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A DECISION PROCEDURE FOR CERTAIN DISJUNCTION-FREE INTERMEDIATE PROPOSITIONAL CALCULI

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0. Introduction. The decision problem for the Heyting propositional calculus H has been considered and solved by several authors in different ways. A number of these decision procedures are referred to in [**2**] (p. 142, fn. 212). The tableau method (see [**1**], ch.9), essentially dual to Gentzen's method, can also solve this problem. Various results concerning the decision problem for intermediate propositional logics. i.e., propositional logics between the intuitionistic and the classical, are given in [**6**].

In this paper we will present a syntactic decision procedure for the disjunction-free fragment of H. With the help of a result of A. Diego [**3**] we will show that any decision procedure for the disjunction-free fragment of H, and hence also ours, can serve for all finitely axiomatizable disjunction-free intermediate logics. These logics were proved decidable in [**6**] also with the help of Diego's result.

1. The system h_{in}. The language L of h_{in} consists of (1) $\perp$—propositional constant, (2) $\rightarrow$ —logical connective, (3) $p_1, p_2, \ldots$—propositional letters and (4) (,)—parentheses. The set For of formulas over L is the smallest set containing: $\perp, p_1, p_2, \ldots$ and closed under the following formation rule: if A and B are formulas then $(A \rightarrow B)$ is a formula. $P, Q, R, \ldots$ and $A, B, C, D, A_1, B_1, \ldots$ are metavariables ranging over $\{\perp, p_1, p_2, \ldots\}$ and For, respectively. We also suppose that, where there are several occurrences of $\rightarrow$, the first one appearing on the left has the highest priority, i.e., we will write

$$A \rightarrow B \rightarrow \cdots \rightarrow C \rightarrow D \text{ instead of } (A \rightarrow (B \rightarrow (\cdots \rightarrow (C \rightarrow D) \cdots))).$$

For each $A \in$For we define its degree $\mid A \mid \in \omega$ and sets of its antecedent and consequent parts, ant(A), con $(A) \subseteq$For, as follows:

a) $\mid P \mid = 0, \ \mid A \rightarrow B \mid = \mid A \mid + \mid B \mid +1;$

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b) $\mathrm{ant}(P) = \varnothing$, $\mathrm{ant}(A \to B) = \{A\} \cup \mathrm{ant}(B)$;

c) $\mathrm{con}(P) = \{P\}$, $\mathrm{con}(A \to B) = \{A \to B\} \cup \mathrm{con}(B)$.

The axiom schemes of h_{in} are:

$(A1)$ $P \to P$; $(A2)$ $\perp \to P$.

The rule schemes of h_{in} are:

$$\frac{A \to \cdots \to A_n \to A}{A_{i_1} \to \cdots \to A_{i_n} \to A}$$ (P– permutation), where $(i_1, \ldots, i_n)$ is any permutation on the$n$ – tuple $(1, \ldots, n)$

$$\frac{A \to A \to B}{A \to B} \ (C\text{–contraction})$$

$$\frac{A}{B \to A} \ (W\text{–weakening})$$

$$\frac{A \to B \quad C \to D}{A \to (B \to C) \to D} \ (AL\text{–arrow on the left})$$

$$\frac{A \to B \quad B \to C}{A \to C} (TR\text{–transitivity}).$$

The sign $\vdash$ will indicate that A is provable in h_{in}.

Lemma 1. (1) $\vdash A \to A$; (2) $\vdash A \to B \to A$;

(3) $\vdash (A \to B \to C) \to (A \to B) \to A \to C$; (4) $\vdash \perp \to A$.

Proof. (1) By induction on $\mid A \mid$.

(2)
$$\cfrac{\cfrac{A \to A \quad (by\ (1))}{B \to A \to A}\ (W)}{A \to B \to A}\ (P)$$

(3)
$$\cfrac{\cfrac{\cfrac{\cfrac{A \to A \quad (by\ (1)) \qquad \cfrac{(A \to B \to C) \to A \to B \to C \quad (by\ (1))}{B \to (A \to B \to C) \to A \to C}}{A \to (A \to B) \to (A \to B \to C) \to A \to C}\ (AL)}{A \to A \to (A \to B \to C) \to (A \to B) \to C}\ (P)}{A \to (A \to B \to C) \to (A \to B) \to C}\ (C)}{(A \to B \to C) \to (A \to B) \to A \to C}\ (P)$$

(4) By induction on $\mid A \mid \cdot \vdash$

Lemma 2. *The following rules are derivable in h_{in}:*

(1) $$\frac{A \quad A \to B \to C}{B \to C} \ (MP\text{– } modus\ ponens);$$

(2) $$\frac{A_1 \to \cdots \to A_n \to A \quad A \to B}{A_1 \to \cdots \to A_n \to B} \ (GTR\text{–}generalized TR);$$

(3) $$\frac{A_1 \to \cdots \to A_n \to A \quad B \to C}{A_1 \to \cdots \to A_n \to (A \to B) \to C} \ (GAL\text{–}generalized\ AL);$$

(4) $$\frac{A_1 \to \cdots \to A_n \to A \quad B}{B(A \to /A_1 \to \cdots \to A_{\vec{n}})} \ (M\text{—}mix\ with\ respect\ to\ A)$$

where $B(A \to /A_1 \to \cdots \to A_n \to)$ is the formula obtained from B by writing "$A_1 \to \cdots \to A_n \to$" instead of "$A \to$" in all places where A has occurrences as an antecedent part of B.

$$\textit{Proof}: (1)\ \frac{\frac{A}{B\to A}(W)\ A\to B\to C}{\frac{B\to B\to C}{B\to C}(C)}(TR)$$

(2) By induction on n.

(3)
$$\frac{A_1 \to \cdots \to A_n \to A \quad \dfrac{A\to A \quad B\to C}{A \to (A\to B)\to C}(AL)}{A_1 \to \cdots \to A_n \to (A\to B)\to C}(GTR)$$

(4) By induction on the number of occurrences of A in B as antecedent part. $\vdash$

Remark. Modus ponens, with the conclusion of degree ≥ 1, does not reduce the set of theorems of the implication-negation fragment *in* H of the Heyting propositional calculus. The rule (M) is a generalization of (GTR) and of (TR) as well.

The following theorem is an immediate consequence of the lemmas above and well-known facts about the Heyting propositional calculus H (cf. [**1, 5**]).

THEOREM 1. *A is provable in h_{in} iff A is provable in H.*

2. The (TR)–elimination theorem. Following Getzen's proof of the cut-elimination theorem (see [**5**]), an analogous theorem can be proved for the formal system h_{in}.

THEOREM 2. *If A is provable in h_{in}, then A is provable without (TR).*

In fact we should prove a (M)–elimination theorem by a double induction of the degree $\mid A \mid$ of the *cut formula* A of (M) and the *rank* of the proof, appropriately defined.

3. Characterization of h_{in}. THEOREM 3. *$\vdash A$, where $A \equiv A_1 \to \cdots \to A_n \to P$* iff at least one of the following conditions is satisfied:*

(i) $P \in \mathrm{ant}(A)$ or $\bot \in \mathrm{ant}(A)$;

(ii) there exists an i_0, $1 \leq i_0 \leq n$, such that $P \in \mathrm{con}(A_{i_0})$ or $\bot \in \mathrm{con}(A_{i_0})$ and for every formula $B \in \mathrm{ant}(A_{i_0})$ $\vdash A_1 \to \cdots \to A_n \to B$.

Proof. The "only if" part. By induction on the length of the proof of A. If A is an axiom, then condition (i) is satisfied. We will consider the cases when the last step in the proof is made by one of the rule schemes: $(P), (C), (W)$ or (AL). (By Theorem 2 every theorem of h_{in} can be proved by a (TR)–free proof). It is not

*"$\equiv$" is used as short for "is the same as".

difficult to see that the rule schemes $(P), (C)$ and (W) are closed for conditions (i) and (ii), i.e. if the upper formulas of these rules satisfy one of the conditions (i) or (ii), then lower formulas satisfy the same condition. Let us consider the case with the (AL)-rule. We suppose that the last step of the proof of A is

$$\frac{A_1 \to C \quad D \to A_3 \to \cdots \to A_n \to P}{A_1 \to (C \to D) \to A_3 \to \cdots \to A_n \to P}(AL),$$

where $A_2 \equiv C \to D$. By the induction hypothesis, $D \to A_3 \to \cdots \to A_n \to P$ satisfies (i) or (ii). If $P \in \{A_3, \cdots, A_n\}$ or $\perp \in \{A_3, \ldots, A_n\}$, then the derived formula A satisfies (i). If $D \to A_3 \to \cdots \to A_n \to A_n \to P$ satisfies (ii) and $i_0 \in \{3, \ldots, n\}$ then the derived formula has the required property (ii) too. We must also consider the following subcases: (1) $D \equiv P$ or $D \equiv\perp$; (2) $D \equiv D_1 \to \cdots \to D_k \to Q$, where $Q \equiv P$ or $Q \equiv\perp$ and for every $i(1 \leq i \leq k)$, $\vdash D \to A_3 \to \cdots \to A_n \to D_i$. Subcase (1): the derived formula satisfies (ii), as $\vdash A_1 \to C$ (by the induction hypothesis), and $\vdash A_1 \to \cdots \to A_n \to C$ (by (P) and (W)). Subcase (2): from $A_1 \to C$, we derive $A_1 \to \cdots \to A_n \to C$ by (W) and (P). From $A_1 \to C$ and $D \to A_3 \to \cdots \to A_n \to D_i$ $(1 \leq i \leq k)$, we derive $A_1 \to A_2 \to A_3 \to \cdots \to A_n \to D_i$ $(1 \leq i \leq k)$ by (AL). So the derived formula $A_1 \to \cdots \to A_n \to P$ satisfies (ii).

$$\frac{A_1\to\cdots\to A_n\to B_k \qquad Q\to P}{A_1\to\cdots\to A_n\to(B_k\to Q)\to P}\,(GAL)$$

$$\frac{A_1\to\cdots\to A_n\to(B_k\to Q)\to P}{(B_k\to Q)\to A_1\to\cdots\to A_n\to P}\,(P)$$

$$\frac{A_1\to\cdots\to A_n\to B_{k-1} \qquad (B_k\to Q)\to A_1\to\cdots\to A_n\to P}{A_1\to\cdots\to A_n\to(B_{k-1}\to B_k\to Q)\to A_1\to\cdots\to A_n\to P}\,(GAL)$$

$$\frac{\qquad\cdots\qquad}{}\,(P),(C)\,(GAL)$$

$$\frac{A_1\to\cdots\to A_n\to B_1 \qquad (B_2\to\cdots\to B_k\to Q)\to A\to\cdots\to A_n\to P}{A_1\to\cdots\to A_n\to A_n\to A_1\to\cdots\to A_n\to P}\,(GAL)$$

$$\frac{\qquad\cdots\qquad}{A_1\to\cdots\to A_n\to P}\,(P),(C) \qquad \dashv$$

The "if" part. Let us suppose that some formula $A \equiv A_1 \to \cdots \to A_n \to P$ satisfies one of the conditions (i) or (ii). If $A_i \equiv P$ or $A_i \equiv\perp$ for some $i(1 \leq i \leq n)$, then, trivially, $\vdash A$. Let $i_0 = n$ and $A_n \equiv B_1 \to \cdots \to B_k \to Q(Q \equiv P$ or $Q \equiv\perp)$ be. Then we have the following proof of A:

Remark. From the proof above we can see that conditions (i) and (ii), without the parts which are related to the propositional constant $\perp$, characterize the implicational fragment of the Heyting propositional calculus, i.e. the positive implicational propositional calculus (cf. [**2**, p. 140]).

4. Decision procedure for h_{in}. A formula $A \equiv A_1 \to \cdots \to A_n \to P$ called (1) *trivially provable* if there exists an $i(1 \leq i \leq n)$ such that $A_i \equiv P$ or $A_i \equiv\perp$ and (2) *trivially refutable* if there is no $i(1 \leq i \leq n)$ such that $P \in \operatorname{con}(A_i)$ or $\perp \in \operatorname{con}(A_i)$.

Description of the decision tree for A (DT_A). We make DT_A by a procedure based on conditions (i) and (ii) of Theorem 3, as follows:

(I) The formula A is the initial node of the tree DT_A.

(II) If we denote an arbitrary node of the tree DT_A by $C \equiv C_1 \to \cdots \to C_m \to P$, then:

a) if C is trivially provable or trivially refutable, then C is the maximal element of the tree;

b) if a) is not satisfied, then there exists a formula $B \in \mathrm{ant}(C)$ such that $P \in \mathrm{con}(B)$ or $\perp \in \mathrm{con}(B)$. From C the tree will ramify into the nodes $C_1 \to \cdots \to C_m \to D$, for every $D \in \mathrm{ant}(B)$. In this case we say that *the ramifying from node C is made in accordance with formula B.*

The length $l(DT_A)$ of DT_A is $k-1$, where k is the number of nodes of the longest branch of DT_A (i.e. of the branch with the maximal number of nodes).

Obviously, in the general case the description of the decision tree above does not define only one tree. In other words, by this description to every formula A corresponds a finite collection $\{DT_A\}$ of trees.

The tree DT_A will be called the tree of *positive (negative)* decision for A, if $l(D_A) \leq k$ where k is fixed and depens on A, and all its maximal elements are trivially provable (if $l(DT_A) > k$ or there is a maximal element which is trivially refutable). This k is determined precisely by the following theorem.

THEOREM 4. *$\vdash A$ iff there exists a tree DT_A of positive decision for A such that $l(DT) \leq \sum\limits_{B \in \mathrm{ant}(A)} \mid B \mid$.*

First, we will prove the following lemma.

LEMMA 3. *If $DT_{A\to B}$ and $DT_{C\to D}$ are the trees of positive decision for $A \to B$ and $C \to D$, respectively, then there exists a tree of positive decision $DT_{A\to(B\to C)\to D}$ for $A \to (B \to C) \to D$ such that $l(DT_{A\to(B\to C)\to D}) \leq l(DT_{A\to C}) + l(DT_{C\to D}) + 1$.*

Proof. We are going to construct the tree $DT_{A\to B} \circ DT_{C\to D}$ which is a tree of positive decision for $A \to (B \to C) \to D$ and $l(DT_{A\to B} \circ DT_{C\to D}) \leq l(DT_{A\to B}) + l(DT_{C\to D}) + 1$. The tree $DT_{C\to D}$ will be used as a basis for the construction. Let us suppose that $\mid C \mid > 1$. Every node of $DT_{C\to D}$ has the form $C \to D'$ and will be replaced by the formula $A \to (B \to C) \to D'$, and every time that the ramifying from node $C \to D'_1 \to \cdots \to D'_k \to P$ is made in accordance with formula C, we will make one additional ramifying from the corresponding node $A \to (B \to C) \to D'_1 \to \cdots \to D'_k \to P$ to the node $A \to (B \to C) \to D'_1 \to \cdots \to D'_k \to B$ and hence all ramifyings will be made in the same way as in $DT_{A\to B}$, replacing every node of the form $A \to B'$ by the formula $A \to (B \to C) \to D'_1 \to \cdots \to D'_k \to B'$. (The initial node of $DT_{A\to B}$, after the replacement, corresponds to $A \to (B \to C) \to D'_1 \to \cdots D'_k \to B$). When made in such a way, the tree $DT_{A\to B} \circ DT_{C\to D}$ will be a tree of positive decision for $A \to (B \to C) \to D$. The maximal length of $DT_{A\to B} \circ DT_{C\to D}$ could be obtained in the case when the ramifying is made from the last but one node of the longest branch of $DT_{C\to D}$ in accordance with formula C. Then $l(DT_{A\to B}\circ DT_{C\to D}) = l(DT_{A\to B}) + l(DT_{C\to D})$. For $C \equiv Q$, we make the tree $DT_{A\to B}\circ DT_{Q\to D}$ in the same way as above, excluding the case when some of the maximal elements of the tree $DT_{Q\to T}$ has the from $Q \to D'_1 \to \cdots \to D'_k \to Q$. In this case, we make the ramifying from the node $A \to (B \to Q) \to D'_1 \to \cdots \to D'_k \to Q$ (which is obtained by replacing $Q \to D'_1 \to \cdots \to D'_k \to Q$) in accordance with the formula $(B \to Q)$ to the node $A \to (B \to Q) \to D'_1 \to \cdots \to D'_k \to B$ and hence we make all ramifying in the same way as in $DT_{A\to B}$, replacing nodes $A \to B'$ by $A \to (B \to Q) \to D'_1 \to \cdots\ D'_k \to B'$. $DT_{A\to B}\circ DT_{Q\to D}$ will be a tree of positive decision for $A \to (B \to Q) \to D$, and its maximal length can be $l(DT_{A\to B}) + l(DT_{Q\to D}) + 1$. For $C \equiv\perp$ $(l(DT_{\perp\to D}) = 0)$, the corresponding tree of positive decision for $A \to (B \to\perp) \to D$ could be made in a similar way. In this case it could be that $l(DT_{A\to B}\circ DT_{\perp\to D}) = l(DT_{A\to B}) + 1$. $\dashv$

Proof of Theorem 4. The "only if" part. By induction on the length of the proof of A. The most interesting case is when the last step in the proof of A is

$$\frac{B \to B_1 \cdots \to B_m \to P \quad C \to D_1 \to \cdots \to D_k \to Q}{(A \equiv) B \to ((B_1 \to \cdots \to B_m \to P) \to C) \to D_1 \to \cdots \to D_k \to Q}(AL).$$

The tree $DT_{B\to B_1\to\cdots\to B_m\to P}\circ DT_{C\to D_1\to\circ\to D_k\to Q}$ is a tree of positive decision for A (in accordance with Lemma 3), furthermore

$$\begin{aligned} l(DT_{B\to B_1\to\cdots\to B_m\to P}\circ DT_{C\to D_c dots\to D_k\to Q}) &\leq \mid B\mid + \mid B_1\mid + \cdots + \mid B_m\mid + \mid C\mid + \\ &\quad + \mid D_1\mid + \cdots + \mid D_k\mid + 1 \ \text{(by Lemma 3)} \\ &\leq \mid B\mid + \mid B_1\mid + \cdots + \mid B_m\mid + m + 1 + \mid C\mid + \mid D_1\mid + \cdots + \mid D_k\mid = \sum_{D\in \mathrm{ant}(A)} \mid D\mid . \end{aligned}$$

The "if" part is justified completely by Theorem 3. $\dashv$

As an immediate corollary of Theorem 4 we have that not $\vdash A$ iff for every $DT_A, l(DT_A) > \sum_{B\in \mathrm{ant}(A)} \mid B\mid$ or there is a maximal element which is trivially refutable; in other words every DT_A is a tree of negative decision for A.

So by the procedure founded on Theorem 3 for every $A \in$ For we can answer the following question whether A is a theorem in H or not?

Examples 1) $P \to Q \to P$ is trivially provable. So $\vdash P \to Q \to P$.

(2) $(P \to Q \to R) \to (P \to Q) \to P \to R$

$(P \to Q \to R) \to (P \to Q) \to P \to P$ (trivially provable) $\quad (P \to Q \to R) \to (P \to Q) \to P \to Q$

$(\to Q \to R) \to (P \to Q) \to P \to P$ (trivially provable)

So $\vdash (P \to Q \to R) \to (P \to Q) \to P \to R$.

3) $((P \to Q) \to P) \to P$

$$\Big|$$

$$((P \to Q) \to P \to Q$$

(trivially refutable)

So not $\vdash ((P \to Q) \to P) \to P$.

4) $\neg\neg P \to P \equiv ((P \to \bot) \to \bot) \to P$

$$\Big|$$

$$((P \to \bot) \to \bot) \to P \to \bot$$

$$\Big|$$

$$((P \to \bot) \to \bot) \to P \to P \to \bot$$

$$\Big|$$

$$\vdots$$

It is clear that the desicion tree for $\neg\neg P \to P$ is of length greater than $| \neg\neg P \to P |$ (it is in fact infinite), and so not $\vdash \neg\neg P \to P$.

5. Disjunction-free intermediate logics. Let X be any propositional logical system with connectives $\neg, \to, \wedge$ and $\vee$. A *disjunction-free formula* of the logic X is an formula in which the connective $\vee$ does not occur. *The disjunction-free fragment* of the logic X is the set of all disjunction-free formulas which are theorems of X. A propositional logic X is *a disjunction-free logic* if the symbol $\vee$ is definable in the disjunction-free fragment of X, i.e., there is a disjunction-free formula $C(P, Q)$ such that for any disjunction free formulas A and $\vdash_X A \vee B \leftrightarrow C(P/A, Q/B)$, where P and Q are distinct propositional letters that occur in C. $C(P/A, Q/B)$ is the formula obtained from C by substituting A and B for P and Q, respectively, at each occurrence of P and Q in C, and $A \leftrightarrow B \equiv (A \to B) \wedge (B \to A)$. X' is an $\neg, \wedge, \vee$-*extension* of H if all logical connectives of X' are among $\rceil, \to, \wedge, \vee$, and for every formula A : *if* $\vdash_H A$, *then* $\vdash_{X'} A$.

Knowing that $\vdash_H (A \wedge B \to C) \leftrightarrow (A \to B \to C)$, $\vdash_H A \to (B \wedge C) \leftrightarrow (A \to B) \wedge (A \to C)$ and $\vdash_H A \wedge B$ iff $\vdash_H A$ and $\vdash_H B$, the decision procedure given above can be used as a decision procedure for the disjunction-free fragment of H. Furthemore, this procedure is applicable to every finitely axiomatizable disjunction-free $\neg, \wedge, \vee$-extension of H.

By a result of A. Diego (cf.[**3, 6, 7**] and [**4**, p. 80]) the set S of all disjunction-free propositional formulas built out of $P_1, \dots, P_k$, which are non-equivalent in H, is finite, and so the conjuction of all the instances $A(Q_1/C_1, \dots, Q_m/C_m)$ of the formula A, denoted by $\&\{P_1, \dots, P_k\}A$, where $Q_1, \dots, Q_m$ are all the propositional letters of A and $C_1, \dots, C_m \in S$, is also finite. Furthermore, according to [**7**] the construction of S given in [**3**] is recursive.

Let H be the Heyting propositional calculus formulated as in [**1**, p. 433] with modus ponens as the only rule of inference.

THEOREM 5. *If X is a finitely axiomatizable disjunction-free $\neg, \wedge, \vee$-extension of H which is obtained by adding the formulas $A, \dots, A_n$ to H as axiom schemes, then: $\vdash_X A$ if $\vdash_H (\&\{P_1, \dots, P_k\}(A_1 \wedge \dots \wedge A_n)) \to A$, where $P_1, \dots, P_k$ are all the propositional letters of A.*

Proof. Let C be $\&\{P_1, \dots, P_k\}(A_1 \wedge \dots \wedge A_n)$. If $\vdash_H C \to A$, then it is clear that $\vdash_X A$. Converse: let m be the length of the proof π_A for A in X. For $m = 0$, A is an axiom of X, and so $\vdash_H C \to A$. Let us suppose that the last step of the proof π_A is: $\frac{B \quad B \to A}{A}$, and let $Q_1, \dots, Q_j, P_1, \dots, P_k$ be all the propositional letters that occur in π_A. If $\pi_A(Q_1, \dots, Q_j, P_1, \dots, P_k)$ is a proof for A in X, then it is easy to see that $\pi_A(Q_1/P_1, \dots, Q_j/P_1, P_1, \dots, P_k)$ is a proof for A in X of the same length as π_A. By the induction hypothesis we have $\vdash_H C \to B(Q_1/P_1, \dots, Q_j/P_1, P_1, \dots, P_k)$ and $\vdash_H C \to B(Q_1/P_1, \dots, Q_j/P_1, P_1, \dots, P_k) \to A$; hence $\vdash_H C \to A$. ⊣

Note that the given decision procedure enables us to reconstruct a proof when the examined formula is a theorem.

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