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A NOTE ON LOCALLY ALMOST PARACOMPACT SPACES

Ilija Kovačević

In [1] the author has introduced and studied the class of locally almost paracompact spaces.

A space X is *locally almost paracompact* iff each point of X has an open neighbourhood U such that $\overline{U}$ is α-almost paracompact (a subset A of a space X is *α-almost paracompact* iff for every X-open cover of A there exists an X-locally finite family of X-open sets, which refines it and the X-closures of whose members cover the set A).

In [2] the author has introduced and studied the class of locally nearly paracompact spaces.

A space X is *locally nearly paracompact* iff each point of X has an open neighbourhood U such that $\overline{U}$ is α-nearly paracompact (a subset A of a space X is *α-nearly paracompact* iff for every X-regularly open cover of A there exists an X-locally finite family of X-open sets, which refines it and covers A).

We know that every locally nearly paracompact space is locally almost paracompact (Theorem 5, [2]). The converse of this is not necessarily true. There exists a space X which is locally almost paracompact but not locally nearly paracompact (Example 3, [2]). We know that every almost regular almost paracompact space is nearly paracompact (a space X is *almost regular* iff for every regularly closed set F and any point $x \notin F$, there exist disjoint open sets contianing F and x respectively, [3]).

We can show that the similar statement for locally almost paracompact and locally nearly paracompact space is true, i. e. every almost regular locally almost paracompact space is locally nearly paracompact. (see Theorem 2).

First we show the next theorem.

Theorem 1. *Let A be any nonempty subset of the almost regular space X. Then the following are equivalent:*

(*a*) *A is α-nearly paracompact.*

(*b*) *For every X-regularly open cover of A there exists an X-locally finite family of X-regularly open sets which refines it and covers A.*

(*c*) *A is α-almost paracompact.*

(*d*) *For every X-reglarly open cover of A, there exists an X-locally finite family of X-open sets, which refines it and the X-closures of whose members cover the set A.*

(e) For every X-regularly open cover of A there exists an X-locally finite family of X-regularly closed sets which refines it and covers A.

Proof. $(a) \Leftrightarrow (b)$ Obvious.

$(b) \Rightarrow (c)$ Let A be an α-nearly paracompact subset of X and $\mathcal{U}$ any X-open covering of A. By Lemma 3 in [2] there exists an X-locally finite family $\mathcal{V}$ of X-open sets which refines $\mathcal{U}$ and is such that $A \subset \cup\{\overline{V^0}: V \in \mathcal{V}\}$. Therefore $A \subset \cup\{\overline{V}: V \in \mathcal{V}\}$, i. e. A is α-almost paracompact.

$(c) \Rightarrow (d)$ Obvious.

$(d) \Rightarrow (e)$ Let $\mathcal{U}$ be any X-regularly open cover of A. For each $x \in A$, there exists $U_x \in \mathcal{U}$ such that $x \in U_x$. Since X is almost regular, for each $x \in A$, there exists a regularly open set U_x^* such that $x \in U_x^* \subset \overline{U_x^*} \subset U_x$.
Now, $\mathcal{U}^* = \{U_x^*: x \in A\}$ is an X-regularly open cover of A. By assumtion there exists an X-locally finite family $\mathcal{V}$ of X-open sets which refines $\mathcal{U}^*$ and the X-closures of whose members cover the set A. Consider the familly,
$\mathcal{V}^* = \{\overline{V}: V \in \mathcal{V}\}$. $\mathcal{V}^*$ is an X-locally finite family of X-regularly closed sets which refines $\mathcal{U}$ and covers A. Hence the result.

$(e) \Rightarrow (a)$ Let $\mathcal{U}$ be any X-regularly open cover of A. Then there exists an X-locally finite family $\mathcal{V}$ of X-regularly closed sets which refines $\mathcal{U}$ and covers A. Since $\mathcal{V}$ is X-locally finite, for each $x \in X$, there exists an X-open set G_x such that $x \in G_x$ and G_x intersects finitely many members of $\mathcal{V}$. Then $\alpha(G_x) = \overline{G}_x^0$ is a regularly open set containing x which intersects finitely many members of $\mathcal{V}$. Let $\mathcal{G} = \{\alpha(G_x): x \in X\}$. It is an X-regularly open covering of A, hence there exists an X-locally finite family $\mathcal{B}$ of X-regularly closed sets which refines $\mathcal{G}$ and covers A. Now, for each $V \in \mathcal{V}$, let $V^* = X \setminus \cup\{B: B \in \mathcal{B}, B \cap V = \Phi\}$. Clearly, V^* is an X-open set containing V and $B \cap V^* \neq \Phi \Leftrightarrow B \cap V \neq \Phi$. Since, $\mathcal{V}$ refines $\mathcal{U}$, then for each $V \in \mathcal{V}$, there exists $U_V \in \mathcal{U}$ such that $V \subset U_V$. Let $\mathcal{A} = \{V^* \cap U_V: V \in \mathcal{V}\}$. $\mathcal{A}$ is an X-open X-locally family which refines $\mathcal{U}$ and covers A, hence A is α-nearly paracompact.

Remark 1. $(a) \Leftrightarrow (b)$ and $(c) \Leftrightarrow (d)$ without almost regularity (Lemma 1.3, [1]).

Theorem 2. *Every almost regular locally almost paracompact space is locally nearly paracompact.*

Proof. Let X be any almost regular locally almost paracompact space. Let $x \in X$. Then there exists an open neighbourhod U of x such that $\bar{U}$ is α-almost paracompact. Since X is almost regular, $\bar{U}$ is α-nearly paracompact. Hence X is locally nearly paracompact.

Corollary 1. *Every regular locally almost paracompact space is locally paracompact.*

Proof. Every regular locally almost paracompact space is locally nearly paracompact. Every regular locally nearly paracompact space is locally paracompact.

REFFRENCES

[1] I. Kovačević, *Locally almost paracompact spaces*, Review of Research, Faculty of Science, Univ. Novi Sad, **10**(1980), 85—91.

[2] I. Kovačević, *Locally nearly paracompact spaces*, Publ. Inst. Math. (Belgrade) **29(43)**(1981), 117—124.

[3] M. K. Singal, S. P. Arya, *On nearly paracompact spaces*, Mat. vesnik **6(21)** (1969), 3—16.

Odsek za matematiku
Tehnički fakultet
21000 Novi Sad
Jugoslavija