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CORRECTION OF THE PAPER „GRAPHS WITH GREATEST NUMBER OF MATCHINGS"

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In [1] Lemma 10 and therefore also Theorem 5 are erroneous. We give now the correction of these statements. The present notation and terminology is the same as used in [1].

Lemma 10 (corrected). *If* $n>8$ *then*

$$C_4(1,\ 1)\,P_{n-8}(n-8,\ 1)\,C_4 \succ C_a(1,\ 1)\,P_{n-a-b}(n-a-b,\ 1)\,C_b$$

for all values of a *and* b ($a \geqslant 3,\ b \geqslant 3$), *provided* $n-a-b>0$.

Proof. By applying eq. (1) from [1] to the graph

$C_a(1,\ 1)\,P_{n-a-b}(n-a-b, 1)\,C_b$ we get

$$(1)\qquad p(C_a(1,\ 1)\,P_{n-a-b}(n-a-b,\ 1)\,C_b,\ k)=p(C_a(1,\ 1)\,P_{n-a},\ k)+$$
$$+p(C_a(1,\ 1)\,P_{n-a-b}\dot{+}P_{b-2},\ k-1)=p(C_a(1,\ 1)\,P_{n-a},\ k)+$$
$$+p(P_{n-b}\dot{+}P_{b-2},\ k-1)+p(P_{n-a-b}\dot{+}P_{a-2}\dot{+}P_{b-2},\ k-2).$$

It has been demonstrated elsewhere (cf. Lemma 6 of [1]) that for $1 \leqslant j \leqslant m-1$,

$$(2)\qquad P_{m-2}\dot{+}P_2 \succ P_{m-j}\dot{+}P_j.$$

Therefore, assuming that the parameter a has a fixed value, the expression (1) will be maximal for $b-2=2$, i. e. $b=4$. A completely analogous reasoning leads also to the condition $a=4$. This proves Lemma 10.

In the above proof it has been tacitly assumed that the graph $P_{n-a-b-1}$ exists, i. e. that $n-a-b>0$. This detail has been overlooked in [1].

For the case $n-a-b=0$ we have the following result.

Lemma 11. *If* $n \geqslant 7$ *and* $3 \leqslant a \leqslant n-3$, *then* $C_4(1,\ 1)\,C_{n-4} \succ C_a(1,\ 1)\,C_{n-a}$.

Proof. For $n=7$, 8 and 9 the validity of Lemma 11 can be checked by direct calculation. Using eq. (1) from [1] one can easily verify the following identity.

$$(3)\qquad p(C_a(1,1)C_{n-a}, k) = p(C_a(1,1)C_{n-a-1}, k) + p(C_a(1,1)C_{n-a-2}, k-1).$$

Eq. (3) enables one to prove Lemma 11 by induction on the number n of vertices.

Lemma 12. For $n \geqslant 9$, $C_4(1,1)C_{n-4} \succ C_4(1,1)P_{n-8}(n-8,1)C_4$.

Proof. By applying eq. (1) from [1] to the graph $C_4(1,1)C_{n-4}$ one obtains

$$(4)\qquad p(C_4(1,1)C_{n-4}, k) = P(C_4(1,1)P_{n-4}, k) + \\ + p(P_4 \dot{+} P_{n-6}, k-1) + p(P_2 \dot{+} p_{n-6}, k-2).$$

Subtracting eq. (1) for $a=b=4$ from (4), we get

$$(5)\qquad p(C_4(1,1)C_{n-4}, k) - p(C_4(1,1)P_{n-8}(n-8,1)C_4, k) = \\ = p(P_4 \dot{+} P_{n-6}, k-1) + p(P_2 \dot{+} P_{n-6}, k-1) - p(P_2 \dot{+} P_{n-4}, \\ k-1) - p(P_2 \dot{+} P_2 \dot{+} P_{n-8}, k-2).$$

Since

$$p(P_4 \dot{+} P_{n-6}, k-1) = p(P_2 \dot{+} P_2 \dot{+} P_{n-6}, k-1) + p(P_1 \dot{+} P_1 \dot{+} P_2 \dot{+} P_{n-8}, \\ k-2) + p(P_1 \dot{+} P_1 \dot{+} P_1 \dot{+} P_{n-9}, k-3),$$

$$p(P_2 \dot{+} P_{n-6}, k-2) = p(P_2 \dot{+} P_2 \dot{+} P_{n-8}, k-2 + p(P_1 \dot{+} P_2 \dot{+} P_{n-9}, k-3),$$

$$p(P_2 \dot{+} P_{n-4}, k-1) = p(P_2 \dot{+} P_2 \dot{+} P_{n-6}, k-1) + \\ + p(P_1 \dot{+} P_1 \dot{+} P_2 \dot{+} P_{n-8}, k-2) + p(P_1 \dot{+} P_2 \dot{+} P_{n-9}, k-3),$$

we further transform the expression (5) into

$$p(C_4(1,1)C_{n-4}, k) - p(C_4(1,1)P_{n-8}(n-8,1)C_4, k) \\ = p(P_1 \dot{+} P_1 \dot{+} P_1 \dot{+} P_{n-9}, k-3) = p(P_{n-9}, k-3),$$

which is evidently non-negative for all values of k. This proves Lemma 12.

The Lemmas 10—12 together with the other results obtained in [1] enable the formulation of

Theorem 5 (corrected) *The greatest matching equivalence class exists in the set* $\Gamma(n,2)$ *for all* $n \geqslant 4$. *For* $n=8$ *and* $n \geqslant 10$ *the greatest matching equivalence class in* $\Gamma(n,2)$ is $\{C_4(1,1)C_{n-4}\}$

In Fig. 1 are presented the bicyclic graphs with the greatest number of matchings, having $n=5$, 6, 7, 8 and 9 vertices.

Proof. Having in mind the results of [1] it is sufficient to verify that

$$(6)\qquad C_4(1,1)C_5 \sim Q(4,2,7)$$

and that for $n \geqslant 10$,

$$(7)\qquad C_4(1,1)C_{n-4} \succ Q(4,2,n-2).$$

Applying eq. (1) from [1] to the graph $Q(4, 2, n-2)$ one obtains

$$p(Q(4, 2, n-2), \mathrm{k}) = \mathrm{p}(C_4(1, 1)\, P_{n-4}, \mathrm{k}) + p(P_{n-2}, k-1) =$$
$$= p(C_4(1, 1)\, \mathrm{P}_{n-4}, k) + p(P_4 \dot{+} P_{n-6}, \mathrm{k}-1) + p(P_3 \dot{+} P_{n-7}, k-2).$$

Combinig this identity with eq. (4) we get

$$(8) \qquad p(C_4(1, 1)\, C_{n-4}, k) - p(Q(4, 2, n-2), k) =$$
$$= p(P_2 \dot{+} P_{n-6}, k-2) - p(P_3 \dot{+} P_{n-7}, k-2).$$

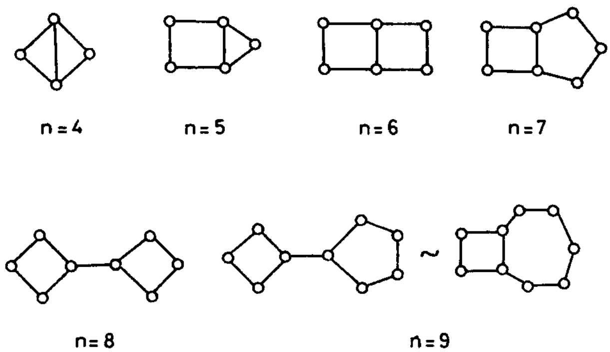

Fig. 1

If $n=9$, the graphs $P_2 \dot{+} P_{n-6}$ and $P_3 \dot{+} P_{n-7}$ are isomorphic and therefore (6) holds. If $n \geqslant 10$, the expression (8) is non-negative for all values of k because of (2). Therefore (7) holds.

Theorem 5 is thus proved.

REFERENCES

[1] I. Gutman, *Graphs with greatest number of matchings*, Publ. Inst. Math. **27(41)** (1980), 67—76.

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