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A NOTE ON TWO-UNIT STANDBY SYSTEMS

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Formulation of the problem. The limit behaviour of redundant systems with repair and preventive repair has been investigated many times during the last few years (for instance in [1], [2], [3], [4]). In [4], the limit behaviour of the two-unit system with quick repair and quick preventive repair was found. Here, we shall investigate the behaviour of the two-unit system when either inspection occurs rarely and repair is rapid, or inspection often occurs and preventive repair is rapid.

As usual, our two-unit standby system satisfies the following conditions:

— the standby unit is unloaded;

— after repair and preventive repair completion, a unit recovers its function completely;

— after repair or preventive repair, a unit is in the standby state;

— all switchover times occurring is connection with repair or inspection are instantaneous;

— the standby unit begins to work the very moment the working unit goes from the operative state to repair, or preventive repair;

— the repair time distribution and the preventive repair time distribution are independent of the failure time distribution or the inspection time distribution, and are respectively $G(x)$, $V(x)$, $F(x)$, $U(x)$ (they all have finite mathematical expectations).

In this paper, we consider only the rigid inspection strategy (i.e. when a time for inspection comes, a unit undergoes inspection independantly of the state of the other unit).

Let $\Phi(x)$ be a time without failure distribution function of our system. Then, the corresponding Laplace-Stieltjes transform $\mathscr{F}(s)$ is ([1], [4]):

$$\mathscr{F}(s)=d_1(s)+d_2(s)-(1-d_1(s)-d_2(s))\,(d_1(s)(1-c_2(s)+c_1(s))+$$
$$+d_2(s)(1-b_2(s)+b_1(s)))\,((1-b_1(s))(1-c_2(s))-b_2(s)\,c_1(s))^{-1}$$

where

$$\mathcal{S}(s)=\int_0^\infty e^{-sx}\,d\,\Phi(x), \qquad d_1(s)=\int_0^\infty e^{-sx}\,\overline{U(x)}\,d\,F(x),$$

$$d_2(s)=\int_0^\infty e^{-sx}\,\overline{F(x)}\,d\,U(x), \qquad b_1(s)=\int_0^\infty e^{-sx}\,G(x)\,\overline{U(x)}\,d\,F(x),$$

$$b_2(s)=\int_0^\infty e^{-sx}\,V(x)\,\overline{U(x)}\,d\,F(x), \qquad c_1(s)=\int_0^\infty e^{-sx}\,G(x)\,\overline{F(x)}\,d\,U(x),$$

$$c_2(s)=\int_0^\infty e^{-sx}\,V(x)\,\overline{F(x)}\,d\,U(x)$$ (we use the notation $\overline{F(x)}=1-F(x)$).

Let us suppose that the distribution functions of the life of a unit $F(x)$ and of the preventive repair time $V(x)$ are fixed, and the inspection time distribution function $U_n(x)$ and repair time distribution function $G_n(x)$ change with the sequence $\{n\}$, so that the following conditions are satisfied:

a)
$$\int_0^\infty \overline{F(x)}\,d\,U_n(x)\xrightarrow[n\to\infty]{}0$$
$$\int_0^\infty \overline{G_n(x)}\,d\,F(x)\xrightarrow[n\to\infty]{}0$$

The conditions a) mean that the inspection is rare and the repair is quick.

On the other hand, we can suppose that the distribution function of the life of a unit $F(x)$ and of the repair time $G(x)$ are fixed, and the inspection time distribution function $U_n(x)$ and preventive repair time distribution $V_n(x)$ change with the sequence $\{n\}$, so that the following conditions are satisfied $\left(\text{a limit } S=\lim\limits_{n\to\infty}\int_0^\infty x\,d\,U_n(x) \text{ exists}\right)$:

b)
$$\int_0^\infty \overline{U}_n(x)\,d\,F(x)\xrightarrow[n\to\infty]{}0$$
$$\int_0^\infty \overline{V}_n(x)\,d\,F(x)\xrightarrow[n\to\infty]{}0$$

i.e. the inspection often occurs and preventive repair is quick.

According to those conditions, the Laplace-Stieltjes transforms where $U_n(x)$, $G_n(x)$ or $V_n(x)$ appear will obtain indexes.

Let us denote by τ the random variable which corresponds to the time without failure of the two-unit system. Then the following theorems hold:

Theorem 1. *In the two-unit system on the conditions* a) *of rare inspection and quick repair*

$$\lim_{n\to\infty} P\{\alpha_n \tau < t\} = 1 - \exp(-t/M),$$

where $M = \int_0^\infty x\,dF(x)$, *and* α_n *is a sequence tending to zero,* $\alpha_n = (1 - b_{1n}(0))(1 - c_{2n}(0)) - b_{2n}(0)\,c_{1n}(0)$.

Our limit distribution is the same as for the two-unit system without inspection [2], so that, in case the condition a) is valid, we don't need the inspection at all, because it does not have any influence on our system.

Proof. If we denote by $p_{dn}(s)$ and $p_n(s)$ the denominator and the numerator of $\mathcal{S}_n(s)$ respectively, then (with some simple transformations) we have:

$$p_n(s) = k_1(s) \int_0^\infty e^{-sx} \overline{F(x)}\, d\, U_n(x) + k_2(s) \int_0^\infty e^{-sx} \overline{G_n(x)}\; \overline{U_n(x)}\, dF(x) +$$

$$+ k_3(s) \int_0^\infty e^{-sx} G_n(x)\, \overline{F(x)}\, d\, U_n(x)$$

where:

$$k_1(s) = \int_0^\infty e^{-sx} \overline{G_n(x)}\; \overline{U_n(x)}\, dF(x) + \int_0^\infty e^{-sx} \overline{V(x)}\; \overline{F(x)}\, d\, U_n(x) -$$

$$- \int_0^\infty e^{-sx} V(x)\, \overline{F(x)}\, d\, U_n(x) \int_0^\infty e^{-sx} \overline{G_n(x)}\; \overline{U_n(x)}\, dF(x) +$$

$$+ \int_0^\infty e^{-sx} G_n(x)\, \overline{F(x)}\, d\, U_n(x) \int_0^\infty e^{-sx} V(x)\, \overline{U_n(x)}\, dF(x) +$$

$$+ \int_0^\infty e^{-sx} (\overline{V(x)} - \overline{G_n(x)})\, \overline{U_n(x)}\, dF(x) \Big(\int_0^\infty e^{-sx} \overline{F(x)}\, d\, U_n(x) +$$

$$+ \int_0^\infty e^{-sx} \overline{U_n(x)}\, dF(x) - 1 \Big) + \int_0^\infty e^{-sx} \overline{U_n(x)}\, dF(x)$$

$$k_2(s) = \int_0^\infty e^{-sx} \overline{U_n(x)}\, dF(x) \Big(1 - \int_0^\infty e^{-sx} V(x)\, \overline{F(x)}\, d\, U_n(x) \Big)$$

$$k_3(s) = \int_0^\infty e^{-sx} \overline{U_n(x)}\, dF(x) \Big(\int_0^\infty e^{-sx} \overline{V(x)}\; \overline{U_n(x)}\, dF(x) - 1 \Big).$$

Let

$$p_n^1(s)=k_1(s)\int_0^\infty \overline{F(x)}\,d\,U_n(x)+k_2(s)\int_0^\infty \overline{G_n(x)}\;\overline{U_n(x)}\,d\,F(x)+$$

$$+k_3(s)\int_0^\infty G_n(x)\,\overline{F(x)}\,d\,U_n(x),$$

$$\alpha_n=k_1(0)\int_0^\infty \overline{F(x)}\,d\,U_n(x)+k_2(0)\int_0^\infty \overline{G_n(x)}\;\overline{U_n(x)}\,d\,F(x)+$$

$$+k_3(s)\int_0^\infty G_n(x)\,\overline{F(x)}\,d\,U_n(x)=p_n^1(0),$$

then we have the following

L e m m a 1. $\lim_{n\to\infty} p_n(\alpha_n s)/\alpha_n=1$ *uniformly on s on every limited interval*[1], *or, equivalently,*

$$\lim_{n\to\infty}[\alpha_n-p_n^1(\alpha_n s)+p_n^1(\alpha_n s)-p_n(\alpha_n s)]/\alpha_n=0$$

uniformly on s.

P r o o f. First, we show

(1) $$\lim_{n\to\infty}[\alpha_n-p_n^1(\alpha_n s)]/\alpha_n=0$$

uniformly on s. We have

$$|\alpha_n-p_n^1(\alpha_n s)|=\Big|\int_0^\infty \overline{F(x)}\,d\,U_n(x)\;(k_1(0)-k_1(\alpha_n s))+$$

$$+\int_0^\infty \overline{G_n(x)}\;\overline{U_n(x)}\,d\,F(x)\,(k_2(0)-k_2(\alpha_n s))+$$

$$+\int_0^\infty G_n(x)\,\overline{F(x)}\,d\,U_n(x)\,(k_3(0)-k_3(\alpha_n s))\Big|$$

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[1] In the sequel, whenever we say that the convergence is uniform on s, it should be understood that it is uniform on s on every limited interval.

and:

$$
\begin{aligned}
|k_1(0)-k_1(\alpha_n s)| = \Big| & \int_0^\infty (1-e^{-\alpha_n sx})\,\overline{G_n(x)}\;\overline{U_n(x)}\,dF(x) + \\
& + \int_0^\infty (1-e^{-\alpha_n sx})\,\overline{V(x)}\;\overline{F(x)}\,dU_n(x) - \int_0^\infty (1- \\
& - e^{-\alpha_n sx})\,V(x)\,\overline{F(x)}\,dU_n(x) \int_0^\infty e^{-\alpha_n sx}\,\overline{G_n(x)}\;\overline{U_n(x)}\,dF(x) - \\
& - \int_0^\infty (1-e^{-\alpha_n sx})\,\overline{G_n(x)}\;\overline{U_n(x)}\,dF(x)\cdot\int_0^\infty V(x)\,\overline{F(x)}\,dU_n(x) + \\
& + \int_0^\infty (1-e^{-\alpha_n sx})\,G_n(x)\,\overline{F(x)}\,dU_n(x) \int_0^\infty \overline{V(x)}\;\overline{U_n(x)}\,dF(x) + \\
& + \int_0^\infty (1-e^{-\alpha_n sx})\,\overline{V(x)}\;\overline{U_n(x)}\,dF(x) \int_0^\infty e^{-\alpha_n sx}\,G_n(x)\,\overline{F(x)}\,dU_n(x) + \\
& + \int_0^\infty (1-e^{-\alpha_n sx})\,(\overline{V(x)}-\overline{G_n(x)})\,\overline{U_n(x)}\,dF(x)\,\Big(\int_0^\infty \overline{F(x)}\,dU_n(x) + \\
& + \int_0^\infty \overline{U_n(x)}\,dF(x) - 1\Big) + \int_0^\infty e^{-\alpha_n sx}\,(\overline{V(x)} - \\
& - \overline{G_n x)})\,\overline{U_n(x)}\,dF(x)\,\Big(\int_0^\infty (1-e^{-\alpha_n sx})\,\overline{F(x)}\,dU_n(x) + \\
& + \int_0^\infty (1-e^{-\alpha_n sx})\,\overline{U_n(x)}\,dF(x)\Big) + \int_0^\infty (1-e^{-\alpha_n sx})\,\overline{U_n(x)}\,dF(x)\Big| \leqslant \\
& \leqslant \alpha_n s\Big\{\int_0^\infty x\,\overline{G_n(x)}\;\overline{U_n(x)}\,dF(x) + \int_0^\infty x\,\overline{V(x)}\;\overline{F(x)}\,dU_n(x) + \\
& + \int_0^\infty x\,V(x)\,\overline{F(x)}\,dU_n(x)\cdot\int_0^\infty e^{-\alpha_n sx}\,\overline{G_n(x)}\;\overline{U_n(x)}\,dF(x) + \\
& + \int_0^\infty x\,\overline{G_n(x)}\;\overline{U_n(x)}\,dF(x) \int_0^\infty V(x)\,\overline{F(x)}\,dU_n(x) + \\
& + \int_0^\infty x\,G_n(x)\,\overline{F(x)}\,dU_n(x) \int_0^\infty \overline{V(x)}\;\overline{U_n(x)}\,dF(x) +
\end{aligned}
$$

$$+\int_0^\infty x\,\overline{V(x)}\,\overline{U_n(x)}\,dF(x)\int_0^\infty e^{-\alpha_n sx}G_n(x)\cdot\overline{F(x)}\,dU_n(x)+$$

$$+\int_0^\infty x\,|\,\overline{V(x)}-\overline{G_n(x)}\,|\,\overline{U_n(x)}\,dF(x)\Big(\int_0^\infty \overline{F(x)}\,dU_n(x)+$$

$$+\int_0^\infty \overline{U_n(x)}\,dF(x)-1\Big)+\int_0^\infty e^{-\alpha_n sx}\,|\,\overline{V(x)}-$$

$$-\overline{G_n(x)}\,|\,\overline{U}_n(x)\,dF(x)\Big(\int_0^\infty x\,\overline{F(x)}\,dU_n(x)+\int_0^\infty x\,\overline{U}_n(x)\,dF(x)\Big)+$$

$$+\int_0^\infty x\,\overline{U}_n(x)\,dF(x)\Big\}\leqslant 16\,\alpha_n s\int_0^\infty x\,dF(x)$$

(Here, and in the sequel, using integration by parts of Stieltjes integral, we do the following estimation:

$$\int_0^\infty x\,\overline{F(x)}\,dU_n(x)=[x\,\overline{F}(x)\;U_n(x)]_0^\infty-\int_0^\infty U_n(x)\,d(x\,\overline{F}(x))=$$

$$=-\int_0^\infty U_n(x)\,\overline{F}(x)\,dx+\int_0^\infty x\,U_n(x)\,dF(x)\leqslant 2\int_0^\infty x\,dF(x),$$

and $\lim\limits_{x\to\infty} x\,\overline{F}(x)\,U_n(x)=0$ is valid, because mathematical expectation of a failure time is finite, and therefore

$$x(1-F(x))=x\int_x^\infty dF(t)\leqslant\int_x^\infty t\,dF(t)\xrightarrow[x\to\infty]{}0);$$

$$|\,k_2(0)-k_2(\alpha_n s)|=\int_0^\infty (1-e^{-\alpha_n sx})\,\overline{U}_n(x)\,dF(x)\Big(1-\int_0^\infty V(x)\,\overline{F}(x)\,dU_n(x)\Big)+$$

$$+\int_0^\infty e^{-\alpha_n sx}\,\overline{U}_n(x)\,dF(x)\int_0^\infty (1-e^{-\alpha_n sx})\,V(x)\,\overline{F}(x)\,dU_n(x)\leqslant$$

$$\leqslant \alpha_n s\Big\{\int_0^\infty x\,\overline{U}_n(x)\,dF(x)\Big(1-\int_0^\infty V(x)\,\overline{F}(x)\,dU_n(x)\Big)+$$

$$+\int_0^\infty e^{-\alpha_n sx}\,\overline{U}_n(x)\,dF(x)\int_0^\infty x\,V(x)\,\overline{F}(x)\,dU_n(x)\Big\}\leqslant$$

$$\leqslant 3\,\alpha_n s\int_0^\infty x\,dF(x);$$

$$|k_3(0)-k_3(\alpha_n s)|=\int_0^\infty (1-e^{-\alpha_n sx})\,\overline{U_n}(x)\,dF(x)\cdot\left|\int_0^\infty \overline{V(x)}\,\overline{U_n}(x)\,dF(x)-1\right|+$$

$$+\int_0^\infty e^{-\alpha_n sx}\,\overline{U_n}(x)\,dF(x)\int_0^\infty (1-e^{-\alpha_n sx})\,\overline{V}(x)\,\overline{U_n}(x)\,dF(x)\leqslant$$

$$\leqslant \alpha_n s\left\{\int_0^\infty x\,\overline{U_n}(x)\,dF(x)\cdot\left|\int_0^\infty \overline{V}(x)\,\overline{U_n}(x)\,dF(x)-1\right|+\right.$$

$$\left.+\int_0^\infty e^{-\alpha_n sx}\,\overline{U_n}(x)\,dF(x)\int_0^\infty x\,\overline{V}(x)\,\overline{U_n}(x)\,dF(x)\right\}\leqslant$$

$$\leqslant 2\,\alpha_n s\int_0^\infty x\,dF(x).$$

Consequently

$$|\alpha_n-p_n^1(\alpha_n s)|\leqslant 16\,\alpha_n s\int_0^\infty x\,dF(x)\left\{\int_0^\infty \overline{F(x)}\,dU_n(x)+\int_0^\infty \overline{G_n}(x)\,\overline{U_n}(x)\,dF(x)+\right.$$

$$\left.+\int_0^\infty G_n(x)\,\overline{F(x)}\,dU_n(x)\right\}.$$

Owing to the conditions a), the expression in brackets tends to zero, and so, (1) is proved.

Let us show now, that $\lim\limits_{n\to\infty}[p_n^1(\alpha_n s)-p_n(\alpha_n s)]/\alpha_n=0$ uniformly on s:

$$p_n^1(\alpha_n s)-p_n(\alpha_n s)=k_1(\alpha_n s)\int_0^\infty (1-e^{-\alpha_n sx})\,\overline{F(x)}\,dU_n(x)+$$

$$+k_2(\alpha_n s)\int_0^\infty (1-e^{-\alpha_n sx})\,\overline{G_n}(x)\,\overline{U_n}(x)\,dF(x)+$$

$$+k_3(\alpha_n s)\int_0^\infty (1-e^{-\alpha_n sx})\,G_n(x)\,\overline{F}(x)\,dU_n(x)\leqslant$$

$$\leqslant \alpha_n s\left\{k_1(\alpha_n s)\int_0^\infty x\,\overline{F(x)}\,dU_n(x)+\right.$$

$$+k_2(\alpha_n s)\int_0^\infty x\,\overline{G_n}(x)\,\overline{U_n}(x)\,dF(x)+$$

$$\left.+k_3(\alpha_n s)\int_0^\infty x\,G_n(x)\,\overline{F}(x)\,dU_n(x)\right\}\leqslant$$

$$\leqslant \alpha_n s \left\{ k_1(\alpha_n s)\left(A_n^1 \int_0^\infty \overline{F}(x)\, d\, U_n(x) + \int_{A_n^1}^\infty x\overline{F(x)}\, d\, U_n(x)\right) + \right.$$

$$+ k_2(\alpha_n s)\left(A_n^2 \int_0^\infty \overline{G_n}(x)\, d\, F(x) + \int_{A_n^2}^\infty x d\, F(x)\right) +$$

$$\left. + k_3(\alpha_n s)\left(A_n^3 \int_0^\infty \overline{F}(x)\, d\, U_n(x) + \int_{A_n^3}^\infty x\overline{F(x)}\, d\, U_n(x)\right)\right\}.$$

Let $A_n^1 = A_n^3 = \left(\int_0^\infty \overline{F}(x)\, d\, U_n(x)\right)^{-1/2}$; $A_n^2 = \left(\int_0^\infty \overline{G_n}(x)\, d\, F(x)\right)^{-1/2}$, then the items in brackets near $k_i(\alpha_n s)$, $i = 1, 2, 3$ tend to zero and consequently, $\lim_{n\to\infty} [p_n^1(\alpha_n s) - - p_n(\alpha_n s)]/\alpha_n = 0$ uniformly on s. The lemma is proved.

The denominator p_{d_n} of $\mathscr{S}_n(s)$ is:

$$p_{d_n}(s) = 1 - \int_0^\infty e^{-sx}\overline{U_n(x)}\, d\, F(x) - \int_0^\infty e^{-sx}\overline{F(x)}\, d\, U_n(x) +$$

$$+ \int_0^\infty e^{-sx}\overline{G_n(x)}\;\overline{U_n(x)}\, d\, F(x) + \int_0^\infty e^{-sx}\overline{V(x)}\;\overline{F(x)}\, d\, U_n(x) +$$

$$+ \int_0^\infty e^{-sx} G_n(x)\,\overline{U_n(x)}\, d\, F(x) \int_0^\infty e^{-sx} V(x)\,\overline{F(x)}\, d\, U_n(x) -$$

$$- \int_0^\infty e^{-sx} V(x)\,\overline{U_n(x)}\, d\, F(x) \int_0^\infty e^{-sx} G_n(x)\,\overline{F(x)}\, d\, U_n(x) =$$

$$= 1 - \int_0^\infty e^{-sx}\overline{U_n(x)}\, d\, F(x) - \int_0^\infty e^{-sx}\overline{F(x)}\, d\, U_n(x) + q_n(s).$$

The following chain of equalities is obvious:

$$\alpha_n = p_n(0) = p_{d_n}(0) = q_n(0),$$

because of $\int_0^\infty \overline{U_n(x)}\, d\, F(x) + \int_0^\infty \overline{F(x)}\, d\, U_n(x) = 1$ and $\mathscr{S}_n(0) = 1$. We have now that

$$\lim_{n\to\infty} q_n(\alpha_n s)/\alpha_n = 1$$

is valid, uniformly on s. We shall not give here the corresponding proof, because for that we have only to use method similar to the method used for Lemma 1.

We denoted by τ the random variable which corresponds to the time without failure of our two-unit system, and $\Phi(t)$ is its distribution function. Then $P\{\alpha_n \tau < t\} = \Phi(t/\alpha_n)$, $\int_0^\infty e^{-st} d\Phi(t/\alpha_n) = \mathcal{S}_n(\alpha_n s)$, and

$$\mathcal{S}_n(\alpha_n s) = \frac{p_n(\alpha_n s)}{1 - \int_0^\infty e^{-\alpha_n sx} \overline{U_n(x)}\, dF(x) - \int_0^\infty e^{-\alpha_n sx} \overline{F(x)}\, d\, U_n(x) + q_n(\alpha_n s)} =$$

$$\frac{p_n(\alpha_n s)/\alpha_n}{\left[1 - \int_0^\infty e^{-\alpha_n sx} \overline{U_n(x)}\, dF(x) - \int_0^\infty e^{-\alpha_n sx} \overline{F(x)}\, d\, U_n(x)\right]/\alpha_n + q_n(\alpha_n s)/\alpha_n} \xrightarrow[n\to\infty]{} \frac{1}{1+sM},$$

where $M = \int_0^\infty x dF(x)$. We have:

$$M = \lim_{n\to\infty} \left(\int_0^\infty x\, \overline{U_n(x)}\, dF(x) + \int_0^\infty x\, \overline{F(x)}\, d\, U_n(x)\right) =$$

$$= \int_0^\infty x dF(x) + \lim_{n\to\infty} \left(-\int_0^\infty x\, U_n(x)\, dF(x) + \int_0^\infty x\, \overline{F(x)}\, d\, U_n(x)\right),$$

and $0 \leqslant \lim\limits_{n\to\infty} \left| -\int_0^\infty x\, U_n(x)\, dF(x) + \int_0^\infty x\, \overline{F(x)}\, d\, U_n(x)\right| \leqslant$

$$\leqslant \lim_{n\to\infty} \left(B_n \int_0^\infty U_n(x)\, dF(x) + \int_{B_n}^\infty x dF(x) + B_n \int_0^\infty \overline{F(x)}\, d\, U_n(x) + \right.$$

$$\left. + \int_{B_n}^\infty x\, \overline{F(x)}\, d\, U_r(x)\right)$$

Let us choose $B_n = \left(\int_0^\infty U_n(x)\, dF(x)\right)^{-1/2} = \left(\int_0^\infty \overline{F(x)}\, d\, U_n(x)\right)^{-1/2}$, ($B_n \xrightarrow[n\to\infty]{} \infty$ because of the condition a)), then items in brackets all tend to zero as $n \to \infty$.

Thus, we have that the limit distribution function of a time without failure is exponential, i.e.

$$\lim_{n\to\infty} P\{\alpha_n \tau < t\} = 1 - \exp(t/M).$$

Theorem 2. *In the two-unit system when the inspection often occurs and the preventive repair is quick (conditions* b)), *we have* $\lim_{n\to\infty} P\{\alpha_n \tau < t\} = 1 - \exp(t/S)$ *where* $S = \lim_{n\to\infty} \int_0^\infty x\, d\, U_n(x)$, *and* α_n *is a sequence tending to zero,* $\alpha_n = (1 - b_{1n}(0))\,(1 - c_{2n}(0)) - b_{2n}(0)\; c_{1n}(0)$.

Proof. We shall not give here the detailed proof of Theorem 2, but just the main lines, because in this proof we use the same technique as in the proof of Theorem 1.

As before, $p_{dn}(s)$ and $p_n(s)$ are the denominator and the numerator of $\mathcal{S}_n(s)$; put $\alpha_n = p_{dn}(0) = p_n(0)$. We have:

$$p_n(s) = k_1(s) \int_0^\infty e^{-sx}\, \overline{U_n(x)}\, d\,F(x) + k_2(s) \int_0^\infty e^{-sx}\, \overline{G(x)}\; \overline{U_n(x)}\, d\,F(x) +$$

$$+ k_3(s) \int_0^\infty e^{-sx}\, \overline{V_n(x)}\; \overline{F(x)}\, d\,U_n(x) + k_4(s) \int_0^\infty e^{-sx}\, \overline{V_n(x)}\; \overline{U_n(x)}\, d\,F(x),$$

where:

$$k_1(s) = \int_0^\infty e^{-sx}\, \overline{G(x)}\; \overline{U_n(x)}\, d\,F(x) \left(1 - \int_0^\infty e^{-sx}\, (V_n(x) + 1)\, \overline{F(x)}\, d\,U_n(x)\right) +$$

$$+ \int_0^\infty e^{-sx}\, \overline{V_n(x)}\; \overline{U_n(x)}\, d\,F(x) \int_0^\infty e^{-sx}\, (G(x) + 1)\, \overline{F(x)}\, d\,U_n(x) +$$

$$+ \int_0^\infty e^{-sx}\, \overline{G(x)}\; \overline{F(x)}\, d\,U_n(x)$$

$$k_2(s) = \int_0^\infty e^{-sx}\, \overline{F(x)}\, d\,U_n(x) \left(2 - \int_0^\infty e^{-sx}\, (V_n(x) + 1)\, \overline{F(x)}\, d\,U_n(x)\right)$$

$$k_3(s) = \int_0^\infty e^{-sx}\, \overline{F(x)}\, d\,U_n(x)$$

$$k_4(s) = \int_0^\infty e^{-sx}\, \overline{F(x)}\, d\,U_n(x) \left(\int_0^\infty e^{-sx}\, (G(x) + 1)\, \overline{F(x)}\, d\,U_n(x) - 1\right),$$

and

$$p_{dn}(s) = 1 - \int_0^\infty e^{-sx}\, \overline{U_n(x)}\, d\,F(x) - \int_0^\infty e^{-sx}\, \overline{F(x)}\, d\,U_n(x) + q_n(s),$$

where

$$q_n(s) = \int_0^{\infty} e^{-sx}\,\overline{G(x)}\,\overline{U_n(x)}\,dF(x) + \int_0^{\infty} e^{-sx}\,\overline{V_n(x)}\,\overline{F(x)}\,dU_n(x) +$$

$$+\int_0^{\infty} e^{-sx}\,G(x)\,\overline{U_n(x)}\,dF(x) \int_0^{\infty} e^{-sx}\,V_n(x)\,\overline{F(x)}\,dU_n(x) -$$

$$-\int_0^{\infty} e^{-sx}\,V_n(x)\,\overline{U_n(x)}\,dF(x) \int_0^{\infty} e^{-sx}\,G(x)\,\overline{F(x)}\,dU_n(x)$$

We state here the following lemma (without proof):

Lemma 2. $\lim_{n\to\infty} p_n(\alpha_n s)/\alpha_n = 1$, $\lim_{n\to\infty} q_n(\alpha_n s)/\alpha_n = 1$ *uniformly on* s.

Then, taking into account Lemma 2, we have that $\mathcal{S}_n(\alpha_n s) \xrightarrow[n\to\infty]{} (1+sS)^{-1}$ which means that $\lim_{n\to\infty} P\{\alpha_n \tau < t\} = 1 - \exp(-t/S)$, where

$$S = \lim_{n\to\infty} \left[\int_0^{\infty} x\,\overline{U_n(x)}\,dF(x) + \int_0^{\infty} x\,\overline{F(x)}\,dU_n(x)\right] = \lim_{n\to\infty} \int_0^{\infty} x\,dU_n(x).$$

REFERENCES

[1] Б. В. Гнеденко, М. Динич, Ю. Наср, *О надёжности дублированной системы с восстановлением и профилактическим обслуживанием*, Изв. АН. СССР Техническая Кибернетика, 1 (1975), 66—71.

[2] Б. В. Гнеденко, Ю. К. Беляев, А. Д. Соловёв, *Математические методы в теоррии надёжности*, Наука, Москва, 1965.

[3] Б. В. Гнеденко, И. И. Махмуд, *О длительности безотказной работы дублированной системы с восстановлением и профилактикой*, Изв. АН. СССР Техническая Кибернетика, 3 (1976), 86—91.

[4] S. Janjić, *Limit Theorems for Two-Unit Standby Redundant Systems with Rapid Repair and Rapid Preventive Maintenance*, Publ. Inst. Math. (Beograd) 29 (43) (1981), 75—87.

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