PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 36 (50), 1984, pp. 57–65

ON NONLINEAR EQUATIONS OF EVOLUTION IN BANACH SPACES

Stanislav Szufla

Abstract. The paper contains an existence theorem and a Kneser-type theorem for the problem $x' = A(t)x + f(t,x)$, $x(0) = x_0$, where $\{A(t)\}_{t\in[0,d]}$ is a family of linear operator generating an evolution operator $U(t,s)$, and f is a continuous function satisfying a Kamke condition with respect to the measure of noncompactness.

In this paper we shall give an existence theorem for mild solutions of the Cauchy problem

$$x' = A(t)x + f(t,x), \;\; x(0) = x_0, \tag{1}$$

where $\{A(t)\}_{t\in[0,d]}$ is a family of closed linear operators in a Banach space E and f is a continuous function with values in E. Moreover, using the Browder-Gupta connectedness principle [4], we shall show that the set of these solutions is a compact R_δ, i.e. it is homeomorphic to the intersection of decreasing sequence of compact absolute retracts. Let us remark that our existence proof differs strongly from those in known papers concerning (1) (see e.g. [2], [3], [8–10], [14]).

Let $Q = \{(t,s): 0 \le s \le t \le d\}$, $B = \{x \in E: \|x - x_0\| \le b\}$, and let $L(E)$ denote the space of all bounded linear operators in E. We assume that $\{A(t)\}$ generates an evolution operator $U: Q \to L(E)$ with the following properties

(U1) the function $(t,s) \to U(t,s)$ is continuous on Q;

(U2) $U(t,s)U(s,r) = U(t,r)$ and $U(t,t) = I$ for all (t,s), $(s,r) \in Q$;

(U3) there exists a continuous function $p: [0,d] \to R_+$ such that

$$\|U(t,s)\| \le \exp\int_s^t p(r)dr \quad \text{for all} \quad (t,s) \in Q.$$

Let us recall some definitions:

AMS Subject Classification (1980): Primary 34 G 05.

A function $u\colon [0,a] \to E$ is called a mild solution of (1) if u is continuous and satisfies

$$u(t) = U(t,0)x_0 + \int_0^t U(t,s)f(s,u(s))ds \tag{2}$$

for all $t \in [0,a]$ (cf. [2]).

A function $h\colon (0,d] \times R_+ \to R_+$ is called a Kamke function if (i) $h(t,\cdot)$ is continuous for almost every $t \in [0,d]$ and $h(\cdot,r)$ is measurable for every $r \in R_+$;

(ii) for every bounded subset Z of $(0,d] \times R_+$ there exists a function m_Z defined on $(0,d]$ such that $h(t,r) \le m_Z(t)$ for $(t,r) \in Z$ and m_Z is integrable on $[c,d]$ for every small $c > 0$;

(iii) for each c, $0 < c \le d$, the identically zero function is the only absolutely continuous function on $[0,c]$ which satisfies $u'(t) = h(t,u(t))$ almost everywhere on $[0,c]$ and such that $D_+u(0) = u(0) = 0$ (cf. [7]).

For any bounded subset X of E the Hausdorff measure of noncompactness of X – denoted $\beta(X)$ – is defined to be the infimum of $\varepsilon > 0$ such that X has a finite ε-net in E. For properties of β see [15].

Moreover, denote by μ the Lebesgue measure in R.

Our fundamental result is given by the following

THEOREM 1. *Assume that* 1° f *is a bounded continuous function from* $[0,d] \times B$ *into* E; 2° q *is a function from* $(0,d] \times R_+$ *into* R_+ *such that* $(t,r) \to p(t)r + q((t,r)$ *is a Kamke function;* 3° *for any subset* X *of* B *and for any* $\varepsilon > 0$ *there exists a closed subset* J_ε *of* $[0,d]$ *such that* $\mu([0,d]\backslash J_\varepsilon) < \varepsilon$ *and*

$$\beta(f(T \times X)) \le \sup_{t \in T} q(t,\beta(X))$$

for each closed subset T *of* J_ε.

Then there exists at least one mild solution of (1) defined on a subinterval J *of* $[0,d]$.

REMARK. It can be easily verified that, in the case when q is nondecreasing in r, the condition 3° holds whenever $f = f_1 + f_2$, where f_1 is a completely continuous function and $\|f_2(t,x) - f_2(t,y)\| \le q(t, \|x-y\|)$ for all x, $y \in B$ and for a.e. $t \in [0,d]$.

Proof. Let us put $k(t,s) = \exp \int_s^t p(r)dr$, $K = \sup\{k(t,s)\colon (t,s) \in Q\}$ and $M = \sup\{\|f(t,x)\|\colon 0 \le t \le d,\ \ x \in B\}$. We choose a number a such that $0 < a \le d$ and

$$\|U(t,0)x_0 - x_0\| + M\int_0^t k(t,s)ds \le b \qquad \text{for all } t \in [0,a]. \tag{3}$$

Let $J = [0,a]$. Denote by C the Banach space of continuous function $J \to E$ with the usual supremum norm $\|\cdot\|_c$, and let $\tilde{B} \subset C$ be the subset of those function with values in B. We introduce a mapping F defined by

$$F(x)(t) = U(t,0)x_0 + \int_0^t U(t,s)f(s,x(s))ds \qquad (x \in \tilde{B},\ \ t \in J).$$

In view of $(U1')$ and (3), from the inequalities

$$\|F(x)(t) - F(x)(\tau)\| \leq \|U(T,0)x_0 - U(\tau,0)x_0\| + M\int_0^\tau \|U(t,s) - U(\tau,s)\|ds + \\ + KM(t-\tau)$$

$$\|F(x)(t) - x_0\| \leq \|U(t,0)x_0 - x_0\| + M\int_0^t k(t,s)ds \quad (x \in \tilde{B},\ \ 0 \leq \tau \leq t \leq a)$$

it follows that $F(\tilde{B})$ is an equicontinuous subset of $\tilde{B}$. On the other hand, if x_n, $x \in \tilde{B}$ and $\lim \|x_n - x\|_c = 0$, then by 1°, $(U1')$ and the Lebesgue dominated convergence theorem we get $\lim_{n\to\infty} F(x_n)(t) = F(x)(t)$ for $t \in J$. From this we deduce that F is a continuous mapping $\tilde{B} \to \tilde{B}$.

For any positive integer n we define a function u_n by

$$u_n(t) = \begin{cases} x_0 & \text{if } 0 \leq t \leq a_n \\ U(t-a_n,0)x_0 + \int\limits_0^{t-a_n} U(t-a_n,s)f(s,u_n(s))ds & \text{if } a_n \leq t \leq a \end{cases}$$

where $a_n = a/n$. Then $u_n \in \tilde{B}$ and

$$u_n(t) = F(u_n)(r_n(t)), \tag{4}$$

where

$$r_n(t) = \begin{cases} 0 & \text{if } 0 \leq t \leq a_n \\ t - a_n & \text{if } a_n \leq t \leq a \end{cases}.$$

Since the set $F(\tilde{B})$ is equicontinuous, we have

$$\lim_{n\to\infty} \|u_n - F(u_n)\|_c = 0. \tag{5}$$

Put $V = \{u_n : n = 1,2,\dots\}$ and $W = F(V)$. For simplicity we introduce the following notation:

$$V(t) = \{x(t): x \in V\},\quad \int_T U(t,s)f(s,V(s))ds = \left\{\int_T U(t,s)f(s,x(s))ds : x \in V\right\}.$$

It is clear from (5) that the sets V, W are equicontinuous and

$$\beta_c(V) = \beta_c(W) \quad \text{and} \quad \beta(V(t)) = \beta(W(t)) \ \text{ for all } \ t \in J. \tag{6}$$

Hence, by Ambrosetti's lemma [1; Th. 2.3], the function $t \to v(t) = \beta(V(t))$ is continuous on J.

Let us fix τ, t, $0 < \tau < t \leq a$. First we shall show that

$$\beta\left(\int_\tau^t U(t,s)f(s,V(s))ds\right) \leq \int_\tau^t k(t,s)q(s,v(s))ds. \tag{7}$$

By the Scorza-Dragoni theorem, for a given $\varepsilon > 0$ there exists a closed subset D_ε of J such that $\mu(J \backslash D_\varepsilon) < \varepsilon$ and the function q is uniformly continuous on $D_\varepsilon \times [0, b]$ Choose $\delta > 0$ in such a way that

$$|q(s_1, r_1) - q(s_2, r_2)| < \varepsilon \quad \text{and} \quad |k(t, s_1) - k(t, s_2)| < \varepsilon$$

for s_1, $s_2 \in D_\varepsilon$, r_1, $r_2 \in [0, b]$ satisfying $|s_1 - s_2| < \delta$ and $|r_1 - r_2| < \delta$, and choose η such that $0 < \eta < \delta$ and $|v(s_1) - v(s_2)| < \delta$ for s_1, $s_2 \in J$ with $|s_1 - s_2| < \eta$. We divide the interval $[\tau, t]$ into n parts

$$\tau = t_0 < t_1 < \cdots < t_n = t$$

in such a way that $t_i - t_{i-1} < \eta$ for $i = 1, \ldots, n$. Let $D_i = [t_{i-1}, t_i] \cap D_\varepsilon$ and $V_i = \{x(s) \colon x \in V, s \in D_i\}$. In virtue of Ambrosetti's lemma [1; Th. 2.2] we have

$$\beta(V_i) = \sup\{\beta(V(s)) \colon s \in D_i\} = v(s_i), \tag{8}$$

where $s_i \in D_i$. Moreover, by 3°, we may choose a closed subset J_ε of J such that $\mu(J \backslash J_\varepsilon) < \varepsilon$ and

$$\beta(f(T \times V_i)) \leq \sup_{s \in T} q(s, \beta(V_i)) \tag{9}$$

for each closed T of J_ε and $i = 1, \ldots, n$. Let

$$P = [\tau, t] \cap D_\varepsilon \cap J_\varepsilon, \;\; S = [\tau, t] \backslash P \;\text{ and }\; T_i = D_i \cap J_\varepsilon.$$

Then

$$\int_\tau^t U(t,s)f(s,V(s))ds \subset \int_P U(t,s)f(s,V(s))ds + \int_S U(t,s)f(s,V(s))ds,$$

and therefore

$$\begin{aligned}\beta&\left(\int_\tau^t U(t,s)f(s,V(s))ds\right) \\ &\leq \beta\left(\int_P U(t,s)f(s,V(s))ds\right) + \beta\left(\int_S U(t,s)f(s,V(s))ds\right).\end{aligned} \tag{10}$$

Further,

$$\int_P U(t,s)f(s,V(s))ds \subset \sum_{i=1}^n \int_{T_i} U(t,s)f(s,V(s))ds \subset \sum_{i=1}^n \mu(T_i)\overline{\text{conv}} Y_i,$$

where $Y_i = \{U(t,s)f(s,y): s \in T_i,\ y \in V_i\}$. Since the set $\{U(t,s): s \in T_i\}$ is compact, it is clear that

$$\beta(Y_i) \leq \sup_{s \in T_i} \|U(t,s)\| \beta(f(T_i \times V_i)).$$

Thus, by (U3), (8) and (9), there exist α_i, $\tau_i \in T_i$ such that

$$\beta(Y_i) \leq k(t,\alpha_i) q(\tau_i, v(s_i)).$$

Consequently,

$$\beta\left(\int_P U(t,s)f(s,V(s))ds\right) \leq \sum_{i=1}^{n} \mu(T_i) k(t,\alpha_i) q(\tau_i, v(s_i)). \tag{11}$$

On the other hand, by 2°, there exists an integrable function $m: [\tau, t] \to R_+$ (dependent only on τ, t) such that

$$q(s,r) \leq m(s) \text{ for } \tau \leq s \leq t \text{ and } 0 \leq r \leq b.$$

Therefore

$$\mu(T_i) k(t,\alpha_i) q(\tau_i, v(s_i)) \leq \int_{T_i} k(t,s) q(s,v(s)) ds + \varepsilon \int_{T_i} m(s) ds + K\varepsilon\mu(T_i),$$

and hence, owing to (11),

$$\beta\left(\int_P U(t,s)f(s,V(s))ds\right) \leq \int_\tau^t k(t,s)q(s,v(s))ds + \varepsilon \int_\tau^t m(s)ds + K\varepsilon(t-\tau) \tag{12}$$

Furthermore, as $\|U(t,s)f(s,x(s))\| \leq KM$ for all $x \in \tilde{B}$ and $s \in J$, we have

$$\beta\left(\int_S U(t,s)f(s,V(s))ds\right) \leq KM\mu(S). \tag{13}$$

From (10), (12) and (13) it follows that

$$\beta\left(\int_\tau^t U(t,s)f(s,V(s))ds\right)$$
$$\leq \int_\tau^t k(t,s)q(s,v(s))ds + \varepsilon \int_\tau^t m(s)ds + K\varepsilon(t-\tau) + KM\mu(S).$$

Since $\mu(S) < 2\varepsilon$ and the above inequality holds for every $\varepsilon > 0$, we obtain (7).

Consider now the function w defined by

$$w(s) = \sup\{\|f(s,x) - f(s,y)\| : x, y \in B, \|x - x_0\| \leq c(s),\ \|y - x_0\| \leq c(s)\},$$

where $c(s) = \min(b, \sup_{0 \leq r \leq s} \|U(r,0)x_0 - x_0\| + KMs)$. The function w is a modification of the function introduced by Olech in [11]. We shall prove that w is lower equicontinuous on $(0,a)$ and continuous at 0. For given $s \in (0,a)$ and $\varepsilon > 0$ there are x, $y \in B$ such that

$$\|x - x_0\| \leq c(s),\ \|y - x_0\| \leq c(s) \quad \text{and} \quad w(s) - \varepsilon/2 \leq \|f(s,x) - f(s,y)\|.$$

As f and c are continuous, there exists $\delta < 0$ such that

$$\|f(r,u) - f(s,x)\| \leq \varepsilon/4 \quad \text{and} \quad \|f(r,z) - f(s,y)\| \leq \varepsilon/4$$

for all $r \in J$, $u, z \in B$ with $|r - s| \leq \delta$, $\|u - x\| \leq \delta$ and $\|z - y\| \leq \delta$, and there exists $\eta > 0$ such that $|c(r) - c(s)| \leq \delta$ for all $r \in J$ with $|r - s| \leq \eta$. Hence, putting

$$u_r = \frac{c(r)}{c(s)}(x - x_0) + x_0 \quad \text{and} \quad z_r = \frac{c(r)}{c(s)}(y - x_0) + x_0,$$

we have $\|u_r - x_0\| \leq c(r)$, $\|z_r - x_0\| \leq c(r)$, $\|u_r - x\| \leq \delta$, $\|z_r - y\| \leq \delta$

$$\begin{aligned} w(s) - \varepsilon/2 &\leq \|f(s,x) - f(s,y)\| \leq \|f(s,x) - f(r,u_r)\| \\ &+ \|f(r,u_r) - f(r,z_r)\| + \|f(r,z_r) - f(s,y)\| \leq w(r) + \varepsilon/2, \end{aligned}$$

so that $w(s) \leq w(r) + \varepsilon$ for $r \in J$ with $|r - s| \leq \eta$. This proves that w is lower semicontinuous at s. The continuity of w at 0 is an immediate consequence of the fact that f and c are continuous and $w(0) = c(0) = 0$.

From (4) and the definitions of c and w it follows that

$$\|u_n(s) - x_0\| \leq c(s) \ \text{ for } \ s \in J \ \text{ and } \ n = 1, 2, \dots,$$

and

$$\left\| \int_\tau^t U(t,s) f(s, u_m(s)) ds - \int_\tau^t U(t,s) f(s, u_n(s)) ds \right\| \leq K \int_\tau^t w(s) ds$$

for $m, n = 1, 2, \dots$. Hence

$$\beta \left(\int_\tau^t U(t,s) f(s, V(s)) ds \right) \leq K \int_\tau^t w(s) ds. \tag{14}$$

Since for any $x \in \tilde{B}$

$$F(x)(t) = U(t,\tau) F(x)(\tau) + \int_\tau^t U(t,s) f(s, x(s)) ds,$$

we have

$$\beta(F(V)(t)) \le \|U(t,\tau)\|\beta(F(V)(\tau)) + \beta\left(\int_\tau^t U(t,s)f(s,V(s))ds\right).$$

Consequently, by (6) and $(U3)$,

$$v(t) \le \exp\left(\int_\tau^t p(s)ds\right) v(\tau) + \beta\left(\int_\tau^t U(t,s)f(s,V(s))ds\right).$$

In view of (7) and (14), this implies that

$$\begin{aligned} v(t)-v(\tau) \le {} & \left(\exp\int_0^t p(s)ds - \exp\int_0^\tau p(s)ds\right)\exp\left(-\int_0^\tau p(s)ds\right)v(\tau) + \\ & \min\left(K\int_\tau^t w(s)ds, \exp\left(\int_0^t p(s)ds\right)\int_\tau^t \exp\left(-\int_0^\tau p(r)dr\right)q(s,v(s))ds\right) \end{aligned} \tag{15}$$

for $0 < \tau < t \le a$.

In particular, from (15) it follows that

$$v(t)-v(\tau) \le N\left(\exp\int_0^t p(s)ds - \exp\int_0^\tau p(s)ds\right) + K\int_\tau^t w(s)ds \ \text{ for } \ 0 \le \tau \le t \le a,$$

where

$$N = \max_{r\in J} v(r)\exp\left(-\int_0^r p(s)ds\right),$$

which proves that the function v is absolutely continuous on J. This fact, plus (15) implies the inequality

$$v'(\tau) \le p(\tau)v(\tau) + \min(Kw(\tau), q(\tau, v(\tau))) \ \text{ for almost every } \ \tau \in J. \tag{16}$$

Obviously $v(0) = \beta(W(0)) = \beta(\{x_0\}) = 0$.

By 2° and Lemma 1 from [11], the function $z = 0$ is the only absolutely continuous function satisfying almost everywhere the equation

$$z' = p(t)z + \min(Kw(t), \ q(t,z))$$

and the initial condition $z(0) = 0$. Hence, applying the theorem on differential inequalities (cf. [5], [12]), from (16) we deduce that $v(t) = 0$ for all $t \in J$. Therefore, by (5) and Ambrosetti's lemma [1; Th. 2.3] we obtain

$$\beta_c(V) = \beta_c(W) = \sup_{t\in J} v(t) = 0,$$

i.e. V is relatively compact in C. Consequently, we can find a subsequence (u_{n_j}) of (u_n) which converges in C to a limit u. In view of (5), this implies that $\|u - F(u)\|_c = \lim_{j\to\infty} \|u_{n_j} - F(u_{n_j})\|_c = 0$. Thus $u = F(u)$, i.e. u is a solution of (2).

The next result is a Kneser type theorem for (1).

THEOREM 2. *Suppose that the assumptions* $1°$ – $3°$ *are fulfilled and in addition the function* q *is nondecreasing in* r. *Then the set of all mild solutions of* (1) *on* J *is a compact* R_δ.

Proof. Let us put

$$\rho(x) = \begin{cases} x, & \text{for } x \in B \\ x_0 + b(x - x_0)/\|x - x_0\|, & \text{for } x \in E\backslash B \end{cases}$$

and

$$g(t,x) = f(t, \rho(x)) \quad \text{for} \quad (t,x) \in J \times E.$$

Then g is a continuous function from $J \times E$ into E and $\|g(t,x)\| < M$ for $(t,x) \in J \times E$. Moreover, as

$$\rho(X) \subset x_0 + \cup_{0\le\lambda\le1} \lambda X,$$

we have $\beta(\rho(X)) \le \beta(X)$ for any bounded subset X of E. Since the function $r \to q(t,r)$ is nondecreasing, from this we deduce that the function g satisfies $3°$.

Consider the mapping G defined by

$$G(x)(t) = U(t,0)x_0 + \int_0^t U(t,s)g(s,x(s))ds \quad (x \in C,\ \ t \in J).$$

Similarly as for F in the proof of Theorem 1, it can be shown that G is a continuous mapping $C \to \tilde{B}$ and the image $G(C)$ is equicontinuous. Further, for any positive integer n, we define a mapping G_n by

$$G_n(x)(t) = G(x)(r_n(t)) \quad (x \in C,\ \ t \in J),$$

where

$$r_n(t) = \begin{cases} 0 & \text{if } 0 \le t \le a/n \\ t - a/n & \text{if } a/n \le t \le a. \end{cases}$$

It can be easily verified (see e.g. [19]) that

(i) G_n is continuous:

(ii) $\lim_{n\to\infty} G_n(x) = G(x)$ uniformly in $x \in C$;

(iii) $I - G_n$ in a homeomorphism $C \to C$.

Now we shall show that $I - G$ is a proper mapping, that is

$$(I - G)^{-1}(Y) \text{ is compact for any compact subset } Y \text{ of } C. \tag{17}$$

Let Y be a given compact subset of C, and let (u_n) be an infinite sequence in $(I - G)^{-1}(Y)$. Since $u_n - G(u_n) \in Y$ for $n = 1, 2, \ldots$, we can find a subsequence (u_{n_j}) of (u_n) and $y \in Y$ such that

$$\lim_{j\to\infty} \|u_{n_j} - G(u_{n_j}) - y\|_c = 0.$$

Putting $V = \{u_{n_j} : j = 1, 2, \dots\}$ and repeating the argument (with slight modifications) from the proof of Theorem 1, we infer that the set V is relatively compact in C. This proves (17).

Applying now Theorem 7 from [4], we conclude that the set $(I - G)^{-1}(0)$ is compact R_δ. As $\|G(x)(t)\| \le b$ for all $x \in C$ and $t \in J$, $(I - G)^{-1}(0)$ is equal to the set of all mild solutions of (1) on J.

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Institute of Mathematics,
A. Mickiewicz University,
Poznan, Poland

(Received 12 12 1983)