PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 25 (39), 1979, pp. 183—196

ON ALMOST PRODUCT AND ALMOST DECOMPOSABLE MANIFOLDS

J. P. Srivastava, S. P. Rai, S. L. Yadav

(Received March 4, 1978)

***Abstract*: In the present paper we have studied an almost product and almost decomposable manifold and obtained some results by making uses of vector bilinear functions.**

1. Introduction

Let us consider an n-dimensional manifold V_n of differentiability class C^{r+1} endowed with real vector valued function f such that for arbitrary vector fields X, Y, Z etc.

(1.1) a $$\overline{\overline{X}}=X$$

where

(1.1) b $$\overline{X}=f(X).$$

Suppose g be a given positive definite Riemannian metric of V_n such that

(1.2) $$g(\overline{X}, Y)=F(X, Y)$$

where F is a tensor of type (0,2).

A manifold V_n for which (1.1) a and

(1.3) $$g(\overline{X}, Y)=g(X, \overline{Y})$$

holds is called an almost product manifold.

In an almost product manifold we have

(1.4) a $$F(X, Y)=F(Y, X)$$

(1.4) b $$F(\overline{X}, Y)=F(X, \overline{Y})$$

(1.4) c $$\underset{X}{(Df)}(\overline{Y})=-\overline{\underset{X}{(Df)}(\overline{Y})}$$

let D be a Riemannian connection in this manifold, then

$$D_X Y - D_Y X = [X, Y],$$

$$(D_X g)(Y, Z) = 0.$$

A 1-form α in an almost product manifold is said almost decomposable if

$$\alpha((D_X f)(Y)) - (D_Y f)(X) - (D_{\bar{X}} \alpha)(Y) + (D_X \alpha)(\bar{Y}) = 0$$

A vector field V in an almost product manifold is to be almost decomposable if the Lie derivative of f with respect to V vanishes

$$(L_V f)(X) = 0$$

2. Let us consider a vector valued bilinear function $L(X, Y)$ given by

(2.1) a $$L(X, Y) = D_{\bar{X}} \bar{Y} + D_X Y + \overline{D_{\bar{Y}} X} + \overline{D_Y \bar{X}}$$

(2.1) b $$L(X, Y) = (D_{\bar{X}} f)(Y) + \overline{(D_Y f)(\bar{X})} + \overline{D_{\bar{X}} Y} + \overline{D_{\bar{Y}} X} + D_X Y + D_Y X$$

Let us define

(2.2) a $$A(X, Y) = D_X Y + D_Y \bar{X}$$

(2.2) b $$A(X, Y) = D_X Y + D_Y X + \overline{(D_Y f)(\bar{X})}$$

Theorem 2.1). *In an almost product manifold we have*

(2.3) a $$A(\bar{X}, \bar{Y}) = D_{\bar{X}} \bar{Y} + \overline{D_{\bar{Y}} X}$$

(2.3) b $$A(X, Y) + A(\bar{X}, \bar{Y}) = L(X, Y)$$

Proof. Barring X and Y in (2.2) a and using (1.1) a we have (2.3) a. Again adding (2.2) a and (2.3) a we have (2.3) b.

Let us consider a vector valued bilinear function $M(X, Y)$ given by (Mishra 1970)

(2.4) a $$M(X, Y) = D_{\bar{X}} \bar{Y} + D_X Y - \overline{D_{\bar{X}} Y} - \overline{D_X \bar{Y}}$$

(2.4) b $$M(X, Y) = (D_{\bar{X}} f)(Y) - \overline{(D_X f)(\bar{Y})}$$

Let us define

(2.5) a $$B(X, Y)=D_X Y-\overline{D_X \bar{Y}}$$

(2.5) b $$B(X, Y)=-\overline{(D_X f)(Y)}$$

then we have

$$A(X, Y)-A(Y, X)=B(X, Y)-B(Y, X)$$

Theorem (2.2). *In an almost product manifold, we have*

(2.6) a $$B(\bar{X}, \bar{Y})=D_{\bar{X}} \bar{Y}-\overline{D_{\bar{X}} Y}$$

(2.6) b $$\overline{B(X, \bar{Y})}+B(X, Y)=0$$

(2.6) c $$\overline{B(\bar{X}, Y)}+B(\bar{X}, \bar{Y})=0$$

and consequently

(2.7) $$B(X, Y)+B(\bar{X}, \bar{Y})=M(\bar{X}, \bar{Y})$$

(2.8) $$\overline{B(\bar{X}, Y)}+\overline{B(X, \bar{Y})}=-M(X, Y).$$

Proof. Barring X and Y in (2.5) a and using (1.1) a we have (2.6) a. Now. Barring Y in (2.5) a we have

$$B(X, \bar{Y})=D\bar{Y}-\overline{D Y}$$

$$\Rightarrow \overline{B(X, \bar{Y})}=\overline{D_X \bar{Y}}-D_X Y=-B(X, Y)$$

Barring X and Y in (2.6) b and using (1.1) a we have (2.6) c. From (2.5) b we have

$$B(X, Y)+B(\bar{X}, \bar{Y})=-\overline{(D_X f)(Y)}-\overline{(D_{\bar{X}} f)(Y)}=-\overline{(D_X f)(Y)}+\overline{\overline{(D_{\bar{X}} f)(Y)}}$$

$$=(D_{\bar{X}} f)(Y)-\overline{(D_X f)(Y)}=M(X, Y)$$

Adding (2.6) b and (2.6) c and using (2.7) we have (2.8).

Theorem (2.3). *In an almost product manifold we have*

(2.9) a $$\overline{L(\bar{X}, Y)}=\overline{L(X, \bar{Y})}=L(Y, X)$$

(2.9) b $$L(X, Y)=L(\bar{X}, \bar{Y}).$$

Proof. Now

$$L(X, Y) = D_{\bar X}\bar Y + D_X Y + \overline{D_{\bar Y} X} + \overline{D_Y \bar X}$$

$$\overline{L(\bar X, Y)} = \overline{D_{\bar X}\bar Y} + \overline{D_{\bar X} Y} + \overline{\overline{D_{\bar Y}\bar{\bar X}}} + \overline{\overline{D_Y X}}$$

$$= \overline{D_X \bar{\bar Y}} + \overline{D_{\bar X} Y} + D_{\bar Y}\bar X + D_Y X$$

$$= D_{\bar Y}\bar X + D_Y X + \overline{D_{\bar X} Y} + \overline{D_X \bar{\bar Y}}$$

$$= L(Y, X).$$

Similarly $L(Y, X) = \overline{L(X, \bar{\bar Y})}$.
Now

$$L(\bar X \bar Y) = D_{\bar{\bar X}}\bar{\bar Y} + D_{\bar X}\bar Y + \overline{D_{\bar Y}\bar{\bar X}} + \overline{D_{\bar Y}\bar{\bar{\bar X}}}$$

$$= D_X Y + D_{\bar X}\bar Y + \overline{D_Y \bar{\bar X}} + \overline{D_{\bar Y} X}$$

$$= L(X, Y).$$

Corollary.

$$L(X, Y) = L(\bar X, \bar Y) = \overline{L(Y, \bar X)} = \overline{L(\bar Y, X)}.$$

Theorem (2.4). *We have*

(2.10) a $$M(X, Y) = M(\bar X, \bar Y)$$

(2.10) b $$M(X, Y) = -\overline{M(\bar X, Y)}$$

(2.10) c $$M(X, Y) = -\overline{M(X, \bar Y)}.$$

Proof. We have from (2.7)

$$B(X, Y) + B(\bar X, \bar Y) = M(X, Y).$$

Barring X and Y

$$B(\bar X, \bar Y) + B(X, Y) = M(\bar X, \bar Y)$$

implies

$$M(X, Y) = M(\bar X, \bar Y).$$

Also we have from (2.8):

$$\overline{\overline{B(\bar{\bar{X}}, Y)}}+\overline{\overline{B(\bar{\bar{X}}, \bar{\bar{Y}})}}=-\overline{M(\bar{\bar{X}}, Y)}$$

$$B(X, Y)+B(\bar{\bar{X}}, \bar{\bar{Y}})=-\overline{M(\bar{\bar{X}}, Y)}$$

$$M(X, Y)=-\overline{M(\bar{\bar{X}}, Y)}.$$

Similarly $M(X, Y)=-\overline{M(X, \bar{\bar{Y}})}$.

Let us define

(2.11) a $$'L(X, Y, Z)=F(L(X, Y), \bar{Z})$$

(2.11) b $$'A(X, Y, Z)=F(A(X, Y), \bar{Z})$$

(2.11) c $$'B(X, Y, Z)=F(B(X, Y), \bar{Z})$$

(2.11) d $$'M(X, Y, Z)=F(M(X, Y), \bar{Z}).$$

T h e o r e m (2.5). *In an almost product manifold we have*

(2.12) a $$'L(\bar{X}, Y, Z)='L(X, \bar{Y}, Z)='L(Y, X, \bar{Z})$$

(2.12) b $$'L(X, Y, Z)='L(\bar{X}, \bar{Y}, Z)$$

(2.12) c $$'L(\bar{X}, Y, \bar{Z})='L(X, \bar{Y}, \bar{Z})='L(\bar{Y}, \bar{X}, Z).$$

P r o o f. From (2.9)

$$L(\bar{X}, Y)=L(X, \bar{Y})=\overline{L(Y, X)}$$

we have

$$F(L(\bar{X}, Y, \bar{Z})=F(L(\bar{X}, Y), Z)$$

$$=F(\overline{L(Y, X)}, Z)$$

$$=F(L(Y, X), Z)$$

$$'L(\bar{X}, Y, Z)='L(\bar{X}, Y, Z)='L(Y, X, \bar{Z}).$$

Similarly from (2.9) b and using (2.11) a we have (2.12) b.

Now barring Z in (2.12) a, we have

$$'L(\bar{X}, Y, \bar{Z})='L(X, \bar{Y}, \bar{Z})='L(Y, X, Z)='L(Y, X, Z)$$

from (2.12) b.

C o r o l l a r y. *We have*

$$'L(X, Y, Z)='L(\bar{X}, \bar{Y}, Z)='L(Y, \bar{X}, \bar{Z})='L(\bar{Y}, X, Z).$$

Theorem (2.6). *In an almost product manifold*

(2.13) a $$'B(X, \bar{Y}, \bar{Z}) = -'B(X, Y, Z)$$

(2.13) b $$'M(X, Y, Z) = \ 'M(\bar{X}, \bar{Y}, Z)$$

(2.13) c $$'M(X, Y, Z) = -'M(X, \bar{Y}, \bar{Z})$$

(2.13) d $$'M(X, Y, Z) = -'M(\bar{X}, Y, \bar{Z}).$$

Proof. From (2.6) b using (2.11) c we have (2.13) a.

From (2.10) a, (2.10) b and (2.10) c and using (2.11) d we have (2.13) b, (2.13) c and (2.13) d respectively.

Theorem (2.7). *In an almost product manifold we have*

(2.14) a $$'L(X, Y, Z) = \ 'A(X, Y, Z) + 'A(\bar{X}, \bar{Y}, Z)$$

(2.14) b $$'M(X, Y, Z) = \ 'B(X, Y, Z) + 'B(\bar{X}, \bar{Y}, Z)$$

(2.14) c $$'M(X, Y, Z) = -'B(\bar{X}, Y, \bar{Z}) - 'B(X, \bar{Y}, \bar{Z}).$$

Proof. (2.3) b and using (2.11) a and (2.11) b we have (2.14) a.

Again from (2.7) and (2.8) and using (2.11) c, (2.11) d we have (2.14) b and (2.14) c respectively.

3. Nizenhuis tensor

The Nizenhuis Tensor "N" in an almost product and almost decomposable manifold is defined by

(3.1) a $$N(X, Y) = [\bar{X}, \bar{Y}] + [X, Y] - \overline{[\bar{X}, Y]} - \overline{[X, \bar{Y}]}$$

(3.1) b $$N(X, Y) = (D_{\bar{X}} f)(Y) - (D_{\bar{Y}} f)(X) - \overline{(D_X f)(Y)} + \overline{(D_Y f)(X)}.$$

Theorem (3.1). *In an almost product manifold we have*

(3.2) a $$L(X, Y) - L(Y, X) = N(X, Y)$$

(3.2) b $$M(X, Y) - M(Y, X) = N(X, Y).$$

Consequently

(3.3) a $$A(X, Y) - A(Y, X) + A(\bar{X}, \bar{Y}) - A(\bar{Y}, \bar{X}) = N(X, Y)$$

(3.3) b $$B(X, Y) - B(Y, X) + B(\bar{X}, \bar{Y}) - B(\bar{Y}, \bar{X}) = N(X, Y)$$

(3.3) c $$\overline{B(\bar{Y}, \bar{X})} - \overline{B(\bar{X}, Y)} + \overline{B(Y, \bar{X})} - \overline{B(X, \bar{Y})} = N(X, Y).$$

P r o o f. Left hand side of (3.2) a

$$=(D_{\bar X} f)(Y)+\overline{(D_Y f)(\bar X)}+D_{\bar X} Y+\overline{D_{\bar Y} X}+D_X Y+D_Y X$$

$$-(D_{\bar Y} f)(X)-\overline{(D_X f)(\bar Y)}-\overline{D_{\bar Y} X}-\overline{D_{\bar X} Y}-D_Y X-D_X Y$$

$$=N(X, Y).$$

Similarly (3.2) b can be proved.

Again (3.3) a and (3.3) b follow directly from (3.2) a and (3.2) b respectively. (3.3) c can be proved by considering (2.8) and (3.2) b.

C o r o l l a r y.

$$N(X, Y)=-N(Y, X)=N(\bar X, \bar Y)=-\overline{N(\bar X, Y)}=-\overline{N(X, \bar Y)}.$$

These at once follow from (3.3) c.

T h e o r e m (3.2). *In an almost product manifold we have*

(3.4) a $$A(X, Y)-A(Y, X)+B(\bar X, \bar Y)-B(\bar Y, \bar X)=N(X, Y)$$

(3.4) b $$A(\bar X, \bar Y)-A(\bar Y, \bar X)+B(X, Y)-B(Y, X)=N(X, Y)$$

(3.4) c $$\overline{A(\bar Y, Y)}-\overline{A(X, \bar Y)}+\overline{B(Y, \bar X)}-\overline{B(\bar X, Y)}=N(X, Y)$$

(3.4) d $$\overline{A(Y, \bar X)}-\overline{A(\bar X, Y)}+\overline{B(\bar Y, X)}-\overline{B(X, \bar Y)}=N(X, Y)$$

P r o o f. Left hand side of (3.4) a

$$=D_X Y+D_Y X+\overline{(D_Y f)(\bar X)}-D_Y X-D_X Y-\overline{(D_X f)(\bar Y)}$$

$$-\overline{(D_X f)(\bar{\bar Y})}+\overline{(D_{\bar Y} f)(\bar{\bar X})}$$

$$=N(X, Y) \quad \text{using (1.4) c.}$$

Now (3.4) b, (3.4) c and (3.4) d follow from (3.4) a and

(3.5) $$N(X, Y)=N(\bar X, \bar Y)=-\overline{N(\bar X, Y)}=-\overline{N(X, \bar Y)}$$

respectively.

Let us define

$$'N(X, Y, Z)=F(N(X, Y), \bar Z).$$

Then we have the following theorem.

Theorem (3.3). *In an almost product manifold we have*

(3.6) a $$'L(X, Y, Z)-'L(Y, X, Z)='N(X, Y, Z)$$

(3.6) b $$'M(X, Y, Z)-'M(Y, X, Z)='N(X, Y, Z)$$

Consequently

(3.6) c $$'L(X, Y, Z)+'M(Y, X, Z)='L(Y, X, Z)+'M(X, Y, Z).$$

Proof is obvious.

Theorem (3.4). *We have*

(3.7) a $$'A(X, Y, Z)-'A(Y, X, Z)+'A(\bar{X}, \bar{Y}, Z)-'A(\bar{Y}, \bar{X}, Z)='N(X, Y, Z)$$

(3.7) b $$'B(X, Y, Z)-'B(Y, X, Z)+'B(\bar{X}, \bar{Y}, Z)-'B(\bar{Y}, \bar{X}, Z)='N(X, Y, Z)$$

(3.7) c $$B(\bar{Y}, X, \bar{Z})-'B(\bar{X}, Y, \bar{Z})+ B(Y, \bar{X}, \bar{Z})-'B(X, \bar{Y}, \bar{Z})='N(X, Y, Z)$$

(3.7) d $$'A(X, Y, Z)-'A(Y, X, Z)+ B(\bar{X}, \bar{Y}, Z)- B(\bar{Y}, \bar{X}, Z)='N(X, Y, Z)$$

(3.7) e $$A(\bar{X}, \bar{Y}, Z)-'A(\bar{Y}, \bar{X}, Z)+ B(X, Y, Z)-'B(Y, X, Z)='N(X, Y, Z)$$

(3.7) f $$'A(\bar{Y}, X, \bar{Z})-'A(X, \bar{Y}, \bar{Z})+'B(Y, \bar{X}, \bar{Z})-'B(\bar{X}, Y, \bar{Z})='N(X, Y, Z)$$

(3.7) g $$'A(Y, \bar{X}, \bar{Z})- A(\bar{X}, Y, \bar{Z})+'B(\bar{Y}, X, \bar{Z})- B(X, \bar{Y}, \bar{Z})='N(X, Y, Z)$$

Proof. The results (3.7) a and (3.7) b and (3.7) c follow from (3.3) a, (3.3) b and (3.3) c respectively.

Again the results (3.7) d, (3.7) e, (3.7) f and (3.7) g follow from (3.4) a, (3.4) b, (3.4) c and (3.4) d respectively.

Corollary.

$$'N(X, Y, Z)=-'N(Y, X, Z)='N(\bar{X}, \bar{Y}, Z)=-'N(\bar{X}, Y, \bar{Z})=-'N(X, \bar{Y}, \bar{Z})$$

Proof is obvious from (3.7) c.

Let us define

$$(\underset{X}{D} F)(Y, Z)=F((\underset{X}{D} f)(Y), \bar{Z})$$

then we have

$$'M(X, Y, Z)=(\underset{\bar{X}}{D} F)(Y, Z)-(\underset{X}{D} F)(Y, \bar{Z}).$$

From this we have

$$'M(X, Y, Z)='M(X, Z, Y)$$

then we have the following theorem.

Theorem (3.5). *In an almost product manifold we have*

(3.8) a $\quad 'N(X, Y, Z)+'N(Y, Z, X)+'N(Z, X, Y)=0$

(3.8) b $\quad 'L(X, Y, Z)+'L(Y, Z, X)+'L(Z, X, Y)$
$\quad ='L(X, Z, Y)+'L(Z, Y, X)+'L(Y, X, Z)$

(3.8) c $\quad 'N(X, Y, Z)-'N(X, Z, Y)='M(Z, Y, X)-'M(Y, Z, X)$

(3.8) d $\quad 'N(X, Y, Z)-'N(Y, Z, X)-'N(Z, X, Y)$
$\quad =2'M(X, Y, Z)-2'M(Y, X, Z)$

(3.8) e $\quad 'N(X, Y, Z)-'N(Z, X, Y)+'N(Z, Y, X)$
$\quad =2'M(X, Y, Z)-2'M(Y, X, Z).$

Proof. The proof of (3.8) a, (3,8) c, (3.8) d, (3.8) e follows from (3.6) b and from the fact that M is symmetric in last two slots and (3.8)c follows from (3.6) a and (3.8) a.

Theorem (3.6). *In an almost product manifold we have*

$$(\underset{V}{L} N)(X, Y)=0.$$

Where V is almost decomposable and L_V is the operation of Lie differentiation.

Proof. We have

$$N(X, Y)=[\bar{X}, \bar{Y}]+[X, Y]-\overline{[\bar{X}, Y]}-\overline{[X, \bar{Y}]}$$

implies

$$\begin{aligned}(\underset{V}{L} N)(X, Y)= & -N([V, X], Y)-N(X, [V, Y])+[V, [\bar{X}, \bar{Y}]]\\ & +[V, [X, Y]]-\overline{[V, [\bar{X}, Y]]}-\overline{[V, [X, \bar{Y}]]}\\ = & -[\overline{[V, X]}, \bar{Y}]-[[V, X], Y]+\overline{[\overline{[V, X]}, Y]}\\ & +\overline{[[V, X], Y]}-[\bar{X}, \overline{[V, Y]}]-[X, [V, Y]]\\ & +\overline{[\bar{X}, [V, Y]]}+\overline{[X, \overline{[V, Y]}]}+[V, [\bar{X}, \bar{Y}]]\\ & +[V, [X, Y]]-\overline{[V, [\bar{X}, Y]]}-\overline{[V, [X, \bar{Y}]]}.\end{aligned}$$

Using skew symmetric property of [ ] and using Jacobi identity and the fact that $[V, \bar{X}]=\overline{[V, X]}$ then we have

$$(\underset{V}{L} N)(X, Y)=0.$$

Remark. In an almost product manifold V be almost decomposable iff

$$[V, \bar{X}]=\overline{[V, X]}.$$

Proof is obvious.

4. Almost product and almost decomposable manifold

An almost product manifold in which

$$(\underset{X}{D}f)(Y)=0 \tag{4.1}$$

is satisfied is called an almost product and almost decomposable manifold, we have the following theorem.

Theorem (4.1). *The necessary and sufficient condition that an almost product manifold be an almost product and almost decomposable manifold is*

(4.2) a $\quad B(X, Y)=0$ implies $\underset{X}{D}\bar{Y}=\overline{\underset{X}{D}Y}$

(4.2) b $\quad M(X, Y)=0$

(4.2) c $\quad N(X, Y)=0$

(4.2) d $\quad L(X, Y)=L(Y, X)$ i.e. L is symmetric.

(4.2) e $\quad A(X, Y)=A(Y, X)=\underset{X}{D}Y+\underset{Y}{D}X.$

Proof. First suppose that almost product manifold V_n be almost product and almost decomposable then

$$(\underset{X}{D}f)(Y)=0$$

implies $\quad \underset{X}{D}\bar{Y}-\overline{\underset{X}{D}Y}=0$

implies $\quad \underset{X}{D}Y-\overline{\underset{X}{D}\bar{\bar{Y}}}=0$

implies $\quad B(X, Y)=0.$

Conversely suppose for an almost product manifold

$$B(X, Y)=0$$

implies $\quad \underset{X}{D}\bar{Y}=\overline{\underset{X}{D}Y}$

implies $\quad \underset{X}{D}Y-\overline{\underset{X}{D}\bar{\bar{Y}}}=0$

implies $\quad \underset{X}{D}Y-\overline{(\underset{X}{D}f)(Y)}-\overline{\overline{\underset{X}{D}Y}}=0$

implies $\quad (\underset{X}{D}f)(Y)=0$ implies $(\underset{X}{D}f)(Y)=0$

i.e. the manifold V_n be almost product and almost decomposable.

Now the proof (4.2) b follows from (4.2) a and (2.7). Again the proof (4.2) c follows from (4.2) a and (3.2) b. Also the proof (4.2) d follows from (4.2) c and (3.2) a and lastly the proof of (4.2) e follows from (4.2) a, (4.2) c and (3.4) a.

Now

$$A(X,\,Y)=D_X Y+\overline{D_Y \bar{X}}=D_X Y+\overline{\overline{D_Y X}} \quad \text{from (4.2) a}$$

$$=D_X Y+D_Y X.$$

The above theorem in case of (4.2) c has been proved by Yano (1965) by another method.

T h e o r e m (4.2). *The necessary and sufficient condition that a vector field V be almost decomposable in an almost product and almost decomposable manifold is*

(4.3) a $$D_{\bar{X}} V=D_X \bar{V}=\overline{D_X V}$$

(4.3) b $$\overline{A(\bar{X},\,V)}=A(X,\,V).$$

P r o o f. The necessary and sufficient condition that a vector field V be almost decomposable in an almost product manifold V_n is

$$(L_V f)(X)=0$$

implies $$L_V \bar{X}-\overline{L_V X}=0$$

implies $$[V,\,\bar{X}]-\overline{[V,\,X]}=0$$

implies $$D_V \bar{X}-D_{\bar{X}} V-\overline{D_V X}+\overline{D_X V}=0$$

implies $$(D_V f)(X)+\overline{D_V X}-D_{\bar{X}} V-\overline{D_V X}+\overline{D_X V}=0$$

since V_n be an almost product and almost decomposable

implies $$D_{\bar{X}} V=\overline{D_X V}=D_X \bar{V}. \quad \text{From (4.2) a.}$$

Again now

$$A(X, V) = D_X V + D_V X$$

implies $\overline{A(\overline{X}, V)} = \overline{D_{\overline{X}} V} + D_V \overline{\overline{X}} = \overline{\overline{D_X V}} + D_V \overline{\overline{X}}$. From (4.3) a

$$= D_X V + D_V X.$$

Theorem (4.3). *In an almost product and almost decomposable manifold we have*

(4.4) a $$(D_X A)(Y, Z) = K(X, Y, Z) + K(X, Z, Y) - D_{D_Z X} X - D_{D_Y X} Z$$

(4.4) b $$(D_X A)(Y, Z) + (D_Y A)(Z, X) + (D_Z A)(X, Y) + D_{A(Y,Z)} X + D_{A(Z,X)} Y + D_{A(X,Y)} Z = 0$$

(4.4) c $$(L_X A)(Y, Z) = 2K(X, Z, Y) + 2 D_Z D_Y X - 2 D_{D_Z Y} X$$

where K is the curvature tensor in V_n, given by

(4.4) d $$K(X, Y, Z) = D_X D_Y Z - D_Y D_X Z - D_{[X, Y]} Z$$

Proof. In an almost product and almost decomposable manifold we have

(4.5) $$A(Y, Z) = D_Y Z + D_Z Y.$$

Differentiating along X

$$(D_X A)(Y, Z) + A(D_X Y Z) + A(Y, D_X Z) = D_X D_Y Z + D_X D_Z Y$$

again using (4.5) and (4.4) d we have

$$(D_X A)(Y, Z) = K(X, Y, Z) + K(X, Z, Y) - D_{D_Z X} Y - D_{D_Y X} Z$$

Again from (4.5)

$$(L_X A)(Y, Z) + A([X, Y], Z) + A(Y, [X, Z])$$
$$= [X, D_Y Z] + [X, D_Z Y]$$

$$(L_X A)(Y,Z) = -D_{[X,Y]}Z - D_Z[X,Y] - D_Y[X,Z] - D_{[XZ]}Y + D_X D_Y Z - D_{D_Z Y}X + D_X D_Z Y - D_{D_Y Z}X$$

$$= -D_{[X,Y]}Z - D_Z D_X Y + D_Z D_Y X - D_Y D_X Z + D_Y D_Z X - D_{[X,Z]}Y + D_X D_Y Z - D_{D_Z Y}X + D_X D_Z Y - D_{D_Y Z}X$$

$$= K(X,Y,Z) + K(X,Z,Y) + D_Y D_Z X + D_Z D_Y X - D_{D_Z Y}X - D_{D_Y Z}X$$

$$= K(X,Y,Z) + K(X,Z,Y) + K(Y,Z,X) + 2D_Z D_Y X - 2D_{D_Y Z}X$$

$$= -K(Z,X,Y) + K(X,Z,Y) + 2D_Z D_X X - 2D_{D_Y Z}X$$

$$= 2K(X,Z,Y) + 2D_Z D_Y X - 2D_{D_Y Z}X$$

which is (4.4) c.

Again now

$$(D_X A)(Y,Z) + (D_Y A)(Z,X) + (D_Z A)(X,Y)$$

$$= K(X,Y,Z) + K(X,Z,Y) - D_{D_X Z}Y - D_{D_X Y}Z$$

$$+ K(Y,Z,X) + K(Y,X,Z) - D_{D_Y X}Z - D_{D_Y Z}X$$

$$+ K(Z,X,Y) + K(Z,Y,X) - D_{D_Z Y}X - D_{D_Z X}Y$$

$$= -D_{D_X Z}Y - D_{D_Z X}Y - D_{D_Y X}Z - D_{D_X Y}Z - D_{D_Z Y}X - D_{D_Y Z}X$$

$$= -D_{A(Z,X)}Y \quad -D_{A(X,Y)}Z \quad -D_{A(Y,Z)}X.$$

Acknowledgement. The authors are thankful to Prof. R. S. Mishra for his valuable suggestion.

REFERENCES

[1] R. S. Mishra, *On almost product and almost decomposable manifold*, Tensor N. S. Vol. 21 (1970) pp 255—260).

[2] K. Yano, *Differential Geometry on Complex and almost Complex spaces*, Pergamon Press, New York (1965).

Department of Mathematics
Benaras Hindu University
Varanasi-221005
(India)