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GRAPHS WHICH ARE SWITCHING EQUIVALENT TO THEIR COMPLEMENTARY LINE GRAPHS II

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All connected graphs which are switching equivalent to their complementary line graphs have been found in [1]. Here, we will find the corresponding disconnected

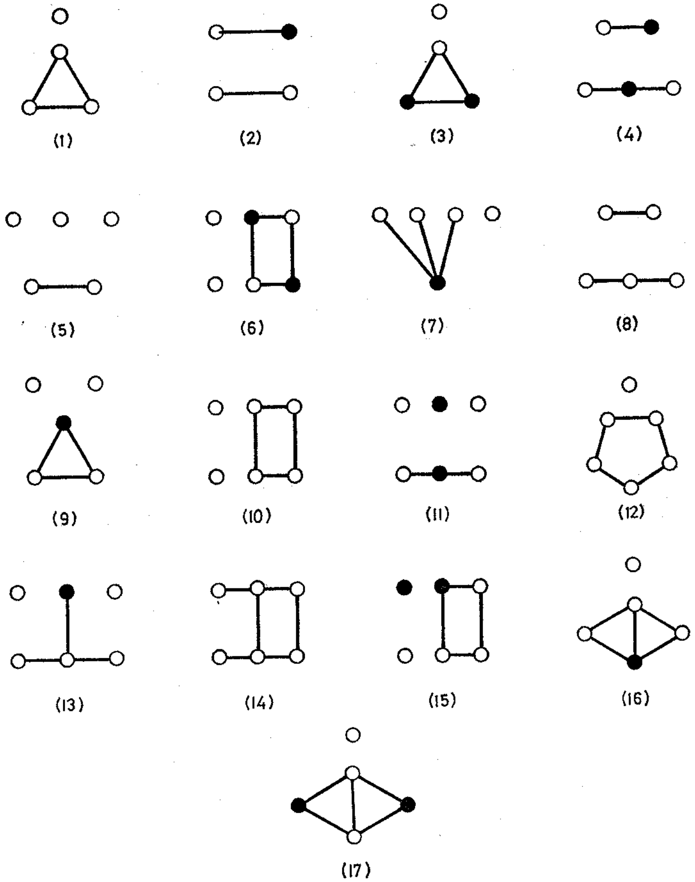


Fig. 1.

* The results of this paper were communicated on the Seventh Yugoslav Congress on Mathematics, Physics and Astronomy, Bečići, 1980.

graphs. Throughout the paper we shall follow the terminology from [2]. For all other facts the reader is refered to [1]. We first list some lemmas (without proofs) which will be used in the further text. Most of them already appeared in [1] but they were restricted to the connected case.

L e m m a 1. *If* $\mathcal{S}(G_c)$ *is a complement of a line graph, then* G_c *does not contain as a colour induced subgraph any of the following graphs* of Fig.* 1.

C o r o l l a r y 1. *If* $K_{1,2} \cup K_2 \subseteq G$, *then* $K_{1,2}$ *and* K_2 *are both monochromatic in* G_c, *but coloured with different colours.*

L e m m a 2. *If* $K_3 \not\subseteq G$ *and* $\mathcal{S}(G_c) = \overline{L(G)}$, *then* G_c *does not contain as a colour induced subgraph any of the graphs of Fig.* 2.

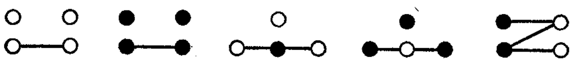


Fig. 2.

L e m m a 3. *If* $\mathcal{S}(G_c) = \overline{L(G)}$, *then the number of colour induced subgraphs of* G_c *equal to any of the graphs of Fig.* 3 *is the same as the number of subgraphs of* G *equal to* C_4.

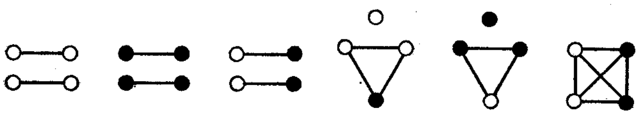


Fig. 3.

L e m m a 4. *If* $\mathcal{S}(G_c) = \overline{L(G)}$, *then the number of colour induced subgraphs of* G_c *equal to any the graphs of Fig.* 4 *(including the graphs obtained by interchanging the colours) is the same as the number of subgraphs of* G *equal to* C_5.

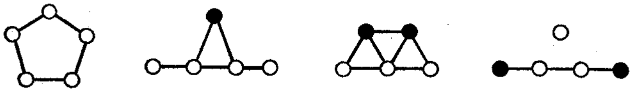


Fig. 4.

L e m m a 5. *If* $\mathcal{S}(G_c) = \overline{L(G)}$, *then* $\Delta(G) = max\ \{p(H_c)\}$, *where* H_c, *as a coloured induced subgraph of* G_c, *is isomorphic to* $K_{m,n}$ *for some* $m, n \geq 0$ *and also coloured properly (in the usual sense).*

In the next lemma an r-matching means a mathching with r-edges involved.

* Of course, the colours in the above graphs may be mutually interchanged.

Lemma 6. *If* $\mathcal{S}(G_c)=\overline{L(G)}$, *then the number of r-matchings of G equals the number of subgraphs of* G_c *which are switching equivalent to* K_r.

Lemma 7. *Suppose* $\mathcal{S}(G_c)=\overline{L(G)}$. *Then we have*

$$q_{BW}=\frac{1}{4}\left(\sum_{i=1}^{p} d_i^2+p-p_B^2-p_W^2\right),$$

where q_{BW} *is the number of BW-type edges of* G_c, *while* p_B *(respectively,* p_W*) being the number of black (white) vertices of* G_c.

From now on we assume that G (if it exists) is the solution of the generalized graph equation $L(G)\sim\overline{G}$. Since G is disconnected, the components of G, when considered in G_c ($\varphi(G_c)=\overline{L(G)}$), may be of all three types, namely B, W and BW. For facility denote by $n(B)$, $n(W)$ and $n(BW)$ the number of components of each of the types. Also, assume that u, $n(B_t)$, $n(W_t)$ refer to trivial ones, while $n(B_{nt})$, $n(W_{nt})$ to nontrivial ones.

Proposition 1. *If* $n(BW)\neq 0$, *then* $n(B_{nt})=n(W_{nt})=0$ *and moreover* $n(BW)<2$.

Proof. According to (2)* we get $n(B_{nt})=n(W_{nt})=0$. If $n(BW)\geqslant 2$, then, by (2) and (4), all BW components are isolated edges. Thus $p(G)>q(G)$, a contradition. ■

Proposition 2. *If* $n(BW)=1$, *then* $n(B_t)+n(W_t)\leqslant 2$.

Proof. Suppose the contrary, i.e. $n(B_t)+n(W_t)>2$.

Case 1: All isolated vertices are coloured by one colour, say, black. According to (5) all black vertices are mutually nonadjacent. Thus, they induce a clique complete subgraph) in $\mathcal{S}(G_c)$ and also $n(B_t)$ out of them have no other neighbours except those in the clique. Since $\mathcal{S}(G_c)=\overline{L(G)}$, it follows that G may be viewed as

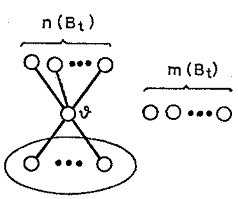


Fig. 5.

* In the further text we shall refer to Lemma 1 only by forbidden subgraphs of Fig .1.

in Fig. 5. We first assume that v is coloured black. Its neighbours must be white by (5), and also mutually nonadjacent. Hence, all cycles of G_c are even. By Lemma 2, the induced ones must be of length 4. The latter contradicts (6). So, let v be coloured white. Then, by (7), at least one neighbour of v that belongs to a hanging edge is coloured white. Consequently, there is a vertex of $\overline{\mathcal{S}(G_c)}$ being nonadjacent to all vertices of the mentioned clique. This implies that some edge of G is nonincident to v or its neighbours. By (2) or (5), this is a contradiction.

Case 2: The isolated vertices are coloured by both colours. We may assume that $n(B_t) \geqslant n(W_t) > 0$. Now, by (1) and (3), G has no triangles. Since $n(B_t) \geqslant 2$, by Lemma 2, all black vertices must be mutually nonadjacent. Hence, the rest of the proof can be carried over in the same way as in the previous case. ∎

Proposition 3. *If* $n(BW)=1$ *and* $n(B_t)+n(W_t)=2$, *then* $n(B_t)=$ $=n(W_t)=1$.

Proof. Suppose the contrary, and assume $n(B_t)=2$, $n(W_t)=0$. Then, according to (1), (3) and (9), if G_c has a triangle all its vertices must be white. By (5) and (10), the black vertices of BW component induce a totally disconnected graph or a star.

Assume first that they induce a star. Now G is of the form as in Fig. 6 with v_1 and v_2 adjacent.

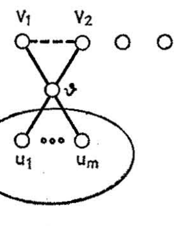


Fig. 6.

Clearly, v, v_1, v_2 must be white. Hence, by (1), all white vertices of G_c are adjacent to v. If $u_1, \ldots, u_m$ are all black, then $\overline{\mathcal{S}(G_c)}$ would be disconnected. Otherwise, applying the same argument, it follows that, say, u_1 is white and has a black neighbour. The latter contradicts (2).

Now assume that all black vertices of the BW component are mutually nonadjacent. In this case G is of the form as in Fig. 6 with v_1 and v_2 being nonadjacent. Next suppose v is black. Then v_1, v_2, $u_1, \ldots, u_m$ are all white and mutually nonadjacent. By (1), it follows that G_c cannot have white triangles implying that G has no triangles. Further, by Lemma 2, all white vertices are also mutually nonadjacent. Thus, all cycles of G are even, and properly coloured. The latter contradicts Lemma 2. So let v be white. If v_1 and v_2 are both black, then, by (7) $u_1, \ldots, u_m$ are all white, while by (11) there are no more white vertices in G_c. Observing now v and all white vertices of G_c, we can conclude by considering $\overline{\mathcal{S}(G_c)}$ that, say, u_1 and u_2 are adjacent and have no neighbours except v. Also, since $\overline{\mathcal{S}(G_c)}$ is connected, some black vertex must be adjacent to some white vertex among

$u_3, \ldots, u_m$. The latter contradicts (2). If, say, v_1 is white, then considering $\overline{\mathcal{S}(G_c)}$ again, it follows that there exists and edge in G being nonincident to v or its neighbours. By (2), both vertices of this edge are white. The latter contradicts either (2) or (8). ■

Propsition 4. *If $n(BW)=1$ and $n(B_t)=n(W_t)=1$, then G_c is given in* Fig. 17(a).

Proof. According to (1) and (3) G has no triangles. Hence, by Lemma 2, all black (white) vertices of BW component induce either a totally disconnected graph or a complete bigraph. The same lemma yields that any vertex of one colour is adjacent to at most one vertex of an other colour. Consider now the isolated vertices. They are clearly nonadjacent in $\mathcal{S}(G_c)$. Since $\overline{\mathcal{S}(G_c)}=L(G)$, there exist two edges in G inducing $2K_2$. By (2) and Lemma 2, these edges are monochromatic in G_c but coloured with different colours. So the vertices of each colour of BW component induce bicomplete graphs. Then, in $\mathcal{S}(G_c)$, the vertices of any of these bicomplete graphs together with the isolated vertex of the same colour induce two cliques with just one vertex in common. Also, notice that the vertices of these two pairs of cliques cover all vertices of $\overline{\mathcal{S}(G_c)}$. Thus, besides the pair of edges already mentioned, all other edges of G are adjacent to just one edge from the observed pair. This implies that the vertices of each colour in BW component induce the trees. Now we easily get that the graph of Fig. 17(a) is a solution to our equation for each $m,n \geqslant 0$. ■

So we have $n(B_t)+n(W_t)=1$. The BW component is now bicyclic, i.e. has two independent cycles. Denote these cycles by C' and C''. Let $l(C')$ and $l(C'')$ be their lengths. Also assume $n(B_t)=1$. In other words let the isolated vertex be black.

Proposition 5. *Cycles C' and C'' cannot be disjoint (without common vertices).*

Proof. Suppose the contrary, i.e. C' and C'' are disjoint. Then, by Lemma 1, they are both monochromatic but coloured with different colours. If C' is black, then, by (1) and (12), $l(C') \neq 3$ and 5, while by (5) or (13), $l(C') < 6$. Thus $l(C')=4$. Next, by (2), $l(C'') \leqslant 4$ while the distance between C' and C'' is one. Owing to (10) all black vertices of BW component are adjacent to some vertex of C'. The same applives to white vertices with respect to C'' if (1), (2) and (8) are inspected. By Lemma 3, we must have $2K_2 \subseteq G$, and also, both copies of K_2 must be equally coloured in G_c. If $l(C'')=4$, the latter is impossible by Lemma 2. So let $l(C'')=3$. Then $K_2 \cup K_3 \subseteq G$, and also the copies of K_2 are monochromatic in G_c but coloured with different colours. Hence, by Lemma 6, there must exist in G a 5-matching. This fact, combined with (5) and (14), enables us to construct few graphs which are not solutions as can be seen by using Lemma 5. ■

So the cycles C' and C'' intersect with each other. Let P be a path of intersection provided also its length is as small as possible. Let $l(P)$ be the length of P. We will address the next possibilities by an ordered triplet $\tau=(l(P),\ l(C'),\ l(C''))$.

Proposition 6. *If $\tau=(0, m, n)$, then G does not exist.*

Proof. Now the graph of Fig. 7 appears in G as an induced subgraph. By Corollary 1 (and also Lemma 3 if $l(C')=l(C'')=3$), $u_1, u_2, \ldots$ belong to one colour class, while $v_1, v_2, \ldots$ to the other. Next, assume $l(C') \leqslant l(C'')$.

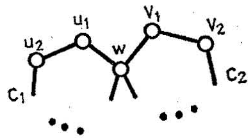


Fig. 7.

Case 1: $l(C')=3$. If u_1, u_2 are black, then, by (1), w must be white. We now easily get a contradiction by (3) or Lemma 3. So let u_1, u_2 be white. By (3) the same holds for w. Owing to symmetry, we may assume $l(C'') \geqslant 4$. If $l(C'')=4$, by (1) and (15), all white vertices must be adjacent to w. Now, $\overline{\mathcal{S}(G_c)}$ is disconnected, a contradiction. If $l(C'')=5$, by using (2) and Lemma 3, it follows that G_c has no more black vertices, and also all additional white vertices are adjacent to the vertices from C'. By (5), at most three additional white vertices may appear and, by Lemma 5, neither of them is adjacent to w. This implies that either u_1 or u_2 is of degree one, a contradiction. If $l(C'') \geqslant 6$, the contradiction follows by (2).

Case 2: $l(C') \geqslant 4$. By Lemma 2, the cycle having all vertices black (except possibly w) must be of the length four. So $l(C')=4$ and also we can take that u_1, u_2, u_3 are black. The same lemma implies that w is also black and that G_c has no more black vertices. If $l(C'')<6$, we easily get a contradiction by using (2) and Lemma 3; for $l(C'') \geqslant 6$ just (2) is sufficient. ■

Proposition 7. *If* $\tau=(1,3,3)$, *then* G *does not exist.*

Proof. Now C_4+x (see Fig. 8) is an induced subgraph of G. The vertices of C_4+x are all white as follows from (1), (3), (16) and (17). By (1), the other white vertices are adjacent to w_1 or w_2, while by (2), each of them has degree one.

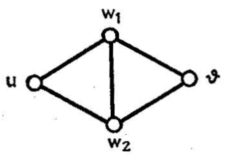


Fig. 8.

By Lemma 3, G contains $2K_2$ as an induced subgraph where both copies of K_2 are equally coloured. Clearly, these copies of K_2 cannot be of W-type. Also, since K_4-x is a subgraph of G, it follows that there exists a vertex in $L(G)$

adjacent to all vertices of C_4. If both copies of K_2 are of BW-type, then each of them is incident to either u or v. But then no vertex of $_c\overline{\mathcal{S}(G_c)}$ is adjacent to all vertices from C_4. If both copies of K_2 are of B-type, then, by (5), G_c has no more black vertices. So an isolated vertex corresponds to a vertex of b $_c\overline{\mathcal{S}(G_c)}$ which is adjacent to all vertices of C_4. Since this vertex has no other neighbours in $_c\overline{\mathcal{S}(G_c)}$ the same holds for vertices w_1, w_2 of Fig. 8. The ıemaining two graphs can be rejected as solutions, for instance, by using Lemma 6. ■

P r o p o s i t i o n 8. *If* $\tau=(1,3,4)$, *then* G_c *is given in Fig.* 17(*b*).

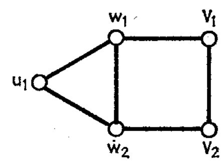


Fig. 9.

P r o o f. Now C_5+x (see Fig. 9) is an induced subgraph of G. We first assume that there exists in G a vertex at distance two from the (unique) triangle of G. Then, by using (1), (3) and (9), it easily follows that this triangle must be white, while the vertex in question black. Next by (1) and (2), all white vertices of G_c not belonging to C_5+x are adjacent to some vertex of the triangle and also, their degrees are one. According to Lemma 4, it follows that the graph as the last one from Fig. 4 appears in G_c. The latter, after eliminating some impossible situations, is rejected by (2). So all vertices of G except the isolated one are adjacent to some vertex of the triangle. If all vertices of the triangle are white, then by Lemma 6, there exists at least one hanging edge at each vertex of the triangle. By Lemma 3, the hanging edge at u_1 is unique and coloured equally as edge v_1v_2. So G can have just one 4-matching by Lemma 6. The graph that now results is not a solution as can be easily checked. So, by Lemma 3, two vertices of the triangle are black while one is white. Suppose next that $deg\ u_1 \geqslant 3$. By (2) and Lemma 3, the hanging edge at u_1 and edge v_1v_2 are both monochromatic but coloured with different colours. Hence, by Lemma 6, there exists a 4-matching in G and also the same lemma implies that w_1 and w_2 each have just one hanging edge. Using Lemma 4, we now obtain two graphs of Fig. 10 as possible solutions. Neither of them is a solution by Lemma 7.

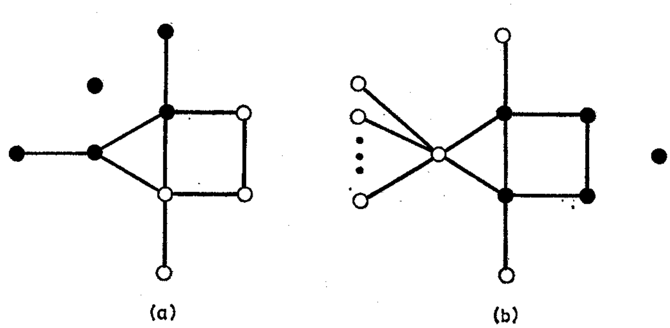


Fig. 10.

Thus let $deg\ u_1=2$. Then we get a graph of Fig. 17(b) which is a solution of our equation for each $m, n \geqslant 0$. ■

P r o p o s i o n 9. *If* $\tau=(1, 3, n)$ *where* n 5, *then G does not exist.*

P r o o f. Now the graph of Fig. 11 is an induced subgraph of G. Using (1), (3) and (9), it follows that the vertices of the triangle are all white while $v_1, v_2, \ldots$ are all black. If $n \geqslant 6$ we have a contradiction owing to (2). When $n=5$, by (2) again, all black vertices except isolated ones are adjacent to v_2. The white ones are, by (1), adjacent to some vertex of the triangle. Hence, v_2 is of degree one in $\overline{\mathscr{S}(G_c)}$, a contradiction. ■

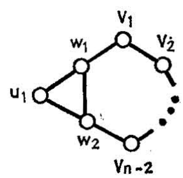
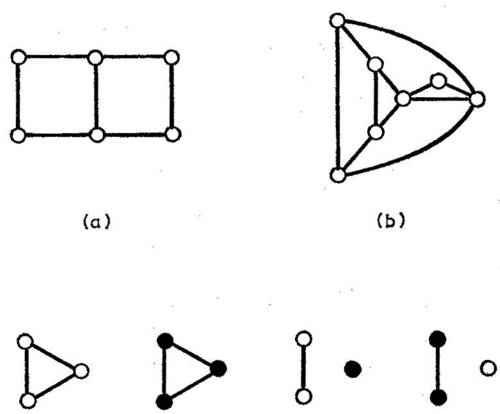


Fig. 11.

P r o p o s i t i o n 10. *If* $\tau=(1,4,4)$, *then G does not exist.*

P r o o f. Now the graph of Fig. 12(a) appears in G as an induced subgraph. Consequently, it follows that the graph Fig. 12(b) is an induced subgraph of $\overline{L(G)}$, or equivalently of $\mathscr{S}(G_c)$. After some appropriate switching of the second graph we get a graph which must be an induced subgraph of G. Since G has no triangles the colourings as in Fig. 12(c) cannot appear in the graph of Fig. 12(b). By direct checking it easily follows that these requirements cannot be satisfied. ■



(a) (b)

(c)

Fig. 12.

Proposition 11. *If* $\tau=(1,4,n)$ *where* $n\geqslant 5$, then G does not exist.

Proof. Suppose first $n=5$. Then the graph of Fig. 13 appears in G as an induced subgraph.

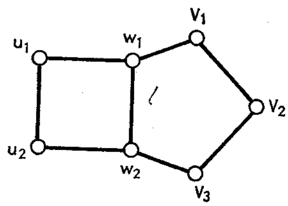


Fig. 13.

By Corollary 1, u_1, u_2 belong to one colour class while v_1, v_2, v_3 to the other. Using (2) it follows that w_1,w_2 belong to the same colour class as u_1,u_2. Assume first u_1, u_2, w_1, w_2 are all black. Then, by Lemma 2, there are no more black vertices. From (2) it follows that all white vertices are adjacent to v_2. But then $\overline{\mathcal{S}(G_c)}$ is disconnected, a contradiction. So let u_1,u_2,w_1,w_2 be all white. Hence, by Lemma 2, all black vertices are adjacent to v_2 and consequently the degree of v_2 in $\overline{\varphi(G_c)}$ is one. Thus there exists and edge at distance one from the graph of Fig. 14. The latter contradicts (2). *If* $n\geqslant 6$, the contradiction follows by (2) and Corollary 1. ■

Proposition 12. *If* $\tau=(1,m,n)$ *where* $m,n\geqslant 5$, *then* G *does not exist.*

Proof. By Corollary 1 the vertices from $C'-P$ and $C''-P$ are equally coloured but belong to different colour classes. Now (2) implies that the vertices from P could not been satisfactorily coloured. ■

Proposition 13. *If* $\tau=(2,4,4)$, *then* G *does not exist.*

Proof. Now $K_{2,3}$ (see Fig. 14) is an induced subgraph of G implying that C_6 is an induced subgraph of $\overline{L(G)}$.

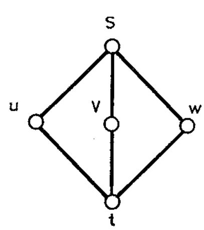


Fig. 14.

Analogously as in the proof of Proposition 10, we get that $3K_2$ must be an induced subgraph of G and moreover each copy of K_2 is of BW-type in G_c. By Corollary 1, any of these copies must be incident to some vertex from the graph of Fig. 14. Further, we easily get that each of them appears as a hanging edge with one of the vertices u, v, w. Now observe the isolated vertex as well. Then we immediately get that $\overline{\mathcal{S}(G_c)}$ contains a subgraph owing to which either s or t must have hanging edges. The latter contradicts Lemma 6. ■

Proposition 14. *If* $\tau=(2,4,5)$, *then* G_c *is given in Fig.* 17(c).

Proof. Now the graph of Fig. 15 appears in G as an induced subgraph.

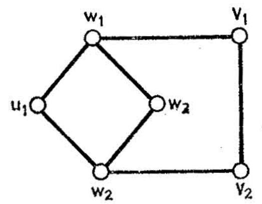


Fig. 15.

By Lemma 3, G_c must contain just one pair of equally coloured edges that induce $2K_2$. Denote these edges by x and y. By Corollary 1 and Lemma 2, both edges are incident to some vertex from the graph of Fig. 15. Also, if some edge is adjacent to x or y, then these three edges cannot induce $K_{1,2}\cup K_2$ in G. Of course, both of the edges x and y cannot be contained in the graph of Fig. 15. So, say, x appears as a hanging edge of some vertex of the graph of Fig. 15. Assuming symmetry, we can suppose that x is incident only to w_1 or u_1 (v_1 is eliminated according to the above observations).

Suppose x is incident to w_1. Now y coincides with v_2w_3. If u_1 or w_2 have any hanging element, we get a contradiction by (2) or Lemma 2. If there exists any hanging element at v_1, then Lemma 6 (combined with some usual reasoning) implies that G has a 4-matching. The latter yields that some vertex of degree greater than one (which is not on the graph of Fig. 15) is adjacent to v_1. Using lemmas from our list, we easily get a contradiction trying to colour this graph. If there are no hanging elements with v_1, we get a graph which can be, by direct checking, rejected as a solution.

We now suppose that x is incident to u_1. By Lemma 2, x cannot be of B-type. If x is of W-type, then either y is a hanging edge at w_2 or coincides with v_1v_2. In both cases, by Lemma 2, there are no hanging elements at vertices w_1 and w_3. If y is the hanging edge at w_2, then, by Corrollary 1, only the hanging edges of B-type, either at v_1 or v_2 could appear. On the other hand, if y coincides with v_1v_2, then the hanging edges of the same type could appear only at w_2. Both possibilities can be eliminated as solutions by using Lemma 4. Finally, if x is of BW-type, we easily get that the graph of Fig. 17(c) is a solution of our equation for each $m, n\geqslant 0$. ■

P r o p o s i t i o n 15. *If* $\tau=(2,4,n)$ *where* $n\geqslant 6$, *then* G *does not exist.*

P r o o f. If $n\geqslant 7$, then, by using (2), (4) and (8) we easily get a contradiction. So let $n=6$. Now the graph of Fig. 16 appears in G as an induced subgraph.

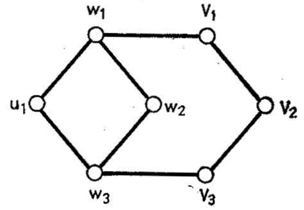


Fig. 16.

Using Corollary 1 it follows that u_1, w_1, w_2, w_3 are coloured by one colour while v_1, v_2, v_3 by the other one. Also, by (2), any vertex of G is adjacent to some vertex of the graph of Fig. 16. Thus $L(G)$ has no vertices of degrees less than two. On the other hand, the degree of v_2 in $\mathscr{S}(G_c)$ is less than two. Namely, it does not have neighbours of the opposite colour, while, by (2), all vertices of the same colour (excluding isolated ones) are adjacent to it. ■

P r o p o s i t i o n 16. *If* $\tau=(l, m, n)$ *where* $l\geqslant 3$, $m, n\geqslant 6$, *then* G *does not exist.*

P r o o f. Now $3K_2\subseteq G$ and we can easily obtain a contradiction by using either (2) or Lemma 3. ■

So it remains to examine the case when $n(BW)=0$.

P r o p o s i t i o n 17. *If* $n(BW)=0$, *then* G_c *is one the graphs of Fig.* 17 *(d, e, f).*

P r o o f. Assume $G_c=G_B\cup G_W$ where $G_B(G_W)$ is coloured black (white). Now we have $L(G_B)\cup L(G_W)=\overline{G}_B\cup\overline{G}_W$. So we shall discuss the following cases:

Case 1. G_B and G_W are both disconnected. Of course, $\overline{G}_B$ and $\overline{G}_W$ are connected. Thus $L(G_B)$ and $L(G_W)$ must have in total two components. The same holds for G_B and G_W with respect to nontrivial components. If either G_B (or G_W) has two nontrivial components, then, by (1) and (8), it must be acyclic. Thus G is acyclic too, a contradiction So each of the graphs G_B and G_W have just one nontrivial component. Therefrom, either $L(G_B)=\overline{G}_B$, $L(G_W)=\overline{G}_W$ or $L(G_B)=$ $=\overline{G}_W$, $L(G_W)=\overline{G}_B$ holds. These two possibilities may be reduced to the following ordinary graph equation: $\varphi^i(H)=H$, where $\varphi=\overline{L}$ while $i=1$ or 2. In [3] it is pointed out that all solutions of the above equation are C_5 and $L(K_{1,3})$ no matter what i is $(i\geqslant 1)$. Thus there are no solutions in this case.

Case 2. G_B and G_W are both connected. Applying similar arguments as in the case above we get the solutions already mentioned in the statement of the proposition.

Case 3. Say, G_B is connected while G_W is disconnected. Assume first that G_W has at least two nontrivial components. Using (1) and (8), it follows that each of them is an isolated edge. Thus $L(G)$ has just one nontrivial component and at least two trivial ones. On the other hand, considering $\overline{\mathcal{S}(G_c)}$, it follows that G_B is complete since G_W is connected and nontrivial. The resulting graph G is not a solution as can be seen by direct verification. If G_W has just one nontrivial component, there are no other solution as can be seen by applying the same argument as in Case 1. ∎

Collecting the above conclusions we arrive to our main result.

T h e o r e m 1. *If G is disconnected and $L(G) \sim \overline{G}$, then G(i.e. G_c) is one of the graphs of Fig.* 17.

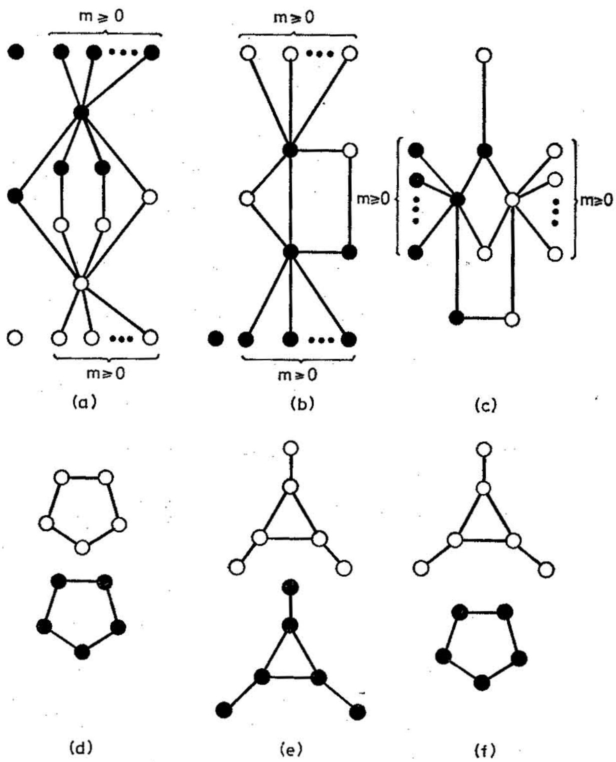


Fig. 17.

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