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ON THE MODULARITY OF SOME ORDERABLE CROSS-LATTICES

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In [1] G. Szász proved that any right-distrubutive orderable cross-lattice is modular. He conjectured that a left-distributive orderable cross-lattice has not to be modular. The main result of our paper shows that this conjecture is wrong.

Theorem. *Let C be an orderable cross-lattice satisfying one of the following distributivity conditions* (see [1], *p.* 464)*:*

1) *C is meet-distributive;*

2) *C is join-distributive;*

3) *C is left-distributive;*

4) *C is right-distributive.*

Then C is modular.

First we prove the following lemma.

Lemma. *Let C be an orderable cross-lattice. Then for all elements a, b of C the relation $a \leq b$ implies the relations*

i) $a \cap b = b \cap a = a$;

ii) $a \cup b = b \cup a = b$.

Proof. If $a \leq b$, then by definition (see (1) and (2) in [1])

$$a \cap b = a \text{ and } b \cup a = b.$$

Since

$$b \cap a \leq a \text{ and } b \leq a \cup b$$

(see [1], p. 463), we have only to prove

$$a \leq b \cap a \text{ and } a \cup b \leq b,$$

or equivalently

$$a \cap (b \cap a) = a \text{ and } b \cup (a \cup b) = b.$$

Actually, from $a \cap b = a$, C_1 and C_5 in [1] we have

$$a \cap (b \cap a) = (a \cap b) \cap (b \cap a) = (a \cap b) \cap a = a \cap a = a.$$

Similarly from $b \cup a = b$, C_2 and C_6 in [1] it follows

$$b \cup (a \cup b) = (b \cup a) \cup (a \cup b) = (b \cup a) \cup b = b \cup b = b.$$

The next proposition is an immediate consequence of the lemma.

Proposition. *Let C be an orderable cross-lattice and $a, b\ c \in C$, $a \leq c$. Then the following implications are right:*

I) $c \cap (b \cup a) = (c \cap b) \cup (c \cap a) \Rightarrow c \cap (b \cup a) = (c \cap b) \cup a;$

II) $(a \cup b) \cap c = (a \cap c) \cup (b \cap c) \Rightarrow a \cup (b \cap c) = (a \cup b) \cap c;$

III) $a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \Rightarrow a \cup (b \cap c) = (a \cup b) \cap c;$

IV) $(c \cap b) \cup a = (c \cup a) \cap (b \cup a) \Rightarrow c \cap (b \cup a) = (c \cap b) \cup a.$

In the case 1), 2), 3) and 4) of our theorem the first parts of implications I) and II, III) and IV), I) and III), II) and IV), respectively are true. Therefore, our theorem is true too.

REFERENCE:

[1] G. Szász, *Constributions to some generalizations of lattices.* Mat. vesnik, **13** (28) 1976, p. 461—464.

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