PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 33 (47), 1983, pp. 225

ON ANTI-INVERSE RINGS

Hisao Tominaga

Following B. Cerović [**1**], a ring $R\,(\neq 0)$ is called an anti-inverse ring if every element x in R has an anti-inverse x^* : $x^*xx^* = x$ and $xx^*x = x$. If R is an anti-inverse ring, then so is every non-zero homomorphic image of R, and $x^2 = x^{*2} = (xx^*)^2 = (x^*x)^2$ and $x = x^{*2}xx^{*2} = x^5$ for any $x \in R$; in particular, R is a strongly regular ring.

The present objective is to prove neatly the following proposition which covers all the results in [**1**].

PROPOSITION. *The following are equivalent:*

(1) *R is an anti-inverse ring.*

(2) *R is a subdirect sum of $GF(2)$'s and $GF(3)$'s*

(3) *R satisfies the polynomial identity $x^3 - x = 0$.*

Proof. Obviously, (2) $\Rightarrow$ (3) $\Rightarrow$ (1). It remains therefore to prove that (1)implies (2). Without loss of generality, we may assume that R is subdirectly irreducible. Then, we can easily see that the strongly regular ring R is a division ring. Now, let x be an arbitrary non-zero element of R. Then, $x^2 = 1$ and $0 = (xx^* - x^*x)^2 = 2(x^2 - x^4) = 2(x^2 - 1)$. Hence, if R is not of characteristic 2 then $x = \pm 1$, and so $R = GF(3)$. On the other hand, if R is of characteristic 2 then $0 = x^4 - 1 = (x - 1)^2$ implies $x = 1$, and so $R = GF(2)$.

REFERENCES

[1] B. Cerović, *Anti-inverse rings*, Publ. Inst. Math. (Beograd) (N.S.) **29 (43)** (1981), 45–48.

Department of Mathematics
Okayama University
Okayama, Japan

(Received 14 07 1982)