s_1^1s_2^0 + b_0s_0^0s_1^1s_2^1 + b_2s_0^1s_1^1s_2^1. \end{aligned}$$

Remark 1. Let $p = 2^n - 1$ (n is a natural number), $\{0, 1\}^n = \{D_0, \dots, D_p\}$, $f : \{0, 1\}^n \rightarrow \{0, 1\}$ be a Boolean function and

$$f(x_1, \dots, x_n) = 0 \quad (4)$$

be a consistent Boolean equation, i.e. $\prod_{i=0}^p f(D_i) = 0$.

In accordance with Theorem 1 one can effectively find all general solutions of (4) in the form

$$X = A_0(f(D_0), \dots, f(D_p))T^{D_0} \cup \dots \cup A_p(f(D_0), \dots, f(D_p))T^{D_p},$$

where $X = (x_1, \dots, x_n)$ and $T = (t_1, \dots, t_n)$.

REFERENCES

- [1] D. Banković, *Formulas of the general solutions of Boolean equations*, Publ. Inst. Math. Beograd **44** (58) (1988), 3–7.
- [2] S. Prešić, *Une méthode de resolution des équations dont toutes les solutions appartiennent à un ensemble fini donné*, C. R. Acad. Sci. Paris Ser. **A 272** (1971), 654–657.
- [3] S. Prešić, *Ein Satz über reproduktive Lösungen*, Publ. Inst. Math. Beograd **14** (28) (1973), 133–136.
- [4] S. Prešić, *All reproductive solutions of finite equations*, Publ. Inst. Math. Beograd (in print).
- [5] S. Rudeanu, *Boolean Functions and Equations*, North-Holland Amsterdam/London and Elsevier, New York, 1974.

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ERRATUM

The reference [4] from above should have appeared as reference [7] in my paper *All general reproductive solutions of Boolean equations*, Publ. Inst. Math. (Beograd) **46** (60) (1989), 13–19, and the reference [7] should have been reference [8].