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0. Introduction

Boolean algebra is called rigid if it has no one non-trivial automorphims.

Already Birkhof [2] arose the question of existence of rigid Boolean algebra. Katetov [5] was first to construct rigid Boolean algebra of the power 2^{ω}. To answer de Groot's and McDowell's [4] question Lozier [6] constructed rigid Boolean algebra of the power $2^{\varkappa}$, for every cardinal $\varkappa \geqslant \omega$. McKenzie and Monk [7] constructed rigid Boolean algebra of power λ, for every strong limit cardinal $\lambda > \omega$. de Groot [3] shows that there exist exactly $2^{2^{\omega}}$ isomorphism types of rigid Boolean algebras of power 2^{ω}, while McKenzie and Monk [7] show that there exist exactly $2^{2^{\varkappa}}$ isomorphism types of rigid Boolean algebras of power $2^{\varkappa}$, where $\varkappa$ is such a regular cardinal that $2^{\lambda} \leqslant \varkappa$, for every $\lambda < \varkappa$.

In this paper we shall show that for every cardinal $\varkappa > \omega$ there exist exactly $2^{\varkappa}$ isomorphism types of rigid Boolean algebras of power $\varkappa$ (there is no rigid Boolean algebra of power 1, 3, 4, 5, ..., ω). This completes the discussion of the given problem (especially, by this we get the answers to problem 8 and 9 of [7]). Besides, we can positively answer on problem 6 of the same paper. At the end we give one answer on problem 7 of the same paper.

1. The Main theorems

Theorem 1.1. *For every cardinal $\varkappa > \omega$ there exist exactly $2^{\varkappa}$ isomorphism types of rigid Boolean algebras of power $\varkappa$.*

Proof: Let $\varkappa$ be a regular uncountable cardinal and let it keep this property until we especially change its properties. Let $S \subseteq \varkappa$ be an arbitrary stationary subset and let $f: S \to \varkappa$ be an arbitrary mapping. Let $S_1 = \{\alpha \in S \mid f(\alpha) < \alpha\}$. If S_1 is stationary, then according to known Lemma about regressive function we know that there exist stationary $S_2 \subseteq S_1$ and $\beta < \varkappa$, so that

$f''(S_2)=\{\beta\}$. Let us assume now that S_1 is not stationary. Then, the set $S_3=S-S_1$ is stationary and for every $\alpha\in S_3$ $f(\alpha)\geqslant\alpha$ holds. Let

$$C=\{\alpha<\varkappa \mid \lim(\alpha)\wedge(\beta\in S_3\wedge\beta<\alpha\rightarrow f(\beta)<\alpha)\}.$$

It is clear that C is a closed and unbounded subset of $\varkappa$ and so $C\cap S_2$ is stationary. We can easily verify that $f(\alpha)<f(\beta)$, for every $\alpha, \beta\in C\cap S_2$, $\alpha<\beta$. So, for an arbitrary stationary $S\subseteq\varkappa$ and arbitrary mapping $f: S\rightarrow\varkappa$ there exist either stationary $S_1\subseteq S$ and $\beta<\varkappa$, so that $f''(S_1)=(\beta\}$, or there exists stationary $S_2\subseteq S$, such that $f\wedge S_2$ is $1-1$ mapping and $f(\alpha)\geqslant\alpha$ for every $\alpha\in S_2$. We shall use this fact later on.

Let $D=\{\alpha<\varkappa \mid \lim(\alpha)\wedge cf(\alpha)=\omega\}$. For every $\alpha\in D$ we fix in advance a strictly increasing continuous function $f_\alpha:\omega+1\rightarrow\varkappa$, such that $f_\alpha(0)=0$ and $f_\alpha(\omega)=\alpha$.

For every subset $S\subseteq D$ by $E(S)$ we denote the set $\{f_\alpha \mid \alpha\in S\}$, which we always consider to be ordered by relation $\prec$ of lexicographical order. The following fact will be useful:

(*) If $(E(S'), \prec)$ is order-isomorphic to a subset of $(E(S''), \prec)$, then $S'-S''$ is non-stationary subset of $\varkappa$ (for the proof see [1, Theorem 5.3. (i)]).

Let $S\subseteq D$ be stationary, then omitting nonstationary subset of S we can get a $S'\subseteq S$ with the property that $\{\gamma\in S' \mid f_\alpha\prec f_\gamma\prec f_\beta\}$ is stationary, for every $\alpha, \beta\in S$, $f_\alpha\prec f_\beta$ (see [1, proof of the Corollary 5.6.]). This fact can be expressed by words that every non-trivial interval in $E(S)$ is stationary.

Let $S\subseteq D$ be stationary and let $E(S)$ have all intervals stationary. By $B(S)$ we denote Boolean algebra of all finite unions of intervals from $E(S)$ of the form $[x, y)$, $x, y\in E(S)\cup\{-\infty, +\infty\}$. So, $|B(S)|=|E(S)|=\varkappa$.

Let $S', S''\subseteq D$ be stationary and let $E(S')$ and $E(S'')$ have minimal elements and all non-trivial intervals stationary. Let us assume that there exists a strictly increasing mapping H: $B(S')\rightarrow B(S'')$. Let us then prove that $S=S'-S''$ is not stationary.

Let us assume the contrary — that $S\subseteq\varkappa$ be stationary. For every $\alpha\in S$ we put $b_\alpha=[0, f_\alpha)\in B(S')$. So, for $f_\alpha\prec f_\beta$. $\alpha, \beta\in S$ we have that $H(b_\alpha)\subset H(b_\beta)$. Since $H(b_\alpha)\in B(S'')$, there exist unique decomposition

$$H(b_\alpha)=\cup\{[x_\alpha^i, y_\alpha^i) \mid i<n(\alpha)\},$$

where $n(\alpha)\in\omega$, x_α^i, y_α^i $E(S'')\cup\{+\infty\}$ and $x_\alpha^i\prec y_\alpha^i\prec x_\alpha^{i+1}$, for every $i<n(\alpha)-1$. Since S is stationary, there exists stationary $T\subseteq S$ and $n<\omega$, so that $n(\alpha)=n$, for every $\alpha\in T$. Without loss of generality we can assume that $T=S$, namely, that $n(\alpha)=n$, for every $\alpha\in S$. Mapping $h: S\rightarrow\varkappa$ is defined by $h(\alpha)=\beta$, if $f_\beta=x_\alpha{}^0$, $\alpha\in S$, $\beta\in S''$. According to the above there exist either stationary $S_1\subseteq S$ and $\beta_0<\varkappa$ so that $h''(S_1)=\{\beta_0\}$ or there exists stationary $S_1'\subseteq S$ such that $h\restriction S$ is $1-1$ mapping and $h(\alpha)>\alpha$ for every $\alpha\in S_1'$ (because of the fact that $\mathrm{dom}(h)\cap\cap\mathrm{rang}(h)=\varnothing$). Let us show that the second case cannot happen. Let us assume the contrary, i. e., that there exists such $S_1'\subseteq S$. Let $\alpha, \beta\subseteq S_1'$ and $f_\alpha\prec f_\beta$. Then $x_\alpha^0\succcurlyeq x_\beta^0$, since $H(b_\alpha)\subset H(b_\beta)$. Then $x_\alpha^0\succ x_\beta^0$, i. e., $f_{h(\alpha)}\succ f_{h(\beta)}$, since $h\restriction S_1'$ is $1-1$ mapping. So, we showed that $(\{f_\alpha \mid \alpha\in S_1'\}, \prec)$ is inversely similar to the subset of $(E(S''), \prec)$. But, this cannot hold, for we can directly check that $\{f_\alpha \mid \alpha \in S_1'\}$ contains uncountable $\prec$-well ordered subset and $E(S'')$ has no uncountable $\succ$-well ordered subset. So, there exists stationary $S_1 \subseteq S$, such that $x_\alpha^0 = x_\beta^0 = x^0$, for every α, $\beta \in S_1$.

Let us consider now a mapping $l: S \to \varkappa$ defined by $l(\alpha) = \beta$, if $\alpha \in S_1$, $\beta \in S''$ and $f_\beta = y_\alpha^0$ and $l(\alpha) = 0$ if $y_\alpha^0 = +\infty$. We know that there exist, either stationary $S_2 \subseteq S_1$ and $\beta_1 < \varkappa$, so that $l''(S_2) = \{\beta_1\}$ or there exists stationary $S_2' \subseteq S_1$, such that $l \restriction S_2'$ is $1-1$ mapping and $l(\alpha) > \alpha$ for every $\alpha \in S_2'$. Let us show that the second case cannot occur. Let us assume the contrary, i.e. that there exists such S_2'. Let α, $\beta \in S_2'$ and $f_\alpha \prec f_\beta$. Then $y_\alpha^0 \preccurlyeq y_\beta^0$, since $H(b_\alpha) \subset \subset H(b_\beta)$. Then $y_\alpha^0 \prec y_\beta^0$, i.e. $f_{l(\alpha)} \prec f_{l(\beta)}$, since $l \restriction S_2'$ is $1-1$ mapping. So, we showed that $(\{f_\alpha \mid \alpha \in S_2'\}, \prec)$ and $(\{f_\beta \mid \beta \in l''(S_2')\}, \prec)$ are isomorphic. Since S_2' is stationary, then, according to (*) we conclude that $l''(S_2')$ is a stationary subset of $\varkappa$. However, this is in contradiction with the fact that $l^{-1} \restriction l''(S_2')$ is a $1-1$ regressive mapping. So, there exists stationary $S_2 \subseteq S_1$ such that $y_\alpha^0 = y_{\alpha'}^0 = y^0$ for every α, $\alpha' \in S_2$.

Repeating this procedure $2n$ times we get a stationary set $S_{2n} \subseteq S_{2n-1} \subseteq \cdots \subseteq S$, such that $x_\alpha^i = x_{\alpha'}^i = x^i$ and $y_\alpha^i = y_{\alpha'}^i = y^i$ for every $i < n$ and $\alpha, \alpha' \in S_{2n}$. It means that $H(b_\alpha) = H(b_{\alpha'})$ for every α, $\alpha' \in S_{2n}$, which is contradictory with $H: B(S') \to B(S'')$ is a strictly increasing mapping. This contradiction proves that $S' - S''$ is nonstationary.

Let $S \subseteq D$ be a stationary set such that every non-trivial interval in $E(S)$ is stationary. Let us prove that $B(S)$ is a rigid Boolean algebra. Let us assume the contrary, i.e., that there exists non-trivial automorphism $H: B(S) \to B(S)$. We can easily find b, $c \in B(S)$, $b \cap c = \varnothing$, such that $H \restriction (B(S) \restriction b)$ is isomorphism of Boolean algebras $B(S) \restriction b$ and $B(S) \restriction c$. Let $S' = \{\alpha \in S \mid f_\alpha \in b\}$, $S'' = \{\alpha \in S \mid f_\alpha \in c\}$, then by assumption S' and S'' are stationary sets and $S \cap S'' = \varnothing$. Analogously to the above we should get a contradiction. So, $B(S)$ is a rigid Boolean algebra.

According to [9] there exists a family $T_\alpha \subseteq D$, $\alpha < \varkappa$ of mutually disjoint stationary subsets. It is also known that there exists a family X_α, $\alpha < 2^\varkappa$ of subsets of $\varkappa$, such that $X_\alpha - X_\beta \neq \varnothing$ for every α, $\beta < 2^\varkappa$, $\alpha \neq \beta$. For every $\alpha < 2^\varkappa$ we put $S_\alpha = \bigcup \{T_\beta \mid \beta \in X_\alpha\}$. The family S_α, $\alpha < 2^\varkappa$ has a property that $S_\alpha - S_\beta$ is stationary for every α, $\beta < 2^\varkappa$, $\alpha \neq \beta$. Without depraving this property we can assume, omitting nonstationary subset of S_α, that every non-trivial interval in $E(S_\alpha)$ is stationary.

According to what was already proved we know that $B(S_\alpha)$, $\alpha < 2^\varkappa$, is a family of power $2^\varkappa$ of mutually nonisomorphic rigid Boolean algebras of power $\varkappa$.

Let us assume now that $\varkappa > \omega$ is a singular cardinal, i.e., that there exists strictly increasing sequence $\varkappa_\alpha$, $\alpha < \lambda = cf(\varkappa)$ of successors with supremum equal to $\varkappa$. Let $S_\alpha \subseteq D(\varkappa_\alpha)$ be stationary subsets, such that $E(S_\alpha)$, for every $\alpha < \lambda$, has all non-trivial intervals stationary. Let us assume that $E(S_\alpha)$, $\alpha < \lambda$ are disjoint and that they have minimal elements. Let $E = \bigcup \{E(S_\alpha) \mid \alpha < \lambda\}$. Let x, $y \in E$, then we put $x \prec y$ if $x \in E(S_\alpha)$, $y \in E(S_\beta)$ and $\alpha < \beta$ or x, $y \in E(S_\alpha)$ and $x \prec y$, where $\prec$ is the lexicographical order of the set $E(S_\alpha)$. We denote by $B(E)$ Boolean algebra of all finite unions of intervals from $(E, \prec)$ of the form $[x, y)$, $x, y \in E \cup \{-\infty, +\infty\}$. So $|B(E)| = |E| = \varkappa$.

Let $B(E)$ and $B(E')$ be two so obtained Boolean algebras and let $H: B(E) \to B(E')$ be isomorphism. Since $E(S_\alpha) \in B(E)$, H maps $B(E) \upharpoonright E(S_\alpha) \cong B(S_\alpha)$ onto $B(E') \upharpoonright H(E(S_\alpha))$. Since $\varkappa_\alpha$, $\alpha < \lambda$ is a strictly increasing sequence, according to the fact that $|B(E) \upharpoonright E(S_\alpha)| = \varkappa_\alpha$ it implies that $H(E(S_\alpha)) = E(S_\alpha') \in B(E')$, for every $\alpha < \lambda$. So isomorphism of Boolean algebras $B(S_\alpha)$ and $B(S_\alpha')$ is induced by H for every $\alpha < \lambda$. By similar arguments we conclude that every algebra of the form $B(E)$ is rigid. This, with already shown, proves that there exists a family of power $\Pi\{2^{\varkappa_\alpha} \mid \alpha < \lambda\} = 2^\varkappa$ of mutually nonisomorphic rigid Boolean algebras of power $\varkappa$. This finishes the proof of the theorem.

Stone space of Boolean algebra of the form $B(S)$ (resp. $B(E)$) is ordered and is obtained from Dedekind's completion of linearly ordered set $(E(S), \prec)$ (resp. $(E, \prec)$) by doubling every nonend-point from $E(S)$ (resp. E).

McKenzie and Monk [7, Problem 6] put the following question: Is there an infinite *BA* with no non-trivial one-one endomorphism?

The answer on this question gives the following theorem which we already proved.

Theorem 1.2. *Let $\varkappa$ be a regular uncountable cardinal. Then there exists a family B_α, $\alpha < 2^\varkappa$, of Boolean algebras, each of power $\varkappa$, so that $B_\alpha \neq B_\beta$, for all $\alpha < \beta < 2^\varkappa$, and every strictly increasing mapping $H: B_\alpha \to B_\beta$, $\alpha, \beta < 2^\varkappa$ must be equal to the identical mapping of the Boolean algebra B_α.*

It is now natural to put a question whether theorem 1.2. also holds for every other cardinal $\varkappa > \omega$. We can answer positively to this question under assumption that $V = L$. More precisely, of $V = L$ we only use the fact that for every successor $\varkappa > \omega$ there exists stationary $S \subseteq \varkappa$ such that $S \cap \alpha$ is nonstationary subset of α for every $\alpha < \varkappa$ and $cf(\alpha) = \omega$ for every $\alpha \in S$. The proof of this can be settled applying the same procedure as in proof of the theorem 1.1. and using the fact that every subset of $E(S)$ of power $< \varkappa$ is equal to the union of countable many of its $\prec$-well ordered subsets (see [1, Lemma 7.1]).

McKenzie and Monk [7, Problem 7] also state the following question: For which infinite cardinals $\varkappa$ do there exists BA's of power $\varkappa$ with no non--trivial onto endomorphisms?

The authors propose this problem in this form because Rieger [8] constructed Boolean algebra without non-trivial onto endomorphisms, but its power is rather large. The following theorem gives, completely enough, the answer to this question.

Theorem 1.3. *Let $\varkappa$ be a regular uncountable cardinal. Then there exists a family B_α, $\alpha < 2^\varkappa$, of Boolean algebras, each of power $\varkappa$, so that $B_\alpha \neq B_\beta$, for all $\alpha < \beta < 2^\varkappa$, and every onto homomorphism $H: B_\alpha \to B_\beta$, $\alpha < \beta < 2^\varkappa$, must be equal to the identical mapping of the Boolean algebra B_α.*

Proof: Let $\varkappa = cf(\varkappa) > \omega$, and $D = \{\alpha < \varkappa \mid cf(\alpha) = \omega\}$. Let $S \subseteq D$ be stationary subset that every non-trivial interval in $E(S)$ is stationary (see the proof of theorem 1.1.). Let us prove that Boolean algebra $B(S)$ defined above has no one non-trivial onto endomorphism. Let $H: B(S) \to B(S)$ be an arbitrary onto endomorphism. Let $b_\alpha=(\cdot, f_\alpha)\in B(S)$, for $\alpha\in S$ It is clear that $\{b_\alpha \mid \alpha\in S\}\cup\{\varnothing, E(S)\}$ is a base of the Boolean algebra $B(S)$. If H is one to one mapping on $\{b_\alpha \mid \alpha\in S\}$ then H is automorphism of Boolean algebra $B(S)$ hence, according to the proof of theorem 1.1 $H=id$. So, we can suppose that there exist $\alpha_0, \beta_0\in S, \alpha_0\neq\beta_0$, so that $H(b_{\alpha_0})=H(b_{\beta_0})$. For example, let $f_{\alpha_0}\prec f_{\beta_0}$, then by assumption $S'=\{\alpha\in S \mid f_{\alpha_0}\prec f_\alpha\prec f_{\beta_0}\}$ is stationary and $H(b_\alpha)=H(b_{\alpha_0})=H(b_{\beta_0})$, for every $\alpha\in S'$. Now, let $\sim$ be an equivalence relation on $E(S)$ defined by: $f_\alpha\sim f_\beta$ iff $H(b_\alpha)=H(b_\beta)$, $\alpha, \beta\in S$. Equivalence classes of $\sim$ are, clearly, convex subsets of $E(S)$. Let $\{f_\alpha \mid \alpha\in T\}$, $T\subseteq S$ be a set of representatives of equivalence classes and let f_{α_0} be the representative for the class $[f_{\alpha_0}](=[f_{\beta_0}])$, i. e. $\alpha_0\in T$. Hence, $S\cap T=\varnothing$. Besides. we can immediately verify that $\{H(b_\alpha) \mid \alpha\in T\}$ is a monotonous base of algebra $B(S)$ and that $H(b_\alpha)\neq H(b_\beta)$ for $\alpha, \beta\in T, \alpha\neq\beta$. Let $\alpha\in S'$ be an arbitrary ordinal. Then there exists representation

$$b_\alpha=\bigcup\{-H(x_\alpha^i)\cap H(y_\alpha^i) \mid i<n(\alpha)\} \qquad (**)$$

where $n(\alpha)\in\omega$, x_α^i, $y_\alpha^i\in\{b_\gamma \mid \gamma\in T\}\cup\{\varnothing, E(S)\}$, $i<n(\alpha)$ and $H(x_\alpha^i)\subsetneq H(y_\alpha^i)\subsetneq H(x_\alpha^{i+1})$, for $i<n(\alpha)-1$. Since S' is stationary there exists stationary $U\subseteq S'$ and $n\in\omega$, so that $n(\alpha)=n$, for every $\alpha\in U$. Let $\alpha, \beta\in U$ and $f_\alpha\prec f_\beta$. Then, according to (**), we conclude that $H(x_\alpha^0)\supseteq H(x_\beta^0)$ must hold.

Let us define $h: U\to\varkappa$ by $h(\alpha)=\beta$, $\alpha\in U$, $\beta\in T$ if $b_\beta=x_\alpha^0$ and $h(x)=0$ if $x_\alpha^0=\varnothing$ (let us remember that $b_\beta=(\cdot, f_\beta)\in B(S)$). Using previous procedure we know that there exists either stationary $U_1\subseteq U$ and $\beta_0<\varkappa$ so that $h''(U_1)=\{\beta_0\}$ or stationary $U_1'\subseteq U$ so that $h\restriction U_1'$ is one to one mapping (and $h(\alpha)>\alpha$, $\alpha\in U_1'$). Let us prove that the second case is impossible. Let us suppose the contrary, i. e. that there exists a stationary $U_1'\subseteq U$ with properties mentioned above. Let $\alpha, \beta\in U_1'$ and $f_\alpha\prec f_\beta$. Then $f_{h(\beta)}\prec f_{h(\alpha)}$, since $b_{h(\alpha)}\subseteq b_{h(\beta)}$ and $h(\alpha)\neq h(\beta)$. It means that $f_\alpha\mapsto f_{h(\alpha)}$, $\alpha\in U_1'$, is inverse isomorphism of linearly ordered sets $(\{f_\alpha \mid \alpha\in U_1'\}, \prec)$ and $(\{f_{h(\alpha)} \mid \alpha\in U_1'\}, \prec)$ what is impossible because of the explanation given in the proof of theorem 1.1.

So, there exists a stationary $U_1\subseteq U$ and $\beta_0<\varkappa$ so that $h''(U_1)=\{\beta_0\}$ which, by definition, means that $H(x_\alpha^0)=H(x_\beta^0)$, for every $\alpha, \beta\in U_1$. Let $\alpha, \beta\in U_1$ and $f_\alpha\prec f_\beta$. Then $H(y_\alpha^0)\subseteq H(y_\beta^0)$, according to the property of the set U_1 and (**). Mapping $l: U_1\to\varkappa$ is defined by $l(\alpha)=\beta$, $\alpha\in U_1$, $\beta\in T$ if $b_\beta=y_\alpha^0$ or $l(\alpha)=0$ if $y_\alpha^0=E(S)$. So, there exists either stationary $U_2\subseteq U_1$ and $\beta<\varkappa$ so that $l''(U_2)=\{\beta_1\}$ or stationary $U_1'\subseteq U_1$ so that $l\restriction U_2'$ is one to one mapping and $l(\alpha)>\alpha$ for $\alpha\in U_2'$. Let us prove that the second case is impossible. Let us assume the contrary, i.e., that $U_2'\subseteq U_1$ has mentioned properties. Let $\alpha, \beta\in U_2'$ and $f_\alpha\prec f_\beta$. Then $f_{l(\alpha)}\prec f_{l(\beta)}$, since $b_{l(\gamma)}\subseteq b_{l(\beta)}$ and $l(\alpha)\neq l(\beta)$. This means that $(\{f_\alpha \mid \alpha\in U_2'\}, \prec)$ is similar to the $(\{f_\alpha \mid \alpha\in l''(U_2')\}, \prec)$. Since U_2' is stationary, according to (*), we conclude that $l''(U_2')$ is stationary what is in contradiction with the fact that $l^{-1}\restriction l''(U_2)$ is one to one regressive mapping. This contradiction proves that there exists stationary $U_2\subseteq U_1$ and $\beta_1<\varkappa$ so that $l''(U_2)=\{\beta_1\}$. This means that $H(y_\alpha^0)=H(y_\beta^0)$, for every $\alpha, \beta\in U_2$.

Repeating this procedure $2n$ times we obtain stationary set $U_{2n}\subseteq U_{2n-1}\subseteq\cdots\subseteq U_1\subseteq S'$ such that $H(x_\alpha^i)=H(x_\beta^i)$ and $H(y_\alpha^i)=H(y_\beta^i)$ for every $i<n$ and

every α, $\beta \in U_{2n}$. According to (**) this means that $b_\alpha = b_\beta$ for every α, $\beta \in U_{2n}$ which is impossible. This contradiction finally proves that $H = id$.

Analogously, we can prove that there is no onto homomorphism $H: : B(S) \to B(S')$ if $S' - S$ is stationary. This shows that the family $B(S_\alpha)$, $\alpha < 2^\varkappa$, constructed in the proof of theorem 1.1 satisfies the condition of theorem 1.3 what finishes the proof.

The natural question arises again: Does theorem 1.3 hold for every cardinal $\varkappa > \omega$. We can positively answer to this question with assumption that for every successor $\varkappa > \omega_1$ there exists stationary set $S \subseteq \{\alpha < \varkappa \mid cf(\alpha) = \omega\}$, such that $S \cap \alpha$ is a nonstationary subset of α for every $\alpha < \varkappa$. This can be proved applying arguments similar to the above and according to the remark after theorem 1.2.

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