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ON NEARLY STRONGLY PARACOMPACT AND ALMOST-STRONGLY PARACOMPACT SPACES*

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In a recent paper [6] M. K. Singal and Asha Mathur have introduced a new class of topological spaces, called nearly compact spaces. These are characterized by the property that "every regular open covering of the space admits a finite subcover" or equivalently, "every open cover admits a finite subfamily of the interiors of the closures whose members cover the space". The class of nearly compact spaces contains the class of compact spaces and is contained in the class of almost-compact spaces. In another paper [5], another class of spaces called almost-paracompact spaces has been introduced. A space is said to be almost-paracompact if for every open cover of the space there exists a locally-finite family of open sets which refines it and of the closures whose members cover the space. In a recent paper [7] M. K. Singal and S. P. Arya have introduced a new class of topological spaces, called nearly paracompact spaces. A space is said to be nearly paracompact if every regular open cover of the space admits a locally-finite open refinement, Clearly, the class of almost-paracompact spaces contains the class of nearly paracompact spaces and also the class of nearly paracompact spaces contains the class of paracompact spaces. In another paper [9] another class of spaces called strongly paracompact spaces has been introduced. A space is said to be strongly paracompact if every open cover of the space admits a star finite open refinement. A very natural question then arises can we introduce two topological properties p_1 and p_2 such that strongly paracompactness $\rightarrow p_1 \rightarrow p_2 \rightarrow$ almost-paracompactness. On examining this question it is found that there exist two properties p_1 and p_2: p_1 — every regular open cover of the space admits a star finite open refinement, p_2 — every open cover of the space admits a star finite family of open sets which refines it and of the closures whose members cover the space.

Spaces with property p_1 we call nearly strongly paracompact and spaces with property p_2 we call almost-strongly paracompact. The class of nearly strongly paracompact spaces will be isolated by showing that there exists (I) a

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space which is nearly strongly paracompact but not strongly paracompact (II) a space which is almost-strongly paracompact but not nearly strongly paracompact (III) a space which is nearly strongly paracompact but not nearly compact (IV) a space which is nearly paracompact but not nearly strongly paracompact.

Nearly strongly paracompact spaces will sometimes be abbreviated as *NSPC* spaces.

1. Definition and characterisations

Definition 1.1. A space X is said to be *nearly strongly paracompact* if every regular open cover of X admits a star finite open refinement.

Definition 1.2. A space X is said to be *almost-strongly paracompact* if every open cover of X admits a star finite family of open sets which refines it and of the closures whose members cover the space X.

Theorem 1.1. *Every strongly paracompact space is nearly strongly paracompact.*

Proof. Let X be any strongly paracompact space. Let $\mathcal{U}=\{U_\lambda:\lambda\in\Lambda\}$ be any regular open covering of X. Since X is strongly paracompact space, there exists a star finite open refinement $\mathcal{V}$ of $\mathcal{U}$. Hence $\mathcal{V}$ is star finite open refinement of $\mathcal{U}$ and thus X is nearly strongly paracompact.

Theorem 1.2. *Every regular nearly strongly paracompact space is strongly paracompact.*

Proof. Let $\mathcal{U}=\{U_\lambda: \lambda\in\Lambda\}$ be any open covering of X. Each $x\in X$ belongs to some open set U_λ. Since the space is regular, there exists a regularly open set V_x for each $x\in X$ such that $V_x\subset U_\lambda$ for some $\lambda\in\Lambda$. Now, $\{V_x: x\in X\}$ is a regular open cover of the nearly strongly paracompact space X. Therefore, there exists a star finite open refinement $\{V_\beta: \beta\in B\}$ of $\{V_x: x\in X\}$. Obviously, then $\{V_\beta: \beta\in B\}$ is a star finite open refinement of $\{U_\lambda: \lambda\in\Lambda\}$ and hence X is strongly paracompact.

Theorem 1.3 *A semi regular space X is nearly strongly paracompact iff it is strongly paracompact.*

Proof. It is similar to the proof of theorem 1.2.

Theorem 1.4. *Every nearly strongly paracompact space is nearly paracompact.*

Proof. It follows easily, since every star finite open family is a locally-finite open family.

Corollary 1.1. *Every nearly strongly paracompact space is almost-paracompact.*

Proof. Every nearly paracompact space is almost-paracompact.

Theorem 1.5. *If X is nearly strongly paracompact space, then for every open covering $\mathcal{U}$ of X there exists a star finite family of open sets which refines it and the interiors of the closures whose members cover the space.*

Proof. Let $\mathcal{U}=\{U_\lambda: \lambda\in\Lambda\}$ be any open covering of X. Then, $\{\alpha(U_\lambda): \lambda\in\Lambda\}$, where $\alpha(U_\lambda)=\overline{U}_\lambda^0$, is a regular open covering of X. Since X is nearly strongly paracompact, there exists a star finite open refinement $\{H_\beta: \beta\in I\}$ of $\{\alpha(U_\lambda): \lambda\in\Lambda\}$, such that $H_\beta\subset\alpha(U_{\lambda(\beta)})$ for some $\lambda(\beta)\in\Lambda$. For each $\beta\in I$, let $M_\beta=H_\beta\setminus[\overline{U}_{\lambda(\beta)}\setminus U_{\lambda(\beta)}]$. Since $H_\beta\subset\alpha(U_{\lambda(\beta)})\subset\overline{U}_{\lambda(\beta)}$, therefore $M_\beta=H_\beta\cap U_{\lambda(\beta)}$. Thus $\{M_\beta:\beta\in I\}$ is a star finite family of open sets which refines $\mathcal{U}$. We shall prove that $\cup\{\alpha(M_\beta):\beta\in I\}=X$. Let $x\in X$. Then $x\in H_\beta$ for some $\beta\in I$. Now $\alpha(M_\beta)=\alpha(H_\beta\cap U_{\lambda(\beta)})=\alpha(H_\beta\cap\overline{U}_{\lambda(\beta)})=\alpha(H_\beta)$. Thus $x\in\alpha(H_\beta)=\alpha(M_\beta)$. Hence $\{M_\beta:\beta\in I\}$ is a star finite family of open sets which refines $\mathcal{U}$ and the interiors of the closures whose members cover X.

Corollary 1.2. *Every nearly strongly paracompact space is almost--strongly paracompact.*

Proof. It follows easily from theorem 1.5.

Lemma 1.1. *If $\{F_\alpha: \alpha\in I\}$ is a star finite closed cover of a space X, then $\{F_\alpha^0:\alpha\in I\}$ is a star finite regularly closed cover of X.*

Proof. It follows easily from lemma 1.1 in [7].

Obviously, every strongly paracompact space is nearly strongly paracompact. But a nearly stıongly paracompact space may fail to be strongly paracompact as is shown by the following example.

Example 1.1. Let $X=\{a_{ij}, a_i, a: i, j=1, 2, 3, \ldots\}$. Let each point a_{ij} be isolated. Let $\{U^k(a_i): k=1, 2, \ldots\}$ be the fundamental system of neighbourhoods of a_i where $U^k(a_i)=\{a, a_{ij}: j\geqslant k\}$ and let the fundamental system of a be $\{V^k(a): k=1, 2, \ldots\}$ where $V^k(a)=\{a, a_{ij}: i\geqslant k, j\geqslant k\}$. Then X is a Hausdorff space which is not regular at a and hence X is not strongly paracompact.

Suppose that X is a strongly paracompact space. Then every open cover of X admits a locally-finite open refinement (every star finite family is a locally-finite family). Since X is a Hausdorff space, therefore X should be regular. It is impossible in contradiction with the fact that X is not regular at the point a, thus X is not strongly paracompact. But X is nearly strongly paracompact (in fact nearly compact), for if $\mathcal{G}=\{G_\lambda: \lambda\in\Lambda\}$ is any regular open covering of X., then $a\in G_{\lambda(a)}$, for some $\lambda(a)\in\Lambda$. Denote by $G_{\lambda(i)}$ that $G_\lambda\in\mathcal{G}$ which contains a_i and $G_{\lambda(ij)}$ that which contains a_{ij}. Then $V^m(a)\subset G_{\lambda(a)}$ for some m. Also, $\alpha(V^m(a))=\{a_m, a_{m+1}, \ldots\}\cup V^m(a)$. Thus $\{G_{\lambda(a)}, G_{\lambda(ij)}, G_{\lambda(i)}: i=1, 2, \ldots, m-1, j=1, 2, \ldots, m-1\}$ is a star finite open refinement of $\mathcal{G}$, hence X is nearly strongly paracompact.

Clearly, every nearly compact space is nearly strongly paracompact. But a nearly strongly paracompact space may fail to be nearly compact as can be seen from the following example.

Example 1.2. Let X be any infinite discrete space. Then X is nearly strongly paracompact, since the family $\{\{x\}: x\in X\}$ is a star finite open refinement of every regular open covering of X. But X is not nearly compact because $\{\{x\}: x\in X\}$ is a regular open cover of X which admits no finite subcover.

However we have the following:

Theorem 1.6. *A space X is nearly compact iff X is nearly strongly paracompact and lightly compact.*

Proof. It is similar to the proof of theorem 1.3 in [7].

By corollary 1.2 every nearly strongly paracompact space is almost-strongly paracompact.

We can show, however, that the converse of corollary 1.2 is not necessarily true. Following example will serve to the purpose.

Example 1.3. Let $X=\{a, b, a_{ij}, b_{ij}, c_i: i, j=1, 2, 3, \ldots,\}$. Let each point a_{ij} and b_{ij} be isolated. Let $\{U^k(c_i): k=1, 2, \ldots\}$ be the fundamental system of neighbourhoods of c_i where $U^k(c_i)=\{c_i, a_{ij}, b_{ij}: j\geqslant k\}$ and let $\{V^k(a): k=1, 2, \ldots\}$ and $\{V^k(b): k=1, 2, \ldots\}$ be that of a and b respectively, where $V^k(a)=\{a, a_{ij}: i\geqslant k, j=1, 2, \ldots\}$ and $V^k(b)=\{b, b_{ij}: i\geqslant k, j=1, 2, \ldots\}$. Then X is not nearly strongly paracompact, since the family consisting of the sets $V^n(a)$, $V^m(b)$, $V^i(c_i)$ for all i and all $\{a_{ij}\}$, $\{b_{ij}\}$ is a regular open covering of X which admits no star finite open refinement. But X is almost-strongly paracompact (in fact almost-compact). To see this, let $\mathcal{G}=\{G_\alpha: \alpha\in I\}$ be any open covering of X. Denote by $G_{\alpha(a)}$, $G_{\alpha(b)}$, $G_{\alpha(a_{ij})}$, $G_{\alpha(b_{ij})}$, $G_{\alpha(i)}$ any members of $\mathcal{G}$ which contain a, b, a_{ij}, b_{ij} and c_i respectively. Then, for some integers n_1, n_2, $V^{n_1}(a)\subset G_{\alpha(a)}$ and $V^{n_2}(b)\subset G_{\alpha(b)}$. Let $n=\max(n_1, n_2)$.

Consider the family

$$\mathcal{G}'=\{G_{\alpha(a)}, G_{\alpha(b)}, G_{\alpha(i)}, G_{\alpha(a_{ij})}, G_{\alpha(b_{ij})}: i=1, 2, \ldots, n-1,$$

$$\{a_{ij}\}, \{b_{ij}\}\notin G_{\alpha(a)}, G_{\alpha(b)} \text{ or } G_{\alpha(i)} \text{ for any } i=1, 2, \ldots, n-1\}.$$

Then $\mathcal{G}'$ is a finite subfamily of $\mathcal{G}$, closures whose members cover X and hence X is almost-strongly paracompact.

By theorem 1.4 every nearly strongly paracompact space is nearly paracompact. We can show, however, that the converse of theorem 1.4 is not necessarily true. Following example will serve the purpose.

Example 1.4. Let $\mathfrak{S}$ be the plane with its usual metric d from which a point o is removed. Let us define the new metric ρ in the following way: if a, b are points on the line trough o $\rho(a, b)=d(a, b)$ and if the line trough a, b does not contain o $\rho(a, b)=d(a, o)+d(o, b)$. It can be easily shown that ρ is a metric and that $\mathfrak{S}$ is connected and does not have the countable base. By theorem 2 in [9] the metric space $\mathfrak{S}$ is not strongly paracompact. Since $\mathfrak{S}$ is the metric space, therefore it is paracompact (in fact nearly paracompact). If $\mathfrak{S}$ was nearly strongly paracompact (since it is regular), by theorem 1.2, $\mathfrak{S}$ would be strongly paracompact. It is impossible.

2. Separation and *NSPC* space

Theorem 2.1. *Every nearly strongly paracompact Hausdorff space is almost-regular.*

Proof. It follows easily from theorem 1.4 and theorem 2.1 in [7].

Theorem 2.2. *In every almost-regular, nearly strongly paracompact space, every pair of disjoint regularly closed sets can be strongly separated.*

Proof. It follows easily from theorem 1.4 and corollary 2.3 in [7].

Theorem 2.3. *For an almost-regular space X with a property: every regular open cover of X admits a star countable refinement, every pair of disjoint regularly closed sets can be strongly separated.*

Proof. Let F_0 and F_1 be any two disjoint regularly closed subsets of X. Then, for any point $x \in F_0$, $\{\bar{x}\}$ is also contained in F_0. Since X is almost-regular, there exists a regular open set O_x such that $x \in O_s$, $\bar{O}_x \subset X \setminus F_1$. For any point $x \in X \setminus F_0$ there exists a regular open set O_x such that $x \in O_x$, $\bar{O}_x \subset X \setminus F_0$. Then $\{O_x : x \in X\}$ is a regular open covering of X. By hypothesis, there exists a star countable open covering $\mathcal{G}$ which refines $\{O_x : x \in X\}$. Let $\mathcal{G}_\alpha$ be a component of the covering $\mathcal{G}$ and $H_\alpha = \cup \{G_\alpha : G_\alpha \in \mathcal{G}_\alpha\}$. By lemma 1 in [9], the sets H_α are open pairwise disjoint, and $\cup H_\alpha = X$. It means, that every H_α is clo-open subset of X. Since every H_α is clo-open subset of X, we have that $F_{\alpha(0)} = H_\alpha \cap F_0$, $F_{\alpha(1)} = H_\alpha \cap F_1$ are disjoint and closed subsets of X. By lemma 2 in [9], for every α the family $\mathcal{G}_\alpha$ is at most countably open covering of the subspace H_α. Let $\mathcal{G}_{\alpha(0)}$ be a subfamily of the component $\mathcal{G}_\alpha$, whose members are sets $G^1_{\alpha(i)}$ intersecting $F_{\alpha(0)}$, and let $\mathcal{G}_{\alpha(1)}$ be a subfamily of the component $\mathcal{G}_\alpha$, whose members are sets $G^2_{\alpha(i)}$ intersecting $F_{\alpha(1)}$. It can be shown that $\overline{G^1_{\alpha(i)}} \subset H_\alpha \setminus F_{\alpha(1)}$ for every $G^1_{\alpha(i)} \in \mathcal{G}_{\alpha(0)}$ and $\overline{G^2_{\alpha(i)}} \subset H_\alpha \setminus F_{\alpha(0)}$, for every $G^2_{\alpha(i)} \in \mathcal{G}_{\alpha(1)}$. Hence $\mathcal{G}_{\alpha(0)}$ and $\mathcal{G}_{\alpha(1)}$ are disjoint families. Let be $U^1_{\alpha(k)} = G^1_{\alpha(k)} \setminus \bigcup_{i \leq k} \overline{G^2_{\alpha(i)}}$ and $U^2_{\alpha(k)} = G^2_{\alpha(k)} \setminus \bigcup_{i \leq k} \overline{G^1_{\alpha(i)}} \cdot F_{\alpha(0)} \subset U_{\alpha(0)} = \bigcup_k U^1_{\alpha(k)}$, $F_{\alpha(1)} \subset U_{\alpha(1)} = \bigcup_k U^2_{\alpha(k)}$ and $U_{\alpha(0)} \cap U_{\alpha(1)} = \varnothing$.

Since $U_{\alpha(0)}$ and $U_{\alpha(1)}$ are disjoint open subset of H_α, and H_α are disjoint open sets of X, therefore $U_0 = \bigcup_\alpha U_{\alpha(0)}$ and $U_1 = \bigcup_\alpha U_{\alpha(1)}$ are disjoint open sets of X, so that $F_0 \subset U_0$, $F_1 \subset U_1$. Hence the result.

3. Subsets and *NSPC* spaces

Theorem 3.1. *Every clo-open subset of a nearly strongly paracompact space is nearly strongly paracompact.*

Proof. Let A be a clo-open subset of a nearly strongly paracompact space X. Let $\{U_\alpha : \alpha \in I\}$ be any relatively open covering of A. Since A is clo-open, therefore each U_α is a regularly open subset of X. Thus $\{U_\alpha : \alpha \in I\} \cup \{X \setminus A\}$ is a regular open covering of X. Therefore there exists a star finite open refinement $\{V_\beta : \beta \in J\}$ of $\{U_\alpha : \alpha \in I\} \cup \{X \setminus A\}$. Then, $\{A \cap V_\beta : \beta \in J\}$ is a star finite open refinement of $\{U_\alpha : \alpha \in I\}$ and hence A is nealy strongly paracompact.

Theorem 3.2. *If $X = \cup \{U_\alpha : \alpha \in I\}$ where $\{U_\alpha : \alpha \in I\}$ is a star finite family of pairwise disjoint clo-open subsets of X, then X is nearly strongly paracompact iff each U_α is nearly strongly paracompact.*

Proof. Only the "if" part need be proved. Let $\{V_\beta: \beta\in J\}$ be any regular open covering of X. Then for each α, $\{V_\beta\cap U_\alpha: \beta\in J\}$ is a regular open covering of U_α. Since U_α is nearly strongly paracompact, there exists a star finite family $\{D_\lambda: \lambda\in K^\alpha\}$ of open (in U_α and hence also in X) subsets of X which covers U_α and refines $\{V_\beta\cap U_\alpha: \beta\in J\}$. Consider the family $\{D_\lambda: \lambda\in K^\alpha, \alpha\in I\}$. Then this is a star finite, open refinement of $\{V_\beta: \beta\in J\}$ and hence X is nearly strongly paracompact.

Theorem 3.3. *If every open subset containing a dense subset A of space X contains a nearly strongly paracompact set containing A, then A is nearly strongly paracompact.*

Proof. Let $\mathcal{U}=\{U_\lambda: \lambda\in\Lambda\}$ be any relatively regular open covering of A. Then A being dense, $U_\lambda=A\cap\alpha(U_\lambda)$ for each λ. Let $U=\bigcup\{\alpha(U_\lambda): \lambda\in\Lambda\}$. Then U is an open set containing the dense set A. Therefore by hypothesis, there exists a nearly strongly paracompact set B such that $A\subset B\subset U$. Since A is dense, B is also dense and hence $\{\alpha(U_\lambda)\cap B: \lambda\in\Lambda\}$ is a relatively regular open covering of B. Since B is nearly strongly paracompact, there exists a star finite (in B) family $\{V_\beta: \beta\in J\}$ of open (in B) subsets of B which covers B and refines $\{\alpha(U_\lambda)\cap B: \lambda\in\Lambda\}$. Now, each $V_\beta=V^*_\beta\cap B$, where V^*_β is an open subset of X. Consider now the family $\mathcal{V}=\{V^*_\beta\cap A: \beta\in J\}$. It is easy to verify that $\mathcal{V}$ is a star finite open (in A) refinement of $\mathcal{U}$ and hence A is nearly strongly paracompact.

Theorem 3.4. *If every open subspace of a space X is dense and nearly strongly paracompact, then every dense subspace of X is nearly strongly paracompact.*

Proof. Let A be a dense subspace of A. Let $\{U_\lambda: \lambda\in\Lambda\}$ be any relatively regular open covering of A. Since A is dense, each $U_\lambda=\alpha(U_\lambda)\cap A$. Let $U=\bigcup\{\alpha(U_\lambda): \lambda\in\Lambda\}$. Then U being open, is dense and nearly strongly paracompact, and $\{\alpha(U_\lambda): \lambda\in\Lambda\}$, is a relatively regular open covering of U. There exists, therefore, a star finite open (in U) refinement $\{V_\beta: \beta\in J\}$ of $\{\alpha(U_\lambda); \lambda\in\Lambda\}$. Then $\{V_\beta\cap A: \beta\in J\}$ is a star finite open (in A) refinement of $\{U_\lambda: \lambda\in\Lambda\}$ and hence A is nearly strongly paracompact.

4. Product and *NSPC* spaces

Theorem 4.1. *The product of a nearly strongly paracompact space and a nearly compact space is nearly strongly paracompact.*

Proof. Let $\mathcal{U}$ be any regular open covering of $X\times Y$, where X is nearly strongly paracompact and Y is nearly compact. Let $(x, y)\in X\times Y$. There exist regularly open subsets V_{xy} and W_{xy} of x and y respectively such that $(x, y)\in V_{xy}\times W_{xy}\subset U$ for some $U\in\mathcal{U}$. Let $I^x=\{x\}\times Y$ for each $x\in X$. Then, $\{W_{xy}: (x, y)\in I^x\}$ is a regular open covering of the nearly compact space Y and therefore there exists a finite subset J^x of I^x such that $\{W_{xy}: (x, y)\in J^x\}$ is a covering of Y. For each $x\in X$, let $V_x=\bigcap\{V_{xy}: (x, y)\in J^x\}$. Since the intersection of finitely many regularly open sets is regularly open, V_x is a regularly open set containing x. Let $\mathcal{V}=\{V_x: x\in X\}$. Then $\mathcal{V}$ is a regular open covering of X. Since X is nearly strongly paracompact, there exists a star finite, open refinement $\mathcal{G}$ of $\mathcal{V}$. Now, for each $G\in\mathcal{G}$, there exists $x(G)\in X$

such that $G \subset V_{x(G)}$. Now, let $\mathcal{H} = \{G \times W_{xy}: G \in \mathcal{G}, (x, y) \in J^x\}$. It is easy to verify that $\mathcal{H}$ is a star finite, open refinement of $\mathcal{U}$ and hence $X \times Y$ is nearly strongly paracompact.

Corollary 4.1. *The product of a nearly strongly paracompact space with compact space is nearly strongly paracompact.*

Corollary 4.2. *The product of a strongly paracompact space with a nearly compact space is nearly strongly paracompact.*

5. Mapping and *NSPC* spaces

A mapping $f: X \to Y$ is said to be *almost-continuous* if the inverse image of every regularly open subset of Y is an open subset of X. f is called *almost-open* if the image of every regularly open subset of X is an open subset of Y [4].

Remark 5.1. It is clear that if $f: X \to Y$ is continuous, then it is almost-continuous. But the converse of this statement may not be true, as the following example shows.

Example 5.1. Let: $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ $\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X\}$. Let i be the identity mapping of (X, τ) onto (X, τ_1). Then i is open and almost-continuous but not continuous.

Theorem 5.1. *If f is one-to-one, almost-continuous and open mapping of a nearly strongly paracompact space X onto a space Y, then Y is nearly strongly paracompact.*

Proof. Let $\{U_\alpha: \alpha \in I\}$ be any regular open covering of Y. Then $f^{-1}(U_\alpha)$ is regular open for each α, since f is almost-continuous and open mapping. Consider now the regular open covering $\{f^{-1}(U_\alpha): \alpha \in I\}$ of X. Since X is nearly strongly paracompact there exists a star finite open refinement $\{A_\beta: \beta \in J\}$ of $\{f^{-1}(U_\alpha): \alpha \in I\}$. Then $\{f(A_\beta): \beta \in J\}$ is a star finite open refinement of $\{U_\alpha: \alpha \in I\}$ and hence Y is nearly strongly paracompact.

Theorem 5.2. *If f is an almost-continuous and open mapping of a nearly strongly paracompact space X onto a space Y such that $f^{-1}(G)$ is compact for each open set $G \subset Y$, then Y is nearly strongly paracompact.*

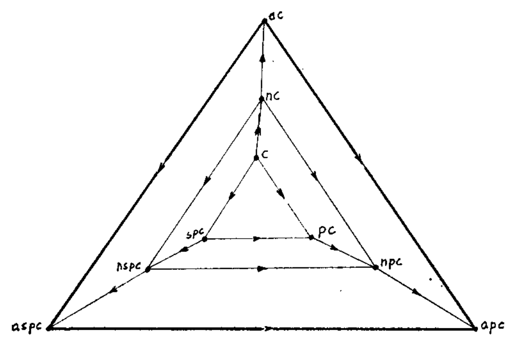
Proof. Let $\{U_\alpha: \alpha \in I\}$ be any regular open covering of Y. Then $f^{-1}(U_\alpha)$ is regular open for each α, since f is almost-continuous and open mapping. Since X is nearly strongly paracompact, there exists a star finite, open refinement $\{A_\beta: \beta \in J\}$ of $\{f^{-1}(U_\alpha): \alpha \in I\}$. Since f is open, $\{f(A_\beta): \beta \in J\}$ is an open covering of Y. Since f is open and $f^{-1}(G)$ is compact for each open set $G \subset Y$, the family $\{f(A_\beta): \beta \in J\}$ is star finite. Thus $\{f(A_\beta): \beta \in J\}$ is a star finite open refinement of $\{U_\alpha: \alpha \in I\}$ and hence Y is nearly strongly paracompact.

Let $\mathcal{W}$ be a fixed covering of a space X. A continuous mapping $f: X \to Y$ is said to be an *$(\mathcal{W}, p)$-mapping* if for every point $y \in Y$ there exists a subfamily $\mathcal{W}_y$ of $\mathcal{W}$ possessing the property p in $\cup \mathcal{W}_y$ and a neighbourhood V_y of y such that $f^{-1}(V_y) \subset \cup \mathcal{W}_y$ [7].

Theorem 5.3. *A space X is nearly strongly paracompact if for each regular open covering $\mathcal{W}$ of X, there exists an $(\mathcal{W}, p)$-mapping of X into some strongly paracompact space Y, where p is the property of being star finite.*

Proof. For every point $y \in Y$, choose a subfamily $\mathcal{W}_y$ of $\mathcal{W}$ which is star finite and on open set V_y containing y such that $f^{-1}(V_y) \subset \cup \mathcal{W}_y$. Then $\{V_y: y \in Y\}$ is an open covering of the strongly paracompact space Y. Therefore, there exists a star finite open refinement $\{U_\alpha: \alpha \in I\}$ of $\{V_y: y \in Y\}$. For each U_α, there is a $V_{y(\alpha)}$ such that $U_\alpha \subset V_{y(\alpha)}$. Let $\mathcal{U}_\alpha = \{f^{-1}(U_\alpha) \cap W: W \in \mathcal{W}_{y(\alpha)}\}$ for each α. Let $\mathcal{U} = \cup \{\mathcal{U}_\alpha: \alpha \in I\}$. Then $\mathcal{U}$ is a star finite open refinement of $\mathcal{W}$. Thus every regular open covering of X has a star finite open refinement and hence X is nearly strongly paracompact.

The following diagram indicates relationships of nearly strongly paracompact spaces with some other classes of topological spaces connected with them.



Here, a = almost, c = compact, p = paracompact, sp = strongly paracompact, n = nearly.

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