PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 31 (45) 1982, 121—128

A LINK BETWEEN ORDERED SETS AND TREES ON THE RECTANGLE TREE HYPOTHESIS

Đuro Kurepa

0. Summary. Certainly the simplest ordered chains and ordered sets are well-ordered sets and trees, respectively.

0:1. We had the opportunity to link these two kinds of ordered sets in particular by associating to every ordered set $(E, \leqslant)$ a tree, labelled

(0:2) $$w(E, \leqslant)$$

and consisting of all well-ordered subsets of $(E, \leqslant)$ and ordered by the relation $\leqslant\cdot$ where

(0:3) $a \leqslant\cdot b \Leftrightarrow a$ *is an initial part of* b; in particular, the empty set v is the first member of $(w(E, \leqslant), \leqslant\cdot)$.

0:4. The tree $(w(E, \leqslant), \leqslant\cdot)$ has interesting properties. In particular, for any ordered chain $(L, \leqslant)$, the tree $w(L, \leqslant)$ reflects some global properties of the chain $(L, \leqslant)$, expressed by the following

0:5. Theorem. *For any infinite totally ordered set $(L, \leqslant)$ the equality (this is a global property of $(L, \leqslant)$)*

(0:6) $$d(L, \leqslant)^{1)} = \operatorname{cel}(L, \leqslant) := c^{2)}$$

holds if and only if every tree $T \subset w(L, \leqslant)$ of cardinality c^+ satisfies

(0:7) $$kT = bT^{3)} \quad (kX \text{ denotes the cardinality of } X).$$

The proof of 0:5 Theorem will be given in section 1.

0:8. We were interested in the problem whether there is a strictly increasing mapping of the tree

$$\sigma(E, \leqslant)^{4)} \text{ into } (E, \leqslant),$$

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[1] $d(L, \leqslant) := \inf kX$, $X \subset L$, $\overline{X} = (L, \leqslant)$, i. e. X is everywhere dense in $(L, \leqslant)$.

[2] $c(L, \leqslant) = \operatorname{cel}(L, \leqslant) := \sup_F kF$, F being any disjoint family of non empty intervals of the chain $(L, \leqslant)$.

[3] For any ordered set $(E, \leqslant)$ we set $b(E, \leqslant) := \sup kD$, D being a degenerated subset of $(E, \leqslant)$, i. e. one in which the comparability relation is transtive. One sees that D is characterized by the property: to be the union of pairwise incomparable chains, i. e., such that each member of any chain is incomparable to each member of another chain.

[4] $\sigma(E, \leqslant)$ is the system of all members of $w(E, \leqslant)$ each bounded in $(E, \leqslant)$; it matters that the empty set is a member of $\sigma(E, \leqslant)$ — the first member of the trees $w(E, \leqslant)$, $\sigma(E, =)$ with respect to the order relation $\leqslant\cdot$.

$(E, \leqslant)$ being any given ordered set. The answer is that *for no* $(E, \leqslant)$ *there should be a strictly increasing mapping of* $\sigma(E, \leqslant)$ *into* $(E, \leqslant)$ (cf. Đ. Kurepa [1964:6] 1.1 Theorem).

0:9. In section 2 we shall prove some statements concerning our tree rectangle hypothesis. In section 3 we shall prove the main theorems of the present paper.

1. Proof of the 0:5 Theorem. 1:1. *(necessity).*

In the opposite case there would be a chain $(L, \leqslant)$ having an infinite cL and a tree $T \subset wL$ such that

1:1:1. $$kT > bT.$$

Then, necessarily, the number kT would be isolated, say $\aleph_{n+1}$ and the levels of T would be $\leqslant \aleph_n$ each. On the other hand, the assumed hypothesis 0:6 implies that there exists a subset B of distinct points

$$b_0, b_1, \ldots, b_m, \ldots \quad (m < \omega_{(c)})$$

such that B is everywhere dense in $(L, \leqslant)$ and $kB = c$; thus

1:1:2. $$\bar{B} = L, \quad kB = c.$$

For every $e \in wL$ and every $x \in e$, let $te(\cdot, x)$ denote the order-type of the set $e(\cdot, x)$ of all members of e, each $< x$.

1:1:3. Lemma. *If* $e, e' \in wL$, $x \in e \cap e'$ *and* $te(\cdot, x) \neq te'(\cdot, x)$, *then* $e \| e'$, *i.e. neither* $e \leqslant \cdot e'$ *nor* $e' \leqslant \cdot e$.

Proof. Let $e := (e_n)_{n<t}$, $e' = (e_{n'}')_{n'<t'}$, and $e_n = x = e_{n'}'$; then $n = te(\cdot, x)$, $n' = te'(\cdot, x)$; by assumption, $n \neq n'$, thus $n < n'$ or $n > n'$. If $n < n'$, then $e_n = x$, $e_n' < e_{n'}' = x$; thus $e_n \neq e_n'$ and therefore $e \| e'$. In a similar way, if $n' < n$, then $e \| e'$.

1:1:4. The assumption 1:1:1 yields that $\gamma T = \omega_{(c)+1}$ and that each level $R_\alpha T$ of T is $\leqslant c$.

1:1:5. For every $x \in e \in T$ let $tT(\cdot, x) := \sup_e te(\cdot, x) := g(x)$ $(x \in e \in T)$; then

1:1:6 $$g(x) < \omega_{(c)+1} := \nu.$$

Proof. In the opposite case there would be an $x \in L$ such that $g(x) \geqslant \nu$. By transfinite induction one should establish a ν-sequence e^n $(n < \nu)$ of members of wL such that the ν-sequence $te^n(\cdot, x)$ $(n < \nu)$ would be strictly increasing; by 1:1:3 this would imply that the ν-sequence e^n $(n < \nu)$ would yield an antichain of cardinality $k\nu = \omega_{(c)+1} = c^+$, which contradicts the assumption 1:1:1.

1:1:7. The assumption 1:1:2 and the 1:1:5 Lemma for an everywhere dense subset B of $(L, \leqslant)$ such that $kB = c$ yield that the number

1:1:8 $\beta := \sup_b t(T, b)$ $(b \in B)$ satisfies $\beta < \nu$ (v. 1:1:5).

1:1:9. Let w_n $(n<\nu)$ be a well-order of all well-ordered 3-point segments $(e_{\alpha\cdot 3}, e_{\alpha\cdot 3+1}, e_{\alpha\cdot 3+2})$ of $e\in T$ such that $te(\cdot, e_{\alpha\cdot 3})>\beta$; the latter relation implies that

$$B\cap w_n=\nu.$$

The set B being everywhere dense in $(L, \leqslant)$ there exists a point $b^n\in B$ located between the first point and the last point of w_n. Let T' be the tree obtained from T just by the latter procedure of inserting a member of B between extremal members of every w_n. T' is a well defined part of $w(L, \leqslant)$. Since $kT'=c^+$ and since $\sup_{b\in B} t(T', b)=\nu$ and $kB=c$, we infer that there exists a point $b\in B$ such that

$$t(T', b)=\nu \quad \text{(v. 1:1:6)}.$$

By the 1:1:3 Lemma we conclude that there exists a subset A' of T' composed of pairwise incomparable elements and that $kA'=c^+$. In other words, if $x, y\in A'$ are distinct, then $x\|y$. Now, for every $z'\in T'$ let z be the element of T obtained from z' by deleting every member x of z' of a rank $>\beta$ such that $x\in B$; obviously, $z\in T$. Moreover, if u', z' are distinct members of A', then $u\|z$ (in the opposite case, if e. g. $u<\cdot z$, then one would have $u'<\cdot z'$ in A' — absurdity, because A' is an antichain). Consequently, the set $A:=\{z' \therefore z\in A'\}$ would be an antichain in T such that $kA=c^+$ which contradicts the assumption 1:1:1. This contradiction ends the proof of the first part of the 0:5 Theorem.

1:2. *Proof of the second part of* 0:5 *(sufficiency).* 1:2:1. In the opposite case, one should have 0:7 and $\neg$0:6 i. e. one should have an infinite chain L such that $dL=c^+$.[1)]

Now, let us consider a complete bipartition — atomization D of L; this would be a decreasing tree of height $\omega_{(c)+1}$ of intervals of L; for any chain $C\subset D$ let $1\,C$ be the well-ordered subset of left end points of intervals — elements of C; then $1\,C\in wL$ and so we have a mapping

$$C\subset D \to 1\,C\in wL \quad (C \text{ being a chain in } (D, \supset)).$$

Let (1:2:2) $\mathrm{p}=\inf 1\,C$ for some chain $C\subset D$; then the number of solutions for C in (1:2:2) is $\leqslant c$ (in the opposite case one would have an inversely well-ordered set of points $\sup C$ of cardinality $>c$ — absurdity).

1:2:3. Let $1D:=\{1\,C \therefore C\subset D,\ C$ being the chain$\}$.

Then, in virtue of (1:2:2) $1\,D$ would be a subtree of wL and $k1D=kD=c^+$. So the tree $1D$ would satisfy, by assumption, the relation (0:7), i.e. $k1D=b1D$. Since the number $b1D$ is regular, there would be a chain or an antichain of $1\,D$ of cardinality $b1D$. The first case being obviously impossible, we infer that there would be an antichain A of $1\,D$ such that $kA=c^+$. For any $a\in A$ let $I(a)\in C\in D$ such that $\inf I(a)=a$; $I(a)$ is a subdivision interval of L in the atomization D of $(L, \leqslant)$; for distinct members a, a' of A the intervals $I(a)$, $I(a')$ would be disjoint; in other words the A-un of intervals $I(a)$ $(a\in A)$ would be a disjointed system of cardinality $kA=c^+$ of non empty intervals of L — absurdity, because cel $L=c<c^+$. Q. E. D.

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1) For any infinite chain $(L, \leqslant)$ we have $dL\in\{cL, (cL)^+\}$ (v. Kurepa [1935:2, 3] p. 121. Theorem 2).

2. On the rectangle hypothesis for trees.

2:1. For a graph (G, R) we define the *global width* of (G, R) as the number
2:1:1 $k_{c'}(G, R) := \sup_A kA$, A being any antichain of graph, i. e. A is any subsystem of G containing no two distinct comparable members.

2:2. The *global length* of (G, R) is the number

2:2:1. $$k_c(G, R) := \sup kL,$$

L being any subset of G such that any 2 members of L are comparable.

2:3. *The rectangle hypothesis* or the *chain × antichain hypothesis* for a graph (G, R) reads:

(2:4) $k(G, R) \leqslant k_c(G, R) \cdot k_{c'}(G, R)$ (cf. Kurepa Đ. [1963:3] nos 3:3, 4:3:3, 4:3:4 and [1964:7]). In the general case, the statement (2:4) is false.

2:5. The most interesting case is the corresponding statement for trees $(T, \leqslant)$:

(2:6) $kT \leqslant k_c(T, \leqslant) \cdot k_{c'}(T, \leqslant)$ *(tree rectangle hypothesis).*

2:7. Theorem. *The tree rectangle hypothesis* [TRH] *is an undecidable statement* (conjectured in Kurepa [1935]; v. also Kurepa [1964:7], [1977:5, 6]; model for ⅂TRH: in Solovay — Tennenbaum [1973]; model for TRH: independently in Jech [1967], Tennenbaum [1968]).

2:7:1. Theorem. *The TRH for trees of cardinality* $\aleph_1$ *is equivalent to the positive answer to the Suslin problem* (Kurepa [1935] p. 106 case b), p. 124 (last passage), p. 132 ($P_4 \Leftrightarrow P_5$).

2:8. Theorem. *The TRH is equivalent to the statement that for every infinite tree* T *one has*

2:9 $$kT = bT.$$

Proof. Necessity: 2:6 ⇒ 2:9. As a matter of fact, for every infinite tree T we have obviously $k_c T \cdot k_{c'} T = bT$; the relation 2:6 implies $kT \leqslant bT$, thus $kT = bT$, i. e. 2:9, because obviously $kT \geqslant bT$.

Sufficiency: 2:9 ⇒ 2:6. Now, this implication is implied by 2:9 and the obvious fact that $bT \leqslant k_c T \cdot k_{c'} T$.

2:10. Theorem. *The TRH is equivalent to the statement that for every infinite totally ordered set* $(L, \leqslant)$

2:10:1 $$T \subset w(L, \leqslant) \;\&\; kT \geqslant \aleph_0 \Rightarrow kT = bT.$$

Proof. 2:10:2. The ⇒ part of 2:10 being obvious, let us prove the ⇐ part.

2:10:3. Now, in virtue of the 0:5 Theorem the equality $kT = bT$ for every infinite $T \subset wL$ implies 0:6.

2:10:4. Since for every infinite tree T one has $kT = bT$ or $kT = (bT)^+$ (Đ. Kurepa [1935:2, 3] p. 105 Th. 1), we have to prove that the implication 2:10:1 implies $kT = bT$ and also TRH.

2:10:5. Let us assume the contrary, i.e. that for some ordinal there exists a tree T_α such that

2:10:6 $$\aleph_\alpha = bT, \quad kT_\alpha = \aleph_{\alpha+1}.$$

2:10:7. Then necessarily the height or the rank γT of T equals $\omega_{\alpha+1}$ and every row $R_n T_\alpha$ has $\leqslant k\,\omega_\alpha$ points. One proves that T_α contains a subtree T of cardinality $\aleph_{\alpha+1}$ such that for every $x \in T$ the set $T[x]$ of all points of T comparable to x has the rank $\omega_{\alpha+1}$ and that x has in T infinitely many next followers and that every chain as well as every antichain of T is $\leqslant \aleph_\alpha$ (cf. Đ. Kurepa [1935:2, 3] p. 109, Th. 2).

2:10:8. Let $\mathcal{N}$ be the system of all nodes of $(T, \leqslant)$[1]; for every node N of $(T, \leqslant)$ let $(N, \leqslant_N)$ be a total order of $\mathcal{N}$ such that N has neither a first nor a last member. The orderings $(T, \leqslant)$, $(N, \leqslant_N)$ $(N \in \mathcal{N})$ yield a total order $(T, \prec)$ of T in the following way:

for $x, y \in T$ let $x \prec y$ mean that either $x \leqslant y$ or that $x \|_{\leqslant} y$ and $x' <_N y'$, where N is the node contained in a row $R_\alpha T$ of minimal index α such that N contains a member $x' < x$ and a member $y' < y$ such that $x' \neq y'$ (cf. the notion of natural order extension of $(T, \leqslant)$ in Đ. Kurepa [1935:2, 3], № 2, p. 87).

2:10:9. Let $(L, \leqslant)$ be a Dedekind completion of $(T, \prec)$. Then obviously

2:10:10. $$\operatorname{cel}(L, \leqslant) = c(T, \prec) = \aleph_\alpha.$$

2:10:11. Let $\mathcal{D}$ be a total bipartion of $(L, \leqslant)$ and E the system of all non singleton intervals occuring in this atomization $\mathcal{D}$ of $(L, \leqslant)$, (cf Đ. Kurepa [1935:2, 3] p. 83 № 3, p. 114). One sees easily that the rank γE of the system $(E, \supset)$ is $\omega_{\alpha+1}$ and that every row of E is $\leqslant \aleph_\alpha$ and that $kE = \aleph_{\alpha+1}$.

2:10:12. Let us consider the system $B\colon = \{E(\cdot, x],\ x \in E\}$. Every $y \in E(\cdot, x]$ is an interval of L; the end points of y are $\inf y$, $\sup y$ and they are distinct.

2:10:13. For every $a \in E$ let $ia\colon = \{\inf y \therefore y \supset a,\ y \in E\}$; then ia is a well-ordered subset of $(L, \leqslant)$; $\inf a$ is the last member of ia.

2:10:14. For any given $a \in E$ the relations $ix = ia$, $x \in E$ have $\leqslant \aleph_\alpha$ solutions.

As a matter of fact, all these solutions constitute a strictly decreasing well-ordered family of intervals of $(L, \leqslant)$ having all just $\inf a$ as its common end point.

2:10:15. If $a, b \in E$ and neither $ia <\cdot\, ib$ nor $ib <\cdot\, ia$, then the intervals a and b of $(L, \leqslant)$ do not overlap, i.e. the sets int a and int b are disjoint.

As a matter of fact, if the intervals a and b overlapped, then one would have $a \subset b$ or $b \subset a$, and consequently $E(\cdot, a] \subset E(\cdot, b]$ or $E(\cdot, b] \subset E(\cdot, a]$, and further $ia <\cdot\, ib$ or $ib <\cdot\, ia$, respectively, contradicting the starting assumption.

2:10:16. Let $W\colon = \{ix \therefore x \in E\}$. Then

2:10:17. $W \subset w(L, \leqslant)$, $kW = kE = \aleph_{\alpha+1}$.

[1] A node of a tree T is every maximal subset X of T such that all members of X have same predecessors in T.

At first, $E=\bigcup i^{-1}\{z\}$ $(z\in W)$; since by 2:10:14 one has $ki^{-1}\{z\}\leqslant\aleph_\alpha$ for every $z\in W$, one has $kE\leqslant kW\cdot\aleph_\alpha$; this relation jointly with $kE=\aleph_{\alpha+1}$ in 2:10:11 implies the requested equality in 2:10:17.

2:10:18. In virtue of the relations 2:10:17 the assumed implication 2:10:1 would yield (put W instead of T) the equality $kW=bW$, i. e. $bW=\aleph_{\alpha+1}$. Since the number $\aleph_{\alpha+1}$ is regular, there would be a degenerated subset $X\subset W$ such that $kX=\aleph_{\alpha+1}$.

2:10:19. X being degenerated the sets $X[a, \cdot)$ are chains in $(w(L, \leqslant), \leqslant\cdot)$; therefore each of them is $\leqslant\aleph_\alpha$; since $X=\bigcup X[a,\cdot)$ $(a\in R_0 X=$ the first row of X) one concludes that $kR_0X=\aleph_{\alpha+1}$. Consequently, the set $R_0X:=A$ would be an antichain of W of cardinality $\aleph_{\alpha+1}$.

If for every $x\in W$ one denotes by x' a member of E such that $ix'=a$, then in virtue of 2:10:15 the system $\{x', x\in W\}$ would be a set of cardinality $\aleph_{\alpha+1}$ of non overlapping intervals of the chain $(L, \leqslant)$, in contradiction with 2:10:11. This contraction ends the proof of 2:10 Theorem $\Leftarrow$.

2:11. Theorem. *The tree rectangle hypothesis TRH implies* 0:6 *for every chain* $(L, \leqslant)$.

Proof. In the opposite case there would be an infinite chain $(L, \leqslant)$ such that $d:=d(L, \leqslant)=c^+$, where $c:=\text{cel}(L, \leqslant)=k\,\omega_\alpha$. Let $\mathfrak{D}$ and E have the same meaning as in 2:10:11. Every subchain of $(E, \supset)$ should be $\leqslant\aleph_\alpha$, therefore $\gamma\mathfrak{D}=\omega_{\alpha+1}$. Now, one has not $\gamma\mathfrak{D}<\omega_{\alpha+1}$ because the set M of all end points of members of E should be of power $\aleph_\alpha$; since M is everywhere dence in $(L, \leqslant)$, one would have $d(L, \leqslant)\leqslant\aleph_\alpha=c$, contradicting the assumption $d=c^+$.

Again $\gamma\mathfrak{D}=\omega_{\alpha+1}$ does not hold either, because according to the TRH one has $k\mathfrak{D}\leqslant k(\mathfrak{D})\ h(\mathfrak{D})=\aleph_\alpha$ (because $k_c(\mathfrak{D})\leqslant\aleph_\alpha$, $k_{c'}(\mathfrak{D})\leqslant\aleph_\alpha$) contradicting the relations $\gamma\mathfrak{D}=\omega_{\alpha+1}$, $k\mathfrak{D}\geqslant k\gamma\mathfrak{D}=\aleph_{\alpha+1}$. Q. E. D.

As a synthesis of theorems 0:5, 2:8 and some of our previous results we have the following

3:1. Main theorem. *The following statements are pairwise equivalent*:

TA — *(Tree alternative). For every ordinal* α *any tree of power* $\aleph_{\alpha+1}$ *is equinumerous to a subchain or to a subantichain* (cf. Đ. Kurepa [1935:2, 3], p.109 Th. 2).

TRH — *Tree rectangle hypothesis (or tree chain × antichain hypothesis): Every tree* T *satisfies* $kT\leqslant k_cT\cdot k_{c'}T$.

$(k=b)$ — *For every infinite tree* T *one has* $kT=bT$.

$(k_{c'}=s)$ — *Every infinite tree* T *satisfies* $k_{c'}T=sT$[1] (v. 3.3 Th. in Đ. Kurepa [1963:3]).

(w) — *For every infinite chain* $(L, \leqslant)$ *every tree* $T\subset(w(L, \leqslant), \leqslant\cdot)$ *of cardinality* $(cel(L, \leqslant))^+$ *satisfies* $kT=bT$.

$(d=c)$ — *Every totally ordered infinite set* $(L, \leqslant)$ *satisfies* 0:6.

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[1] The star number of a graph (G, R) is defined as $sG:=\inf kF$, F running through the system of all families of chains of (G, R) such that $\bigcup F=G$ (v. Đ. Kurepa, [1963:3], № 1.1).

(s_1) *For every family F of sets one has*

$$kF = k_c F k_{c'} F \cdot s_1 F,$$

where F denotes the graph (F, v), $x \cap y : = x \cap y = v$;

$s_1 F : = \sup_x s F(\cdot, x]$, $F(\cdot, x] : = \{y \therefore y \supset x,\ y \in F\}$.

(cf. Đ. Kurepa [1963 : 3] p. 34 Th. 4.3.4).

Proof. The equivalence of the statements TA, TRH, $k = b$ is obvious (cf. also 2 : 11 Th). Further, $TRH \Leftrightarrow k_{c'} = s$ (v. 3.3 Th. p. 30 in Đ. Kurepa [1963 : 3]; $TRH \Leftrightarrow (s_1)$ (v. 4.3.4 Th. p. 34 in Đ. Kurepa [1963 : 3]); $(k = b) \Leftrightarrow (w)$ (v. 2 : 10 Th); $(w) \Leftrightarrow (d = c)$ (s. 0 : 5 Th). Each of the statements: TA, TRH, $k = b$, $k_{c'} = s$, (w), $(d = c)$, (s_1) having been involved at least once in an equivalence, the proof of the Main Theorem is finished.

3 : 2. *Another version of the* Main Theorem. *In the wording of the* 3 : 1 *Theorem it is legitimate to replace everywhere the word tree by the word pseudo-tree.*

4. Denotations

kX = the cardinal number of X; if n is a cardinal then $kn : = n$.

A pseudotree or ramified set is any ordered set $(E, \leqslant)$ in which no member x has two incomparable ancestors $a, b < x$; in other words, for every $x \in E$ the set $E(\cdot, x) : = \{y \therefore y \leqslant x,\ y \in E\}$ is a chain (v. Đ. Kurepa [1935 : 2, 3] pp. 69, 127).

S-un (S being any set or any class) : = any procedure f by which to every member x of S corresponds an object fx (fx may be a number, point, set, structure, ...); in particular, 2-un : = ordered pair, 3-un : = ordered triplet, n-un (for any number n) : = ordered n-tuple = n-sequence. One says: f is an S-un.

$t(X, \leqslant)$ = order-type $(X, \leqslant)$.

v = the vacuous or the empty set.

$\omega_{(n)}$ (n being a given set or cardinal number) is the first ordinal number of cardinality kn.

$\therefore$ means „such that".

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Laze Simića 9
11000 Beograd, Jugoslavija