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ON THE CONNECTION OF RECURRENCE OF METRIC, EIGEN TENSORS AND TRANSITION OF LENGTH IN $W-O_n$ SPACES

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Introduction

The base of the theory of regular general connection was laid in [1] from T. Otsuki. That theory was jointed by one from us with theory of Weyl spaces in [2]. The spaces get on that way is called Weyl-Otsuki ($W-O_n$) spaces and it was proved in [3] that the length of vector V^i resp. V_i is proportional to the Otsuki's differential if either the metric tensor g_{ij} or vector V^i resp. V_i is an eigen vector. This holds in the case if in place of the vector V^i a symmetric tensor T^{ij}, or a skew-symmetric tensor S^{ij} stands too. By this considerations was supposed that the fundamental tensor

$$P_{ij}=P_i^t g_{tj}$$

of the $W-O_n$ space is symmetric, what in followings we will suppose always.

In followings we change some of the above conditions and results, and we ask that from which two of conditions of them give one of the others:

$$\nabla_k g_{ij}=\gamma_k g_{ij}, \tag{1}$$

$$P_i^r P_j^s g_{rs}=\tau g_{ij}, \quad \tau=\tau(x), \quad x^i=x^i(t), \tag{2}$$

$$P_i^r(x) P_j^s(x) g_{rs}(x)=\tau(x) g_{ij}(x), \tag{2'}$$

$$P_i^r V^i=\tau V^r, \quad \tau=\tau(x), \quad x^i=x^i(t), \tag{3}$$

$$\frac{DV}{dt}=\frac{1}{2}\varphi_k(x)\frac{dx^k}{dt}V, \quad V:=\sqrt{g_{ij}V^iV^j}, \quad \frac{DV^i}{dt}=0. \tag{4}$$

In [3] it was proved that from (1), (3) or (1), (2) it follows (4).

Our fundamental results are the following:

If we suppose that (3) and (4) holds, then (1) follows in the case

$$P_j^i=\tau\delta_j^i$$

where τ is an eigen value of vector V^i, i.e. in a general W_n[1]).

1) General W_n means a W_n in which the differential D of tensors of type (p, q) is multiplied with τ^{p+q}.

We prove also that from (2) and (4) it follows

(5) $$\frac{Dg_{ij}}{dt}=\left(\frac{d\tau}{dt}+\varphi_k\frac{dx^k}{dt}\right)g_{ij}.$$

If g_{ij} is an eigen tensor field, i.e. if it holds (2') then from (2') and (4) it follows (1).

On a more or less general way one can transport this theory for vectors $\vec{V}$ in the set of symmetric resp. skew-symmetric tensors T^{ij} resp. S^{ij}. The most important results are expressed in theorems 3—6.

In this paper we shall denote the differential operator D/dt frequently by D, resp. the differential operator d/dt by d.

§ 1. Preliminaries. The basic formulae

It is known that Otsuki's regular general connection Γ consists of two different parts. These are the covariant part $'\Gamma$ and the contravariant part $''\Gamma$. The covariant differential is given for example for a tensor of type (1.1) by

(1.1) $$DV_j^i:=\nabla_k V_j^i\,dx^k\equiv P_a^i P_j^b \overline{D} V_b^a$$

where

(1.2) $$\nabla_k V_j^i:=P_a^i P_j^b V_{b/k}^a;\qquad V_{b/k}^a:=\partial_k V_b^a+{}'\Gamma_{sk}^a V_b^s-{}''\Gamma_{bk}^s V_s^a$$

and

(1.3) $$\overline{D} V_b^a=V_{b/k}^a\,dx^k$$

(see [2] § 1.). A very usefull relation is

(1.4) $$\overline{D}\delta_b^a=({}'\Gamma_{bk}^a-{}''\Gamma_{bk}^a)\,dx^k.$$

For a scalar T it holds

(1.5) $$DT=\overline{D}T=dT.$$

For the complete theory we relate to the fundamental work of T. Otsuki [1].

§ 2. The case of vectors

Le be V^i a vector, the length V of which we define by

(2.1) $$V^2:=g_{ij}V^iV^j.$$

According to (1.5) using (2.1), the symmetry of tensor g_{ij}, (1.2), (1.3) and (1.4) we get

(2.2) $$\frac{DV^2}{dt}\equiv\frac{dV^2}{dt}=\frac{\overline{D}g_{ij}}{dt}V^iV^j-2g_{ij}\frac{\overline{D}\delta_s^i}{dt}V^sV^j+2g_{ij}\frac{\overline{D}V^i}{et}V^j.$$

([3], (3.4)).

Now we suppose that V^i is an eigen vector, i.e. it holds (3) and from it according to

$$P^i_j Q^j_s = \delta^i_s$$

it follows that $\tau^{-1} V^j = V^i Q^j_i$. From (1.1) it follows that for a vector V^i

$$\overline{D}V^j = Q^j_i DV^i. \tag{2.3}$$

Applying the above two relations on (2.2) according to

$$g_{ij} Q^j_a = g_{aj} Q^j_i \tag{2.4}$$

following from the symmetry of P_{ij} we get

$$\frac{DV^2}{dt} = \frac{Dg_{ab}}{dt} \tau^{-2} V^a V^b - 2 g_{ia} \tau^{-2} V^b V^i \frac{D\delta^a_b}{dt} + 2 g_{ia} \frac{DV^a}{dt} V^i \tau^{-1}, \tag{2.5}$$

where we applied the relation analogous to (2.3) on $\overline{D} g_{ij}$ and $\overline{D}\delta^i_j$. If we have a parallel displacement of an eigen vector $V^i (DV^i = 0)$, then it holds

$$\frac{D\delta^m_s}{dt} V^s = \tau \frac{d\tau}{dt} V^m$$

(see [1], (5.8)). Applying it on (2.5) we get

$$\frac{DV^2}{dt} = \left(\frac{Dg_{ab}}{dt} + 2 g_{ab} \frac{d\tau}{dt}\right) \tau^{-1} V^a V^b. \tag{2.6}$$

Now we suppose that (4) holds i.e.

$$\frac{DV^2}{dt} \equiv 2V \frac{DV}{dt} = \varphi_k(x) \frac{dx^k}{dt} V^2. \tag{2.7}$$

Substituting (2.1) and (2.7) in (2.6) according to (1.1) we get

$$\left\{(\nabla_k g_{ab}) \tau^{-2} \frac{dx^k}{dt} - 2 g_{ab} \frac{d\tau}{dt} \tau^{-1} - \varphi_k g_{ab} \frac{dx^k}{dt}\right\} V^a V^b = 0. \tag{2.8}$$

If here V^a are arbitrary, from (3) it follows that

$$P^i_j V^j = \tau \delta^i_j V^j,$$

or

$$P^i_j = \tau \delta^i_j.$$

In this case from (2.7) we have

$$\nabla_k g_{ab} \frac{dx^k}{dt} = 2 g_{ab} \frac{d\tau}{dt} \tau + \varphi_k g_{ab} \frac{dx^k}{dt} \tau^2.$$

If V^i is an arbitrary eigen tensor field, i.e. $P^i_j(x)V^j(x) = \tau(x)V^i(x)$ then according to $\frac{d\tau}{dt} = \partial_k\tau\frac{dx^k}{dt}$ and from the arbitrarity of $\frac{dx^k}{dt}$ we have

$$\nabla_k g_{ab} = \gamma_k g_{ab}, \quad \gamma_k := \partial_k\tau^2 + \varphi_k\tau^2.$$

Thus it follows

Theorem 1. *If V^i is an arbitrary eigen vector field with eigen value $\tau(x)$, and if DV/dt is proportional to V along all curves $C: x^i = x^i(t)$ and all eigen vectors V^i, then the metric tensor g_{ij} is recurrent iff $P^i_j = \tau\delta^i_j$.*

It is known that if $P^i_j = \tau\delta^i_j$ holds, then $W-O_n$ reduces on a general W_n (see introduction).

The Theorem 1 expresses that from the assumptions (3) and (4) follows the relation (1) of paragraph 1.

Now we suppose that the metric tensor g_{ij} and its differential Dg_{ij} are eigen tensors with the same eigen value τ, i.e. it holds (2) and

$$P^a_i P^b_j Dg_{ab} = \tau Dg_{ij}. \tag{2.9}$$

In the case of parallel displacement of vector V^i, from (2.2) (2.1) and (2.7) which is equivalent to (4), applying the relations analogous to (2.3) on $\overline{D} g_{ij}$ and $\overline{D}\delta^i_j$ it follows that

$$\left(\frac{Dg_{ab}}{dt} Q^a_i Q^b_j - 2 g_{s(j} Q^b_{i)} \frac{D\delta^a_b}{dt} Q^s_a\right) V^i V^j = \varphi_k \frac{dx^k}{dt} g_{ij} V^i V^j.$$

Now V^i being arbitrary, so is

$$\frac{Dg_{ab}}{dt} Q^a_i Q^b_j - 2 g_{s(j} Q^b_{i)} \frac{D\delta^a_b}{dt} Q^s_a - \varphi_k \frac{dx^k}{dt} g_{ij} = 0. \tag{2.10}$$

In [2] it was proved, that if a tensor g_{ij} is a symmetric eigen tensor with eigen value τ along a curve C, if tensor P_{ij} is symmetric and if our condition (2.9) is satisfied, then it holds

$$\frac{D\delta^a_b}{dt} g_{as} + \frac{D\delta^a_s}{dt} g_{ab} = g_{bs}\frac{d\tau}{dt} \tag{2.11}$$

([2], Satz 5, relation (4.4)). Using the relations (2.11) and

$$g_{ab} Q^a_i Q^q_j = \tau^{-1} g_{ij} \tag{2.12}$$

following from the fact that Dg_{ij} and g_{ij} are eigen tensors, from (2.10) we get

$$\frac{Dg_{ab}}{dt} Q^a_i Q^b_j - \tau^{-1} g_{ij}\frac{d\tau}{dt} - \varphi_k\frac{dx^k}{dt} g_{ij} = 0.$$

After a contraction with P_a^i, P_b^j using (2) we get

$$\frac{Dg_{ij}}{dt}=\left(\frac{d\tau}{dt}+\tau\varphi_k\frac{dx^k}{dt}\right)g_{ij}. \tag{2.13}$$

This relation is not equivalent to (1) of § 1, because (2.13) holds only along a given curve $C: x^i=x^i(t)$. In the following we suppose that the metric tensor g_{ij} is an eigen tensor not only along a curve C, but it is an eigen tensor field, i.e. it holds (2') of the introduction and $\frac{d\tau}{dt}=\partial_k\tau\frac{dx^k}{dt}$ where $\frac{dx^k}{dt}$ are arbitrary. We also suppose in place of (2.9) the stronger condition:

$$P_i^a P_j^b \nabla_k g_{ab}=\tau \nabla_k g_{ij}. \tag{2.14}$$

It is obvious that from (2.14) it follows (2.9), but the *inverse holds only in the case if in* (2.9) $\frac{dx^k}{dt}$ *can be arbitrarily choosen.*

Since — according to suppositions — (2') of introduction, (2.14) and the arbitrarity of dx^k/dt hold, accordinng to (1.1) from (2.13) we get the relation (1) with

$$\gamma_k:=\partial_k\tau+\varphi_k\tau. \tag{2.15}$$

Thus it holds:

Theorem 2. *If the metric tensor* g_{ij} *is an eigen tensor field, i. e. it is satisfied* (2') $\nabla_k g_{ij}$ *satisfies* (2.14) *and for all vectors* V^i *it holds* (4) *along all curves* $x^i(t)$, *then the metric tensor* g_{ij} *is recurrent, i.e. it holds the relation* (1) *with* (2.15).

§ 3. The case of symmetric tensors

In this paragraph we use relation (2) of § 1, and for symmetric tensors T^{ij} analogous with (3) and (4) of the same paragraph the conditions

$$P_i^r P_j^s T^{ij}=\tau T^{rs}, \quad \tau=\tau(x), \quad x^i=x^i(t), \tag{3.1}$$

$$DT=(\gamma_k dx^k-d\tau)\tau^{-1}T, \quad T:=g_{ij}T^{ij}, \quad DT^{ij}=0, \tag{3.2}$$

where

$$T^{ij}=T^{ji}, \quad g_{ij}P_m^i=g_{im}P_j^i. \tag{3.3}$$

In [3] one from us proved that from (3.3), (1) and (3.1) and from (3.3), (1) and (2) it follows the relation (3.2) if $DT^{ij}=0$ ([3] Satz 4 and Satz 7).

Theorem 3. *From* (3.3), (3.1) *and* (3.2) *if* $\frac{dx^i}{dt}$ *is arbitrary and* $DT^{ii}=0$, *it holds*:

$$(\nabla_k g_{rs}-\gamma_k g_{rs})T^{rs}=0. \tag{3.4}$$

Proof: From the second part of (3.2) according to (1.5) we have

$$\frac{DT}{dt}=\frac{\overline{D}g_{ij}}{dt}T^{ij}+g_{ij}\frac{\overline{D}T^{ij}}{dt}-g_{ij}\left(\frac{\overline{D}\delta_r^i}{dt}T^{rj}+\frac{\overline{D}\delta_r^j}{dt}T^{ri}\right)$$

along the curve $C: x^i=x^i(t)$. Applying the analogous of (2.3) on $\overline{D}T^{ij}$ and $\overline{D}\delta_j^i$ and the analogous of (1.3) on $\overline{D}g_{ij}$ we get

$$DT=Q_i^r Q_j^s(\nabla_k g_{rs})T^{ij}dx^k+g_{ij}\{Q_r^i Q_s^j DT^{rs} -Q_s^i Q_r^l(D\delta_l^s)T^{rj}-Q_s^j Q_r^l(D\delta_l^s)T^{ir}\}. \tag{3.5}$$

Using (3.1) and $DT^{rs}=0$, according to (2.4) we transforme (3.5) in

$$DT=\tau^{-1}(\nabla_k g_{rs})dx^k T^{rs}-g_{is}Q_j^i(D\delta_l^s)T^{rj}Q_r^l-g_{sj}Q_i^j Q_r^l(D\delta_l^s)T^{ir}.$$

Using (3.1) again we get

$$DT=\tau^{-1}\{(\nabla_k g_{rs})dx^k T^{rs}-2g_{is}T^{il}D\delta_l^s\}. \tag{3.6}$$

Applying D/dt on the condition (3.1) it follows the relation

$$g_{ij}\frac{DT^{ij}}{dt}+2g_{rl}T^{rs}\frac{D\delta_s^l}{dt}=\tau g_{ij}Q_a^i Q_b^j\frac{DT^{ab}}{dt}+\frac{d\tau}{dt}T \tag{3.7}$$

(for the complete proof see (4.5) — (4.11) of [3] since (3.1) is equivalent to (4.5) of [3]). Substituting $DT^{ij}=0$ in (3.7) we get

$$2g_{ij}T^{is}\frac{D\delta_s^j}{dt}=T\frac{d\tau}{dt}. \tag{3.8}$$

According to (3.8) from (3.6) we get

$$\frac{DT}{dt}=\tau^{-1}\{(\nabla_k g_{rs})T^{rs}-T\partial_k\tau\}dx^k. \tag{3.9}$$

Substituting DT from (3.2) in (3.9) it follows

$$(\nabla_k g_{rs})dx^k T^{rs}=\gamma_k g_{rs}dx^k T^{rs}$$

and according to the arbitrarity of $\frac{dx^k}{dt}$ we get (3.4) and the proof is finished.

From (3.4) do not follows the validity of the relation (1) of introduction, since T^{rs} is not arbitrarily choosen. But if T^{rs} is in (3.4) arbitrary then (3.1) is independent of T^{rs}, i.e.

$$(P_r^i P_s^j-\tau\delta_r^i\delta_s^j)T^{rs}=0 \tag{3.10}$$

holds for all symmetric tensors T^{rs}. Hence we have

Theorem 4 *If* $P^i_j = \sqrt{\tau}\,\delta^i_j$, *and* (3.1) — (3.3) *hold along all curves* $x^i(t)$ *and for all simmetric tensors* T^{rs}, *then it follows the recurrency* g_{ij}, *i.e. the relation* (1) *holds.*

Proof. From the condition for P^i_j it follows that (1.1) holds for all symmetric tensors T^{ij}, and so is T^{rs} in (3.4) arbitrary. Connsequevently from (3.4) one has the relation (1) Q. E. D.

In the case, if instead of the relation (3.1) it hold the eigen-property of the metric tensor g_{ij} and of Dg_{ij}, then it holds the following

Theorem 5. *If* (2), (2.9) *and* (3.2) *hold along all curves* $C: x^i = x^i(t)$ *and for all symmetric tensors* T^{ij}, *then it follows* (1).

Proof: According to the assumption that (2.9) holds it follows that (2.11) holds too. Using the contraction by $Q^i_r Q^j_s$ and (2) from (2.11) it follows

$$g_{mj} Q^j_s Q^i_r \frac{D\delta^m_i}{dt} + g_{mi} Q^i_r Q^j_s \frac{D\delta^m_j}{dt} = \tau^{-1} \frac{d\tau}{dt} g_{rs}. \tag{3.14}$$

Applying the contraction by T^{rs} according to (2.4) from (3.14) we get

$$g_{ab}\{Q^b_m Q^e_r (D\delta^m_e) T^{ar} + Q^a_m Q^e_s (D\delta^m_e) T^{bs}\} = \tau^{-1} T\, d\tau. \tag{3.15}$$

Using (2), (3.15) and the symmetry of tensors g_{ij} and T^{ij} from (3.5) we get

$$DT = Q^r_i Q^s_j (\nabla_k g_{rs})\, dx^k T^{ij} + g_{rs}\tau^{-1} DT^{rs} - \tau^{-1} T d\tau.$$

Applying the condition that $DT^{rs} = 0$, according to (3.2) we get

$$Q^r_i Q^s_j (\nabla_k g_{rs})\, dx^k T^{ij} = \gamma_k \tau^{-1} g_{ij} T^{ij} dx^k.$$

Since dx^k and $T^{ij} = T^{ji}$ are arbitrary it follows that

$$Q^r_i Q^s_j \nabla_k g_{rs} - \gamma_k \tau^{-1} g_{ij} = 0. \tag{3.16}$$

Contracting the equation (3.16) with $P^i_a P^j_b$ according to (2) we obtain (1) in the form

$$\nabla_k g_{ab} - \gamma_k g_{ab} = 0,$$

and so the theorem is proved.

§ 4. The case of skew-symmetric tensors

In this paragraph we suppose that the metric tensor g_{ij} and Dg_{ij} are eigen tensors with the same eigen value $\tau(x)$, i.e. it hold (2) and (2.9). Now we observe the skew-symmetric tensors $S^{ij} \equiv -S^{ji}$. Let be

$$g_{ijhk} := \frac{1}{2}(g_{ih} g_{jk} - g_{ik} g_{jh}) \tag{4.1}$$

a tensor skew-symmetric in i, j and h, k and

$$S^2 := g_{ijhk} S^{ij} S^{hk} \equiv g_{ih} g_{jk} S^{ij} S^{hk} \tag{4.2}$$

a skalar like in the foregoing observations. S represents the "length" of the tensor S^{ij}.

Applying (1.5) on (4.2) it follows

$$\frac{DS^2}{dt}=\frac{d}{dt}(g_{ir}\,g_{js}\,S^{ij}\,S^{rs}). \tag{4.3}$$

From (4.3) applying (1.1) on Dg_{ij} and according to the skew-symmetry of tensor S^{ij} and the parallel displacement of them by a suitable procedure (for the complete proof see [3], (5.2) — (5.3)) we get

$$\frac{DS^2}{dt}=2\,\frac{\overline{D}\,g_{ir}}{dt}\,g_{as}\,S^{ia}\,S^{rs}-4\,S_{ip}\,Q_k^b\,Q_c^i\,\frac{D\,\delta_b^c}{dt}\,S^{pk}. \tag{4.4}$$

Using the consequences (2.12) resp. (2.11) of (2) resp. (2.9) — which after the assumption hold — on the second term of the right side of (4.4), we have

$$\frac{DS^2}{dt}=2\left(\frac{\overline{D}\,g_{ab}}{dt}-\tau^{-1}\,g_{ab}\,\frac{d\tau}{dt}\right)g_{ts}\,S^{at}\,S^{bs}. \tag{4.5}$$

In followings we suppose that DS/dt is proportional to S if $DS^{ij}/dt=0$, or in a more useful form

$$\frac{DS^2}{dt}=2\,\varphi_k\,\frac{dx^k}{dt}\,S^2,\quad \frac{DS^{ij}}{dt}=0. \tag{4.6}$$

Substituting it in (4.5) we get

$$\left(\frac{\overline{D}\,g_{ij}}{dt}\,g_{ts}-\tau^{-1}\,g_{ij}\,g_{ts}\,\frac{d\tau}{dt}-\varphi_k\,\frac{dx^k}{dt}\,g_{ij}\,g_{ts}\right)S^{it}\,S^{js}=0.$$

According to the skew-symmetry and the arbitrarity of tensor S^{ij} it is equivalent to

$$\left\{\left(\frac{\overline{D}\,g_{ij}}{dt}\,g_{ts}-i/t\right)-j/s\right\}-\tau^{-1}\,\frac{d\tau}{dt}\,\{g_{ij}\,g_{ts}-i/t)-j/s\}$$
$$-\varphi_k\,\frac{dx^k}{dt}\,\{(g_{ij}\,g_{ts}-i/t)-j/s\}=0. \tag{4.7}$$

where i/t means the foregoing expression in them indices i and t are changed. For the differential $\overline{D}$ applied on purely co-resp. contravariant tensors it holds the Leibniz formula and so is

$$\overline{D}\,(g_{ij}\,g_{ts})=(\overline{D}\,g_{ij})\,g_{ts}+g_{ij}\,(\overline{D}\,g_{ts}).$$

According to it from (4.7) we get

$$\frac{\overline{D}}{dt}\,(g_{ij}\,g_{ts}-g_{is}\,g_{jt})=2\,(g_{ij}\,g_{ts}-g_{is}\,g_{jt})\left(\tau^{-1}\,\frac{d\tau}{dt}+\varphi_k\,\frac{dx^k}{dt}\right).$$

Contracting by P_a^i, P_b^j, P_t^r, P_v^s and using the fact that g_{ij} is an eigen tensor we get

$$\frac{D}{dt}(g_{ab}\,g_{rv}-g_{av}\,g_{br})=2\,\tau^2\,(g_{ab}\,g_{rv}-g_{av}\,g_{br})\left(\tau^{-1}\frac{d\,\tau}{dt}+\varphi_k\frac{dx^k}{dt}\right),$$

or

$$\frac{D\,g_{arbv}}{dt}=\left(\frac{d\,\tau^2}{dt}+2\,\tau^2\,\varphi_k\,\frac{dx^k}{dt}\right)g_{arbv}.$$

For the following observation it is necessary to suppose that the metric tensor g_{ij} be not only eigen tensor, but an eigen tensor field satisfying (2'). Now using the formula (1.1) according to the condition that it holds (2') and that dx^k/dt is arbitrary we get

(4.8) $$\nabla_k\,g_{arbv}=\gamma_k\,g_{arbv}$$

where

$$\gamma_k=\partial_k\,\tau^2+2\,\gamma_k\,\tau^2.$$

T h e o r e m 6. *If in a* $W-O_n$ *space metric tensor* g_{ij} *and* Dg_{ij} *are eigen tensors along all curves* $x^i(t)$ *with the same eigen value* $\tau(x)$, *and if* (4.6) *is satisfied for all skew-symmetric tensors* S^{ij}, *then is* g_{abij} *defined by* (4.1) *recurrent, i.e. it holds* (4.8).

We prove still the following two corollaries:

C o r o l l a r y 1. *If in a Weyl space* (4.1), (4.6) *and* (4.8) *are satisfied, then the space reduces on a Riemannian space.*

P r o o f: If we suppose, that the space is a Weyl space, i.e. (1) and $P_j^i=\delta_j^i$ hold, then from (4.8) and (4.1) it follows $\gamma_k\,g_{avrs}=0$ i.e. $\gamma_k=0$, and so according to (1) it is valid $\nabla_k\,g_{ab}=0$.

C o r o l l a r y 2. *If* g_{abrs} *is recurrent, the metric tensor* g_{ij} *is an eigen tensor and the dimension of space is* $n>2$, *then is metric tensor* g_{ij} *also recurrent, or* $\nabla_k\,g_{ij}=0$.

P r o o f: From (4.8) on account of eigen-property of g_{ij} it follows

$$\tau\,(g_{rs}\,\nabla_k\,g_{ij}+g_{ij}\,\nabla_k\,g_{rs}-g_{rj}\,\nabla_k\,g_{is}-g_{is}\,\nabla_k\,g_{rj})$$
$$=\gamma_k\,(g_{ij}\,g_{rs}-g_{is}\,g_{rj}).$$

After a contraction with g^{rs} one has

$$(n-2)\,\nabla_k\,g_{ij}=(\tau^{-1}\,\gamma_k\,(n-1)-g^{rs}\,\nabla_k\,g_{rs})\,g_{ij}$$

and this proves the corollary.

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