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# ON $[m, n]$-RAMIfICATIONS OR $[m, n]$-PSEUDO-TREES OF SETS*[1]

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**0.** *Ramified families* of sets or *pseudo-trees* of sets were defined in Đ. Kurepa [1934 d] p. 112 as any family $S$ of sets such that

(0:1) $$X, Y \in S \Rightarrow X \subset Y \vee X \supset Y \vee X \cap Y = \varnothing.$$

Obviously, the last implication could be equivalently written as

(0:2) $$X, Y \in S \Rightarrow X \subset Y \vee X \supset Y \vee |X \cap Y| < 1.$$

It is natural to generalize the statement (0:2) and to consider, for any cardinal number $n$, the following condition

(0:3) $$X, Y \in S \Rightarrow X \subset Y \vee X \supset Y \vee |X \cap Y| < n.$$

**0:4.** $[m, n]$-*ramifications*. Let $(m, n)$ be any odered pair of cardinal numbers $\neq 0$; a system $S$ of sets is said to be $[m, n]$-ramified or $[m, n]$-pseudotree provided

(0:5) $$(X, Y) \in S^2 \Rightarrow X \subset_m Y \vee Y \subset_m X \vee X|_n Y, \text{ where}$$

(0:6) $$X \subset_m Y : \equiv |X \setminus Y| < m.$$

(0:6′) $$X|_n Y : \equiv |X \cap Y| < n.$$

Consequently, $A \subset_1 B$; means $A \subset B$; also $A|_1 B$ means $A \cap B = \varnothing$.

**0:7.** *Relation* $[m, n]$. For any oriented pair $(m, n)$ of cardinal numbers $>0$ and any oriented pair $(A, B)$ of sets let $A[m, n]B : = A \subset B \vee B \subset_m A \vee A | \vee B$; one says that $A[m, n]$-*ramifies* $B$.

**0:8.** *Graph* $(F, [m. n])$. If $F$ is any family of sets, then we have the graph $(F, [m, n])$, the set of vertices is $F$, and the binary relation $[m, n]$ holds or does not hold for a given $(A, B) \in F^2$. In this way, $[m, n]$-ramifications coincide with complete ramified graphs $(F, [m, n])$ (of course, in general, the graph $(F, [m, n])$ is not a $[m, n]$-ramification because in general case the relation $A[m, n]B$ need not hold for $(A, B) \in F^2$.

**0:9.** *m-chains of sets. n-disjoint set systems.*

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* Presented 1978:08:09:3 at The Colloquium on Topology (Budapest, 1978:08:07:1—08:12:5).

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Analogously, we have the graphs $(F, |_n)$, $(F, \subset_m)$, $(F, \subset_m)$, $(F, \subset_m \vee \supset_m)$, etc. If $(A, B) \in F^2 \Rightarrow A \subset_m B \vee B \subset_m A$, one says that $F$ is an *m-chain of sets* (with respect to $\subset_m$ or $\supset_m$). If $(A, B) \in F^2 \Rightarrow A |_r B$, one says that $F$ is an *m-disjoint set system* or an *m-antichain of sets*. E. g. for a set $S$ the set $(P(S), \subset)$ is a $|S|^+$-chain as well a $|S|^+$-antichain. $[m, n]$-ramifications are natural generalizations of ramifications (i.e. of [1, 1] ramifications); and so for every statement concerning pseudo-trees [trees] one has to examine the corresponding version for $[m, n]$-ramifications ($[m, n]$-trees).

So e. g. we have the following

**0.10.** Lemma. If $(F, [m, n])$ is any $(m, n)$-ramification of decreasing sets, then for every $X \in F$ such that $|X| \geqslant n$ the corresponding principal ideal i.e. the set of all $Y \in F$ such that $Y_m \supset X$ is an $m$-chain. (for the case $[m, n] = [1, 1]$ v. Kurepa **[1935]** condition $C$ pp. 69, 81).

**0:11.** *Convention. Terminology.* The prefix [1, 1] could be deleted; also:

$[m, n]$-ramified $\equiv m$-comparable; [1. 0]-ramified $\equiv$ 1-comparable $\equiv$ comparable,

$[0, n]$-ramified $\equiv n$-disjoint;

[0, 1]-ramified $\equiv$ 1-disjoint $\equiv$ disjoint;

[1, 1]-ramified = ramified (v. Đ. Kurepa **[1934 d]** p. 112).

**0:12.** *Almost disjointness.* In the list of these relations let us remind the relation "*to be almost disjoint*" introduced by W. Sierpiński **[1928]** (v. **[1975]** 719—722).

A set $A$ is *almost disjoint* from the set $B$, symbolically $A\ [a \cdot d]\ B \Leftrightarrow |A \cap B| < |A|, |B|$.

**0:13.** *Almost comparability.* A set $A$ is *almost comparable* $[a \cdot c]$ or *ac* to a set $B$, symbolically $A\ [ac]\ B \Leftrightarrow$ one of the numbers $|A \setminus B|$, $|B \setminus A|$ is $< |A|, |B|$; i.e.

Aac $B \Leftrightarrow \neg |A \setminus B|, |B \setminus A| \geqslant |A|, |B|$.

**0:14.** *Almost ramification.*

0:14:1. Definition. A family $F$ of sets is said to be *almost ramified* $[a \cdot r.]$ or an *almost pseudotree* if any pair of its members are almost disjoint or almost comparable. In particular, $F$ is any *almost chain* [*almost antichain*] if every pair of its distinct members are a. comparable [a. disjoint].

0:14:2. *Almost inclusion tree* is any almost ramified system of sets in which every almost chain $L$ has an own member $x \in L$ such that $x$ is almost contained in every member of $L$.

0:14:3. A set $A$ is *almost contained* in a set $B$, symbolically

$$A \underset{a}{\subset} B \Leftrightarrow |B \setminus A| < |A|.$$

One says also that $B$ almost contans $A$ and one writes $B \underset{a}{\supset} A$.

0:14:4. Dually, one defines almost dual inclusion trees.

Consequently, for any family $F$ of sets we have also the graphs $(F, ac)$ $(F, ad)$, $(F, ar)$, and the following cardinal numbers:

**0:15.** For every family $F$ of sets we have following numbers;

(0:16) $ad\ F := \sup\{|A| \therefore A \subset F,\ A$ is almost disjoint$\}$

(0:17) $ac\ F := \sup\{|L| \therefore L \subset F,\ L$ is almost chain$\}$[1]

(0:18) $ar\ F := \sup\{|R| \therefore R \subset F,\ R$ is almost ramified$\}$.

0:19. It matters to explore especially the case when $F = P(B)$ when $F = G(V)$, $V$ being a topological space and $G(V)$ being the system of all open subsets of the space.

**0:20.** *On some kinds of extremal numbers.*

For a given structure $(E, \rho)$ one has various substructures $(A, P)$; for any kind $P$ of such structures we define the family $(P)$ of all substructures of $(E, \rho)$ of the given kind $P$; further we define following numbers:

(0:21) $i(P) := \inf |X|\ (X \in (P))$

(0:22) $S(P) := \sup |X|\ (X \in (P))$

as well as the following *exhaustion numbers*:

(0:23) $ie(P) := \inf_F |F|$ where $F \subset (P)$, $\cup F = E$ (inf exhaustion number)

(0:24) Se $(P) := \sup\{|F|$ where $F \subset (P)$, $\cup F = E$ (Sup exhaustion number).

E. g. if $P$ means "to be a subchain" then the corresponding inf-exhaustion number was called the star number of $(E, \leqslant)$.

**0:25.** We are interested in particular for the $[m, n]$-subramifications of a given graph $(G, \rho)$ and corresponding numbers

$$\text{ie}[m, n],\ \text{Se}[m, n],\ \sup |X|,\ X \in (P),\ \inf_{x \in (P)} |X|.$$

**0:26.** *Generalizations.* For ann $3-un$ ($\equiv$ oriented triplet) $(m_1, m_2, n)$ of cardinal numbers and any $2-un\,(A, B)$ of sets one could define the relation

$$A\,[m_1, m_2, n]\,B :\equiv A \subset_{m_1} B \vee B \subset_{m_2} A \vee A\,|_n\,B.$$

This case would be again a special case of the following notion:

For any $2 \times k$ matrix $\begin{bmatrix} m_1, \ldots, m_k \\ n_1, \ldots, n_k \end{bmatrix}$ of positive cardinal numbers and any $1+k-$ un $[X_0, X_1, \ldots, X_k]$ of sets one considers the relation

$$X_0 X_1 \cdots X_k \begin{bmatrix} m_1, \ldots, m_k \\ n_1, \ldots, n_k \end{bmatrix} :\equiv (X_0 \subset_{m_1} X_1 \subset_{m_2} X_2 \subset_{m_3} \cdots \subset_{mk} X_k) \vee$$

$$(X_0\,|_{n_1}\,X_1 \vee X_1\,|_{m_1}\,X_2 \vee \cdots X_{k-1}\,|_{nk}\,X_k).$$

Again one has the corresponding $1+k-$ary graph $(F, [\ \ ])$ for any family $F$ of sets.

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[1] $\therefore$ = such that.

**0:27.** *Generalization to ordered sets.* If $(E, \leqslant)$ is any ordered set then the mapping $x \in E \to [a, \cdot)_{(E, \leq)} := \{x \cdot \cdot a \leqslant x \in E\}$ is an isomorphism between $(E, \leqslant)$ and $(\{[a, \cdot)_{(E, \leq)} \cdot \cdot a \in E\}, \supset)$. By convention we transfer terminology of ordered sets $(F \supset)$ to any ordered [quasiordered] sets $(E, \leqslant)$[1]. So we have $[m, n]$-ramifications, $m$-quasichains, $n$-quasiantichains, $n$-disjoint ordered sets, ... not only of the form $(F, \supset)$ but also the ones of the form $(E, \leqslant)$.

**1.** *Binary relation* $\dot{-}_n$ Instead of $|A \dot{-} B| < n$ let us write $A \dot{-}_n B$. In this way for any system $S$ of sets and any cardinal number $n$ we have the symmetrical graph $(S, \dot{-}_n)$. One sets $A \dot{-} B := (A \setminus B) \cup (B \setminus A)$.

**1:1.** Theorem. *If $S$ is any infinite set and $n \in N$, then every complete graph $(G, \dot{-}_{\aleph_0})$, for which $G \subset PS$ has $\leqslant |S|$ members.*

Proof. Let $g \in G \subset P(S)$; then $g \subset S$; since $g \dot{-}_{\aleph_0} y'$ for every $y' \in G$, one has necessarily

(1:2) $y' = (g \setminus X) \cup Y$, where $X$, $Y$ are subsets of $S$, each of cardinality $< \aleph_0$, i.e. $X, Y \in \binom{S}{<\aleph_0} := \{X \cdot \cdot X \subset S, |X| < \aleph_0\}$. Since the last set is of cardinality $|S| + |S|^2 + \cdots = |S|$, the system of all sets (1:2) is $|S|$, $Q \cdot E \cdot D$.

1:1 is a particular case of the following

**1:3.** Theorem. *Let $S$ be any set and $n$ any cardinal number $\leqslant |S|$; if $G \subset P(S)$ and the graph $(G, \dot{-}_n)$ is complete, then $|G| \leqslant \sum_{k<n} S^k$.*

**1:4.** Remark. The theorem 1:1. is to be compared to the corresponding statement concerning complete graphs $(G, |_{\aleph_0})$, $G \subset P(S)$; in this case $|G|$ might be $> |S|$; so is e.g. if $cf|S| = \aleph_0$.

**2.** *Binary relation* $|_n$ *among sets.*

2:1. For any cardinal $n > 0$ and any $2-un(A, B)$ of sets let $A |_n B := |A \cap B| < n$. In particular, $A |_1 B$ means that $A$, $B$ are disjoint.

For any system $S$ of sets one has so the corresponding binary graph $(S, |_n)$.

**2:2.** Remark. Analogously, for any cardinal number $j$ and any system $F$ of $j$ sets one could define the relations $(F)$ to mean $|\cap F| < j$.

**2:3.** Lemma. For any ordinal number $\nu$, any $n \in N$ and any $G \subset P(\omega_\nu)$, one has

(2:4) The graph $(G, |_n)$ is complete $\Rightarrow |G| \leqslant \aleph_\nu$; i.e. if $\{X, Y\}_{\neq} \subset G \Rightarrow Y |_n Y$ then $|G| \leqslant \aleph_\nu$.

Proof. The implication (2:4) is obvious for $n = 1$. Let us assume, by induction argument, that (2:4) holds for every $m < r$, $r$ being a fixed number $< \omega_0$; we are going to prove that (2:4) holds also for $m = r$. In opposite case, there would exist a complete graph $(G, |_r)$ such that

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[1] In my Abstracts p. 46 of Budapest Conference the occuring of the sets $(\cdot, x]_E$ should be replaced by the sets $[x, \cdot)_{(E, \leq)}$.

(i) $G \subset P(\omega_\nu)$, $|G| = \aleph_{\nu+1}$. For every $\beta < \omega_\nu$ let $A_\beta$ be the set of all members of $G$, each containing $\beta$ as a member. Obviously, $A = \bigcup_\beta A_\beta \, (\beta < \omega_\alpha)$ and $|A| \leqslant \Sigma |A_\beta| \, (\beta < \omega_\nu)$. Since by hypothesis $|A| = \aleph_{\nu+1}$ there would be some $\beta < \omega_\nu$ such that $|A(\beta)| = |A| = \aleph_{\nu+1}$. Now, if we set $B := \{X \setminus \{\beta\} \cdots X \in A_\beta\}$, then of course

(ii) $|B| = |A| = \aleph_{\nu+1}$; but $(B, |_{r-1})$ is a complete graph such that $B \subset P(\omega_\nu)$; by assumption hypothesis, $|B| \leqslant \aleph_\nu$, contradicting (ii) that was implied by (i); therefore (i) does not hold.

**3.** *[m, n]-trees.*

3:1. Definition. *A* $[m, n]$-pseudotree $T$ of sets is said to be a *decreasing* $[m, n]$*-tree* of sets if every subset $X$ of $T$ contains a *complete first level* $R_0 X$, i.e. a subset $R_0 X$ so that for every $x \in X$ there exists some $x_0 \in R_0 X$ such that $x_0 \supset_m x_n$.

3:2. By induction procedure like for trees, (v. Kurepa [1935] p. 74) one defines all levels $R_\beta T$ and the hight or rank $\gamma T$ of $T$ as the first ordinal $\nu$ such tha $R_\nu T = R_{\nu+1} T$.

**3:3.** Theorem. *If* $(m, n) \in N_0^2$, *then any decreasing* $[m, n]$*-subtree* $(T, \supset)$ *of* $P(\omega_\nu)$ *is* $\leqslant \aleph_\nu$ $(N_0 := \{0, 1, 2, \ldots\})$.

Proof. We have the standard decomposition

$$T = \bigcup_\alpha R_\alpha T, (\alpha < \gamma T);$$

in virtue of 2:1 Th. every level $R_\alpha T$ is $\leqslant \aleph_\nu$; we claim that the hight $\gamma(T)$ is $< \omega_{\nu+1}$. For every $\beta < \omega_\nu$ let $T(\alpha) := \{X \cdots x \in X \in T\}$. Obviously

$$T = \bigcup_\alpha T(\alpha) \, (\alpha < \omega_\nu) \quad \text{and} \quad |T| \leqslant \sum_\alpha |T(\alpha)| \quad (\alpha < \omega_\nu).$$

We claim that $\gamma T(\alpha) < \omega_{\nu+1}$. Assume $\gamma T(\alpha) \geqslant \omega_{\nu+1}$ for some $\alpha < \omega_\nu$. Assume $\gamma T((\alpha) = \omega_{\nu+1}$ and let

(3:4) $A_\beta \in R_\beta T(\alpha) \, (\beta < \omega_{\nu+1})$; then

$$A_\xi \supset_m A_\eta \text{ for every } \xi < \eta < \omega_{\nu+1}.$$

The sets $A_\beta \setminus A_{\beta+1} \, (\beta < \omega_{\nu+1})$ being disjoint subsets of $W(\omega_\nu)$, there is some $\delta < \omega_{\nu+1}$ such that $A_\beta \setminus A_{\beta+1} = \varnothing \, (\delta < \beta < \omega_{\nu+1})$. Hence for every $\beta > \delta$ the set $A_\beta$ differs of $A_\delta$ by a finite subset of $W(\omega_\nu)$; therefore the system of all $A_\beta \, (\beta < \omega_{\nu+1})$ is $\leqslant \aleph_\nu$, contrarily to hypothesis that the sets (3:4) are pairwise distinct. Thus $\gamma T(\alpha) < \omega_{\nu+1}$ and consequently, $|T(\alpha) \leqslant \aleph_\nu$. ||

**3:5.** Corollary. *If* $n \in N$ *every n-well-ordered subset of* $(P \omega_\nu), \supset)$, *is* $\leqslant \aleph_\nu$; *where we have the following*

**3:6.** *Definition of n-well-ordered systems of sets.* A family $(F, \supset)$ of sets is said to be *n-well-ordered*, if every non empty subfamily $E$ has a *n-least member*, i.e. a member $X_0 \in E$ such that $X_0 \supset_n X$ for every $X \in E$.

**4.** Theorems. We are going to indicate two theorems showing a great difference betwen finite sets and transfinite sets.

**4:1.** Theorem. *If $S$ is any finite set, the system $P(S)$ is $|S|$-disjoint and almost ramified.*

Proof. Let $X$, $Y$ be any distinct subsets of $S$; then $|X\cap Y|<|S|$; in opposite case, the set $Z:=X\cap Y$ would contain $\geqslant|S|$ points. Since $X\neq Y$, there would be a point $u\in(X\setminus Y)\cup(Y\setminus X)$, thus $u\notin Z$; therefore, the subset $Z\cup\{u\}$ would contain $\geqslant|S|+1>|S|$ members-absurdity. Thus $P(S)$ is $|S|$-disjoint. $P(S)$ is also almost ramified because if $X$, $Y\subset S$ are not almost disjoint, one of the numbers $|X|$, $|Y|$ would be $|Z|$, say $|X|=|X\cap Y|$, thus $X\subset Y$, thus $X\setminus Y=\varnothing$.

**4:2.** Theorem. *If $S$ is any transfinite set, the system $P(S)$ is neither $[|S|, |S|]$-ramified nor is the union of less then $|P(S)|$ of $[|S|, |S|]$-ramifications. $P(S)$ is the union of the system of $2^{|S|}$ $[|S|, |S|]$-ramifications. Symbolically, ie $[n, n]P(n)=2\cdot(\mathrm{v}\cdot 0:23)$.*

At first we have the following.

**4:2.** Lemma. *If $S$ is any infinite set, then $P(S)$ contains a system $V$ of cardinality $2^{|S|}$ and has the property that*

(4:3:1) $X$, $Y\in V$, $X\neq Y\Rightarrow|X\setminus Y|=|Y\setminus X|=|X\cap Y|=|S|:=s$; *in other words, the ordered set $(P(S)\supset)$ is equinumerous to an antialmost ramified subsystem.*

Proof of the 4:3 $L$. Since $|S|:=s$ is infinite, thus $s=2s$, there is a decomposition of $S$ in 2 disjoint subsets, say $A$, $B$, of cardinality $s$ each:

$$S=A\cup B,\ A\cap B=\varnothing,\ |A|=|B|=s.$$

Since $s=2s^2$ there is a disjoint decomposition $e$ of $A$:

$$a\in A\rightarrow e(a):=f(a)\cup g(a)\subset_{\rightarrow} A \text{ such that } |f(a)|=s=|g(a)|,$$

$$f(a)\cap g(a)=\varnothing=e(a)\cap e(b) \text{ for any } b\in A\setminus\{a\} \text{ and } \bigcup_{a\in A} e(a)=A.$$

For any $X\subset A$ set

$$h(X):=B\cup\bigcup_x f(x)\cup\bigcup_y g(y),\ (x\in\bigcup_{a\in X} f(a),\ y\in A\setminus\bigcup_{x\in X} f(x)).$$

The system $V:=\{h(X)\ldots X\in A\}$ has all the three requested propert'es (4:3:1).

As a matter of fact, for every distinct $X$, $Y\subset A$ we have $h(X)==h(Y)=B$; further, $X\neq Y$ implies that some $z\in X\dot{-}Y$ ($\equiv$ symmetr c difference of $X$, $Y$), thus $z\in(X\setminus Y)\vee z\in Y\setminus X$. If $z\in X\setminus Y$ then $f(z)\subset h(X), f(z)\cap\cap h(Y)=\varnothing$, thus $h(X)\supset h(Y)\supset f(z)$, hence $|h(X)\setminus h(Y)|=s$ Analogously, $z\in Y\setminus X$ implies $|h(Y)\setminus h(X)|=s$.

4:3:2. Remark. My primitive proof of 4:3 Lemma was completely different; St. Todorčević called my attention to Kuratowski [1966] p. 435 Teor. 3; v. also Kuratowski [1933], p. 218 hint.[4)]

4:3:3. Proof of 4:2 Theorem. Let

(4:3:4) $P(S)=\cup_i T_i\,(i\in I)$, $T_i$ being an $[s, s]$-ramificat'on $\subset P(S)$; we claim that the index set $I$ satisf.es $|I|\geqslant 2^s$. The decomposition (4:3:4) implies the corresponding decomposition of the system $V$ occuring in 5:3 L:

$$V=V\cap P(S)=\cup_{i\in I} V\cap T_i. \text{ Thus}$$

(4:3:5) $|V| \leqslant \Sigma_{i \in I} |V \cap T_i|$;

for no $i \in I$ the set $V \cap T_i$ contains 2 or more distinct members $X$, $Y$ because of (4:3:1) as members of $V$ and because of $X, Y \in T_i \Rightarrow s \notin \{|X \setminus Y|, |Y \setminus X|, |X \cap Y|\}$ for every $i \in I$. In other words, (4:3:5) implies

$$|V| \leqslant \sum_{i \in I} 1 = |I|. \ ||$$

Finally, if $X \in PS$, then the singleton $\{X\}$ is obviously a subchain of $(P(S), \subset)$, the partition $P(S) = \bigcup_X \{X\} (X \in P(S))$ shows that $P(S)$ is the union of a system of cardinality $|P(S)|$ of $[n, n]$-ramifications $\subset (P(S), \subset)$.

**4:4.** Theorem. *If $n$ is any transfinite cardinal, the set system $P(n)$ is not almost ramified. $P(n)$ is the union of an almost antichain $A$ of cardinality* $2^{(n+)}$ *of sets of power $n$ each; thus iear* $P(n) = 1$.

The proof of the first part of the theorem 4:4 is analogous to the proof of 4:2 Theorem. The second part of the 4:4 theorem is the theorem 17 in Tarski [1928].

**4:5.** In connexion with 4:2, 4:4 theorems let us remark that the $[n, n]$-ramifications [almost pseudotrees] $X$ involved might be maximal. On the other hand every maximal $[|n|, |n|]$-pseudotree $R \subset P(n)$ has $\geqslant 2^{\underline{n}} := \sum_{k<n} 2^k$ members. As a matter of fact, if $X \subset S$ and $|X| < n$, then $P(X)$ is an $[n, n]$-ramified (even an $n$-chain and an $n$-antichain); the union $P(x) \cup R$ is also $[n, n]$-ramified; the maximality of $R$ impliee $P(X) \cup R = R$; therefore $|P(X)| \leqslant R$ i.e. $2^{|X|} \leqslant |R|$ and a fortiori $2^n \leqslant |R|$ because $|X|$ may run through all cardinals $< n$.

Further let us remind that $(P(n), \subset)$ contains chains $> n$ (v. W. Sierpiński [1922], [1977] p. 109—112) as well as almost pseudotrees $> n$ (v. Sierpiński [1928], [1977]). Hence when we are dealing with a decomposition $P(S) = \bigcup_{i \in I} T_i$ in $[s, s]$-ramifications [almost pseudotrees] $T_i$, $T_i$ might be quite great parts of $P(S)$; nevertheless, the cardinal number of $I$ is $\geqslant 2^s$. It is every useful to compare this situation of exhausting $P(n)$ by means of $[n, n]$-ramifications [almost ramifications] with the situation when one is exhausting the unit interval by means of sets of measure 0 or with sets of the Baire's category 1. In our present case the statement is decidable (assuming the choice axiom), while either of the last 2 statements is indecidable.

**5.** *Statements $r(n)$ and $ar$ $(n)$ concerning any cardinal number $n$:*

$r(n)$ $P(n)$ is equinumerous to a $[n, n]$-ramification $\subset p(n)$

$ar(n)$ $P(n)$ is equinumeros to an almost ramification $\subset p(n)$.

Further results and extensio will appear elsewhere.

## BIBLIOGRAPHY

Kuratowski, K. [1933] *Topology I,* Monogr. Mat. 3 (Warszawa-Lwow 1933) $X + 285$.

Kuratowski, K. [1966] *Topology* Vol. I, Warszawa 1966 XX+560 (Russian: Moskva 1966, 594 pp.

Kurepa, Đuro, [1934 d] *Tableaux ramifiés d'ensembles,* Comptes rendus, 199, (1934), 112—114

Kurepa, Đuro, [1935] *Ensembles ordonnés et ramifiés,* Thèse, Paris, 1935, Publ. Math. Belgrade, 4 (1935) 1—138.

Kurepa, Đuro, [1939 b] *Sur la puissance des ensembles partiellement ordonnés,* Comptes rendus, Soc. Sci. Varsovie, classes Math. 32, 1939, 61—67, Re-impressed in: Glasnik Mat. — fiz,, Zagreb, 14, 1959, 205—211.

Kurepa, Đuro, *On A-trees,* Publ. Inst. Math. 8 (22) (Beograd 1968) 153—161.

Kurepa, Đuro, [1977 a] *On some hypotheses concerning trees,* Publ. Math, Inst. Math. Beograd, 21 (35) (1977) 99—108.

Kurepa, Đuro, [1977 b] *Ramified sets or pseudotrees,* Publ. Inst. Math. Beograd 22 (36) (1977), 149—163.

Miller, Edwin W., [1943] *A note on Souslin problem.* Amer J. of Math. 65 (1943), 673—678.

Sierpiński, W., [1922] *Sur un problème concernant les sous-ensembles croissants du continu,* Fund. Math. 3 (Warszawa 1922) 109—112. ≡ Oeuvres choisies II (Warszawa 1975) 444—447.

Sierpiński, W., [1928] *Snr une décomposition d'ensembles,* Monatshefte f. Math. u. Phys. 35 (Wien 1928) 239—243; also Oeuvres choisies II, Warszawa 1975 pp. 719—722.

Sierpiński, W., [1937] *Sur les suites transfinies finalement disjointes,* Fundamenta Math. 28 (Warszawa 1937) 115- 119; also Oeuvres choisies III pp. 343—347.

Tarski, Alfred, [1928] *Sur la décomposition des ensembles en sous-ensembles presque disjoints,* Fundamenta Math. 12 (Warszawa 1928) 188—205.

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GEORGES KUREPA, Ensembles ordonnés et ramifieés. Thèse de doctorat, № 2475 Série A № 1609 Paris 1935, pp. 6 + 138 + 2 & *Publications mathématiques de l'Université de Belgrade,* Vol. 4 (Beograd, 1935) pp. 1—138.

| Page | Errata | Corrige |
| --- | --- | --- |
| Title | ordonnées | ordonnés |
| $3^9$ | dans | comme une portion initiale, dans |
| $4_7$ | 9,1908 | 6,1905 |
| $5_{15}$ | Second | Troisième |
| $14_2$ | existance | existence |
| $15_{16}$ | toute | tout |
| $16^3$ | avant | ayant |
| $18_3$ | et suiv. | $-190$ |
| $29^5$ | 2 | 3 |
| $30_{10}$ | continu dans | contenu dans |
| $32^7$ | une | |
| $36_8$ | $AB$ | $G \neq A$ $G \neq B$ |
| $56^6$ | $(1+\lambda)$ | $[1+\lambda)$ |
| $72_{15}$ | $\leqslant$ | $<$ |
| $74_7$ | $+1$ | $+2$ |
| $76_{13}$ | 5 | 10 |
| $102^{12}$ | démont | démontre |
| $106_3$ | $\omega_1$ | $<\omega_1$ |
| $109_8$ | ou | , $a<b$ ou |
| $109_7$ | · | , $a'<b'$ · |
| $110^7$ | $A$ | non vides $A$ |
| $113^3$ | pas | par |
| $113^{10}$ | $8\cdot 2$ | $8\cdot 3\,c$ |
| $118_{11}$ | $\subseteq$ | $\supseteq$ |
| $120_{10}$ | , | , $S$ |
| $128^5$ | $oS$ | $pS$ |
| $130^{17}$ | tableaux | tableau |
| $133^1$ | $P_8$ $P_9$ | $P_8 \rightarrow P_9$ |
| $133_{10}$ | $P_1 \rightarrow (Q)$ | $(Q) \rightarrow P_1$ |
| $134_4$ | inclusion | „être une partie initiale de“ |